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Mean Velocity Modeling of Open-Channel Flow with Submerged Rigid Vegetation

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***Abstract*—**Vegetation affects the mean and turbulent flow structure. It may increase flood risks and sediment transport. Therefore, it is important to develop analytical approaches for the bed shear stress on vegetated bed, to predict resistance caused by vegetation. In the recent years, experimental and numerical models have both been developed to model the effects of submerged vegetation on open-channel flow. In this paper, different analytic models are compared and tested using the criteria of deviation, to explore their capacity for predicting the mean velocity and select the suitable one that will be applied in real case of rivers. The comparison between the measured data in vegetated flume and simulated mean velocities indicated, a good performance, in the case of rigid vegetation, whereas, Huthoff model shows the best agreement with a high coefficient of determination (R^2=80%) and the smallest error in the prediction of the average velocities.

***Keywords*—**Analytic Models, Comparison, Mean Velocity, Vegetation.

I. Introduction

THE development of vegetation in river plays an important role in the flow structure of open channels and may increase flood risks and sediment transport [1], [2]. Thus, an understanding of vegetated flows is necessary to control floods and the ecosystem of the stream [3]. The mean velocity may be useful in the estimation of shear velocities and the bed shear stresses. It is the key factors in estimating the bed load transport and the related scour, deposition, entrainment and bed changes in rivers. For non-vegetation, Velocity distribution is related directly to the bed shear stress; while, for vegetated flow, it's related to the vegetation drag because the vegetation roughness is much larger than the river bed roughness [4].

The effects of vegetation on flow have been studied over the last decades by laboratory experiments with rigid cylinder, flexible vegetation prototypes and natural vegetation [5]. As a result, different experiments in laboratory flumes have been carried out [6]-[10] and several vegetation-resistance methodologies have been proposed to model the effects of vegetation on open-channel flow. Such relations exist that relate the average flow velocity to the hydraulic roughness (Chézy, Darcy-weisbach, Manning, Strickler, Keulegan). However, these relations were not derived to describe the complex interactions of vegetation with flow. These empirical formulas are useful only for describing the resistance of the deeply submerged vegetation [11]. Therefore, new approaches have been derived based on vegetation characteristics (vegetation height h_p, density of vegetation m, diameter of plant stems D, drag coefficient C_D), instead of using a constant roughness coefficient using analytical models. Most of these relationships adopted a two-layer model [12]-[17]. In this approach, the flow domain was divided into two layers, a "vegetation layer" through the vegetation and "the surface layer" surface layer above it (Fig. 1). The flow in each of the two layers was described separately. The logarithmic flow velocity profile is adopted for solving the velocity above the vegetation, and the momentum equation within the vegetated layer. The continuity of the velocity and the shear stress between the two layers is ensured by boundary conditions at the interface. The average velocity (U) over the total depth is given by combination between the mean velocity flow inside (U_1) and above the vegetation (U_2) [12], [18], [10].

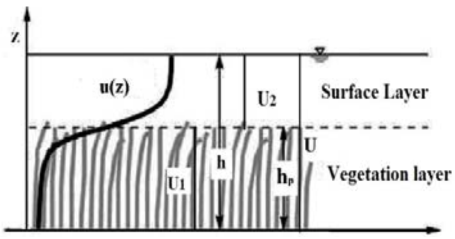

Fig. 1 Velocity profile within and above vegetation

However, the wide variety of vegetation types and hydrodynamic conditions considered in these works make it difficult to compare the results and draw general conclusions. In this context, this study aims to understand and determine the range of validity and applicability of some analytical models in the predicting of the mean flow velocity through submerged vegetation and select the most adequate model. These models are validated using measurement data in flume with rigid vegetation.

II. Theoretical Background

A. Klopstra Model [11]

In the vegetated layer, [11] proposed an analytical expression for the velocity distribution. This method based on

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the momentum equation for the vegetation layer assuming uniform steady flow and using the Boussinesq concept, to describe the shear stress. The analytic solution of this momentum equation gives the velocity distribution in the vegetated layer. The bottom shear stress is neglected behind the vegetation shear stress.

In the surface layer, the velocity follows a logarithmic profile that was derived using Prandtl's mixing length theory. The connection between the boundary conditions at the interface ensures the continuity of the velocity and the shear stress between the two layers, and allows the determination of the logarithmic law parameters and the mean velocity in the surface layer.

The average velocity over the total depth (U) is given by combination between the mean velocity flow inside (U_1) and above the vegetation (U_2):

$$U = \frac{h_p}{h} U_1 + \frac{(h - h_p)}{h} U_2 \tag{1}$$

where U is the average velocity over the total depth, $U_1$is the mean velocity in the vegetation layer, U_2 is the mean velocity in the surface layer, h is the water depth (m) and hp is the vegetation height (m).

Reference [11] determined the total average velocity through submerged vegetation and it is given by:

$$U = \frac{\sqrt{i}}{h} \left[\begin{array}{l} \frac{2}{\sqrt{2A}} \left(\sqrt{C_3 e^{h_p \sqrt{2A}} + u_{v0}^2} - \sqrt{C_3 + u_{v0}^2} \right) \\ + \frac{u_{v0}}{\sqrt{2A}} \ln\left(\frac{(\sqrt{C_3 e^{h_p \sqrt{2A}} + u_{v0}^2} - u_{v0})}{(\sqrt{C_3 e^{h_p \sqrt{2A)}} + u_{v0}^2} - u_{v0})} * \\ \frac{(\sqrt{C_3 + u_{v0}^2} + u_{v0})}{(\sqrt{C_3 + u_{v0}^2} + u_{v0})} \right) + \frac{\sqrt{g(h - (h - (h_p - h_s))}}{\kappa} \\ ((h - (h_p - h_s)) \ln\left(\frac{(h - (h_p - h_s)}{z_0} \right) - \\ h_s \ln\left(\frac{h_s}{z_0} \right) - (h - h_p)) \end{array} \right] \tag{2}$$

where, A is a help variable:

$$A = \frac{mDC_D}{2\alpha} \tag{3}$$

m is the density of vegetation (m^{-2}), D is the diameter of plant stems (m), C_D is the drag coefficient and α is a closure parameter derived from experimental data. The constants C_3 follow from boundary conditions, h is the water depth (m) and hp is the vegetation height (m) and iis the energy gradient. u_{v0} is the characteristic constant flow velocity in non-submerged vegetation:

$$u_{v0} = \sqrt{\frac{2g}{C_D mD}} \tag{4}$$

κ is Von Karman's constant (0.41), hs is the distance between the vegetation top and the surface layer virtual bed (m) and z_0 is the length scale for bed roughness of the surface layer (m).

B. Stone and Shen Model [12]

Using scaling assumption and laboratory data for submerged rigid vegetation, [12], derived an analytical expression for the total average velocity (U) over the total depth and it's given by:

$$U = \sqrt{\frac{2g}{C_D mD}} \sqrt{i} \left(1 - D\sqrt{m}\right) \sqrt{\left(\frac{h}{h_p} - \frac{1}{4} \pi D^2 m \right) \frac{h}{h_p}} \tag{5}$$

m is the density of vegetation (m^{-2}), D is the diameter of plant stems (m), C_D is the drag coefficient, g is the acceleration due to gravity (m/s^2).

C. Van Velzen Model [13]

In the vegetation layer, [13] assumed uniform velocity in the vegetation layer and it's defined by:

$$U_1 = \sqrt{\frac{2gi}{C_D mD}} \tag{6}$$

The flow in the surface layer is described by a logarithmic term:

$$U = U_1 + 18\sqrt{(h - h_p)i} \log\left(\frac{12(h - h_p)}{k_n}\right) \tag{7}$$

Then the total average velocity through submerged vegetation is given by:

$$U = U_1 + 18 * (h - h_p)^{3/2} \frac{\sqrt{i}}{h} * \log(12 * (h - h_p)/k_n) \tag{8}$$

K_n is the roughness height and it's given by the empirical function:

$$k_n = 1.6 h_p^{0.7} \tag{9}$$

D. Baptist Model [14]

Baptist model is based on an analytical solution of the momentum balance of flow through and over vegetation, using the Boussinesq's eddy viscosity approach to determine the Reynolds stress τ and the mixing-length theory for the eddy viscosity [14].

The expression of the velocity in the vegetation layer is given by:

$$U_1 = \frac{L\sqrt{i}}{h_p} \left[\begin{array}{l} 2(u(h_p) - \sqrt{a_v + u_{v0}^2}) \\ + u_{v0} \ln\left(\frac{(u(h_p) - u_{v0})}{(u(h_p) + u_{v0})} * \frac{(\sqrt{a_v + u_{v0}^2} + u_{v0})}{(\sqrt{a_v + u_{v0}^2} - u_{v0})} \right. \end{array} \right] \tag{10}$$

where, i is the energy gradient, h is the water depth (m). h_p is the vegetation height (m), u_{v0} is the characteristic constant flow velocity in non-submerged vegetation, L is the length scale (m), a_v is the integration constant.

$$u_{v0} = \sqrt{\frac{2g}{C_D mD}} \tag{11}$$

$$L = \sqrt{\frac{C_p l}{C_D mD}} \tag{12}$$

$$a_v = \frac{2Lg(h-h_p)}{C_p l \exp(\frac{h_p}{l})} \tag{13}$$

The coefficient C_p is the turbulent intensity, height – averaged over the vegetation height, hp and l is the mixing length.

For the surface layer, Prandtl's mixing length concept is adopted, and the mean velocity is given by:

$$U_2 = \frac{\sqrt{g(h-h_p)i}}{\kappa(h-h_p)} \begin{bmatrix} (h-d)\ln(\frac{h-d}{z_0}) + (h_p-d)* \\ \ln(\frac{h_p-d)}{z_0} - (h-h_p)) \end{bmatrix} \tag{14}$$

where d is the zero-plane displacement (m), which is located at distance from the bed inside the vegetation and z_0 is the roughness length.

E. Huthoff Model [15]

Reference [15] derived an analytical expression for the velocity flow through and over vegetation. In the vegetation layer, the mean velocity is given by:

$$U_1 = U_{r0}\sqrt{\frac{h}{h_p}} \tag{15}$$

h is the water depth (m) and hp is the vegetation height (m), Ur0 is the depth-averaged flow velocity in the resistance layer for emergent resistance elements:

$$U_{r0} = \sqrt{2bgi} \tag{16}$$

b is the drag length:

$$b = \frac{1}{C_D mD} \tag{17}$$

m is the density of vegetation (m^{-2}), D is the diameter of plant stems (m), C_D is the drag coefficient, g is the acceleration due to gravity (m/s^2).

In the upper layer, the velocity is given by:

$$U_2 = U_{r0}(\frac{h-h_p}{s})^{2/3\left(1-(\frac{h}{h_p})^{-5}\right)} \tag{18}$$

with, s is the separation between individual resistance elements:

$$s = \frac{1}{\sqrt{m}} - D \tag{19}$$

The expression for the average velocity of the entire flow depth becomes:

$$U = U_{r0}(\sqrt{\frac{h_p}{h}} + \frac{(h-h_p)}{h}(\frac{h-h_p}{s})^{2/3(1-(\frac{h}{h_p})^{-5})}) \tag{20}$$

F. Yang and Choi Model [16]

Reference [16] used the two layer approach to determine the velocity profile. The velocity is assumed to be uniform in the vegetation and it has been determined by applying a momentum balance. In the upper layer, the velocity profile follows a logarithmic distribution.

The equation of the velocity in the vegetation layer (U_1) is given by:

$$U_1 = \sqrt{\frac{2ghi}{a\,C_D h_p}} \tag{21}$$

In the upper layer, the expression of the velocity (U_2) is given by:

$$U_2 = \frac{C_u u_*}{\kappa}(\frac{h}{(h-h_p)}\ln(\frac{h}{h_p}) - 1) + U_1 \tag{22}$$

with u_* is the shear velocity.

The average velocity over the total depth (U) is given by combination between the mean velocity flow inside (U_1) and above the vegetation (U_2):

$$U = \frac{C_u u_*}{\kappa}(\ln(\frac{h}{h_p}) - (\frac{h-h_p}{h})) + U_1 \tag{23}$$

where a is the density vegetation (m^{-1}), κ is Von Karman's constant (0.41) and $C_\mu$is a constant (C_μ=1 for a ≤ 5 m^{-1}, Cμ=2 for a > 5m^{-1}).

III. Analytic Models Comparison

The use of experimental data flume available in the literature (Table I), concerning the free surface flow in presence of rigid vegetation, allows the verification of the validity and the ability of these models in predicting the mean velocity.

The verification is determined due to a comparison between the measured and simulated mean velocities using the criteria of deviation (the mean error E, the Mean Absolute Error (MAE), the Root-Mean Square Error (RMSE), the Coefficient of determination (R^2) and the standard deviation of the mean error (σ).

$$E = \frac{1}{N}(\sum_{i=1}^{n}(\text{measured}_{\text{value}} - \text{calculated}_{\text{value}}) \tag{24}$$

$$MAE=\frac{1}{N}\left(\sum_{i=1}^{n}\left|measured_{value}-calculated_{value}\right|\right) \quad (25)$$

$$RMSE=\sqrt{\frac{1}{N}\left(\sum_{i=1}^{n}\left(measured_{value}-calculated_{value}\right)^{2}\right)} \quad (26)$$

$$\sigma=\sqrt{\frac{1}{N}\left(\sum_{i=1}^{n}\left(measured_{value}-calculated_{value}-E\right)^{2}\right)} \quad (27)$$

TABLE I
EXPERIMENTS DATA USED FOR VERIFICATION THE PERFORMANCE OF THE MODELS IN THE PREDICTION THE MEAN VELOCITYIN THE CASE OF RIGID VEGETATION

Author(s)	Number of experiments	h(m)	vegetation characteristics			
			h_p(m)	D(m)	m (m^{-2})	C_D
Reference [20]	6	0.368-0.23	0.07-0.14	0.0064	42-388	1.13
Reference [6]	12	0.854-0.164	0.12	0.0064	42-384	1.13
Reference [23]	8	0.05-0.095	0.045	0.0015	2500	1.46
Reference [24]	36	1.98-2.5	0.9-1.5	0.008	64-256	0.96-1
Reference [19]	20	0.073-0.108	0.038	0.0064	3-108	1.4
Reference [12]	92	0.151-0.315	0.124	0.0127	481	1.11
Reference [21]	5	0.6	0.12	0.004	67-1072	1.5
Reference [22]	24	0.088-0.467	0.07-0.139	0.006	250-800	0.61-1

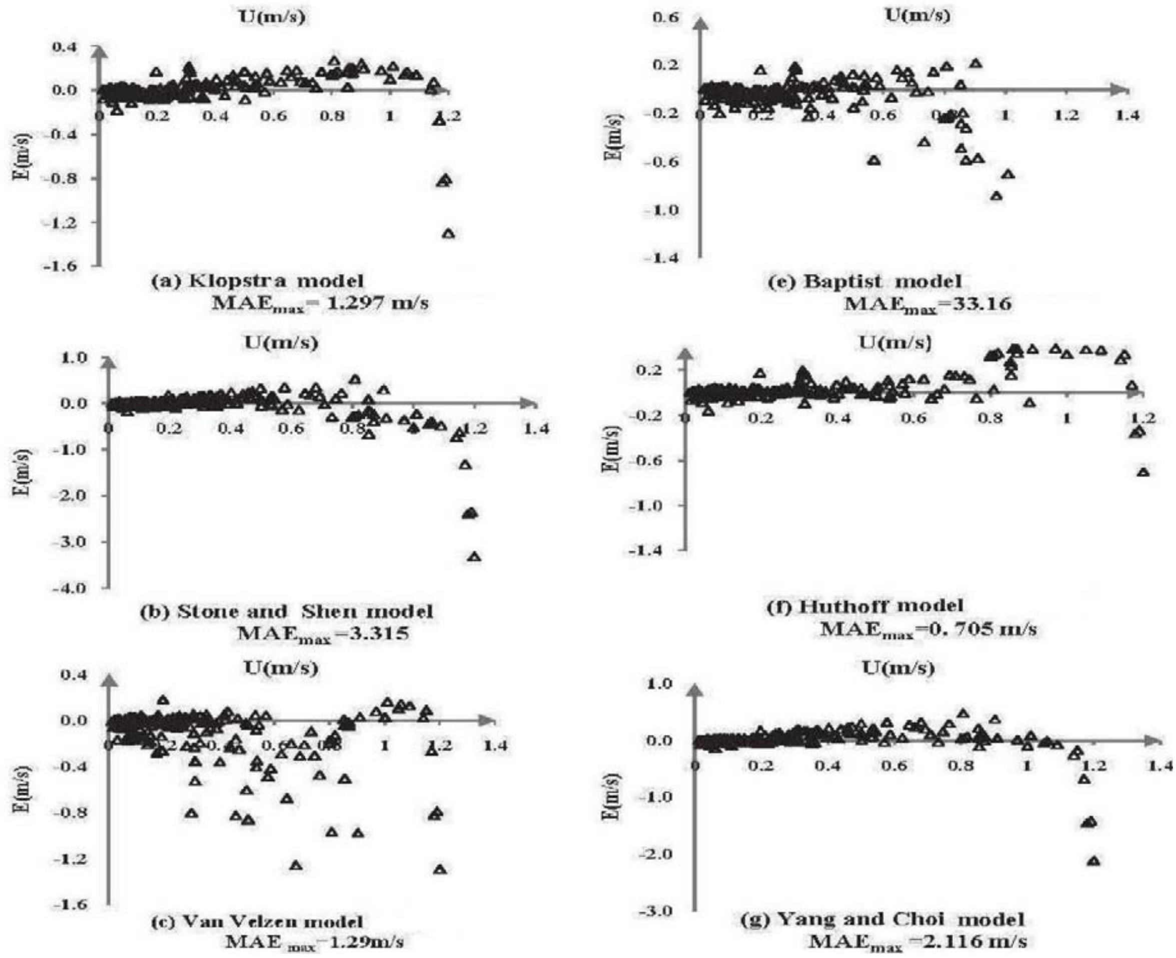

Fig. 2 The mean error between the mean (E) measured velocities and the predicted ones by the different analytic models

IV. MEASURED AND PREDICTED MEAN VELOCITY COMPARISON

The comparison between the measured and simulated mean velocities by different analytical models using data of rigid vegetation is summarized in the Table II. A High value of R^2 and a low value of MAE, RMSE and σ indicate the good performance of the model. Most descriptors show a good performance. However, Baptist model performs less well, with a low value of R^2 (26 %) and High value of (MAE, RMSE and σ). The model of Huthoff shows the best agreement with a high coefficient of determination (80%).

In general, river models are used to set a safety standard, so, it is very important that a method can predict higher velocities as accurate as possible. Therefore, graphs are presented with the mean error between the predicted and measured velocities for each model to investigate under which circumstances the model shows the largest/smallest errors (Fig. 2).

For smaller velocities, more data sets were available. However, the difference in performance of the different descriptors is small. For higher velocities, the prediction of the mean velocities by the different models indicates an under-estimation or over-estimation.

The model of Huthoff shows the smallest error in the prediction of the average velocities. Huthoff model is more adequate in the prediction of the high velocity than the other models (a velocity ≤ 0.8 m/s) and that's very import for flood management. Only for a very low density of vegetation, this model shows an underestimation (Fig. 3).

TABLE II
MEAN VELOCITIES PREDICTED BY ANALYTIC MODELS COMPARED TO THE MEASURED MEAN VELOCITIES IN THE CASE OF RIGID VEGETATION

Author(s)	σ (m/s)	MAE (m/s)	RMSE (m/s)	R^2
Reference [11]	0.148	0.073	0.148	0.793
Reference [12]	0.376	0.138	0.380	0.691
Reference [13]	0.223	0.128	0.252	0.774
Reference [14]	3.039	0.529	3.079	0.260
Reference [15]	0.127	0.068	0.132	0.816
Reference [16]	0.234	0.092	0.235	0.714

The following figure shows a comparison between the measured mean velocities and calculated by the model of Huthoff:

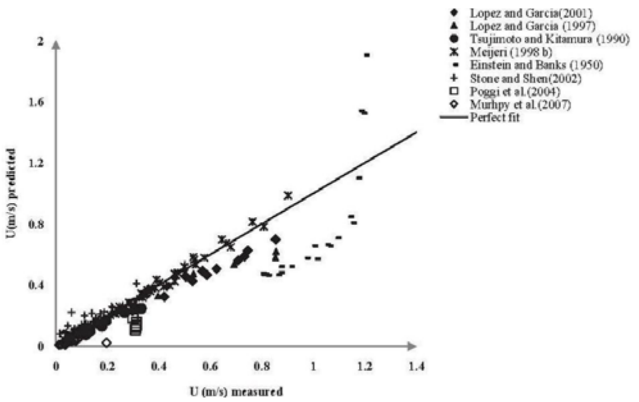

Fig. 3 Measured and calculated mean velocity by [15] model in the case of rigid vegetation

V. CONCLUSION

The verification of the capacity of different analytic models in the prediction of the mean velocity is determined due to a comparison between the measured and simulated mean velocities. This comparison is done through the calculation of the deviation 's criteria (the mean error E, the Mean Absolute Error (MAE), the Root-Mean Square Error (RMSE), the Coefficient of determination (R^2) and the standard deviation of the mean error (σ) and the graphs of mean error.

Most descriptors show a good performance in the case of submerged and rigid vegetation. However, Huthoff model shows the best agreement with a high coefficient of determination (80%). Only, in the case of sparse vegetation, this model indicates an under estimation. That could be explained by the neglect of the bed roughness effect.

In perspective, we will include the model of Huthoff in a computer code (telemac 2D) to predict the mean velocity in flow through submerged vegetation. We will apply the new model in real cases (rivers).

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REFERENCES

[1] C. Liu, and Y-M, Shen, "Flow Structure and Sediment Transport with Impacts of Aquatic Vegetation," J. Hydrodyn., vol.20, no.4, pp. 461-468, 2008.
[2] W. Wu, and Z.HE, "Effects of vegetation on flow conveyance and sediment transport capacity," IJSR, vol. 23, no.3, pp. 247-259, 2009.
[3] J. M. V. Samani, and M. Mazaheri, "An analytical model for velocity distribution in transition zone for channel flows over inflexible submerged vegetation" *J. Agr. Sci. Tech*., vol.11, pp. 573-584, 2009.
[4] A. Defina and A. C. Bixio, "Mean flow and turbulence in vegetated open channel flow," *Water Resour. Res*., vol.7,pp. 1-12, July 2005.
[5] M. Morri, A. Soualmia and P.Belleudy," The Tests of Analytical Models for Prediction the Vertical Velocity Profiles through Submerged Vegetation," *in Conf. 2014. Int. Conf. on Mechanics and Energy, ICME'2014, March 18-20, 2014, Monastir, Tunisia.*
[6] F. López, and M.H. García, "Mean Flow and Turbulence Structure of Open Channel Flow through Non-emergent Vegetation,"*J. Hydraul. Eng*., vol.127, no.5, pp. 392-402, May.2001.
[7] T. Tsujimoto, T. Okada, and K, Kontani, "Turbulent structure of open-channel flow over flexible vegetation," *Progressive Report*. Hydr. Lab., Kanazawa University, Japon, pp. 37-45, 1993.
[8] E. Kubrak, J.Kubrak, and P.M. Rowinski, "Verticalvelocitydistributions through and above submerged, flexible vegetation," *Hydrolog. Sci. J*., vol.53, no.4, pp. 905-920, Aug. 2008.
[9] J. Jarvela, "Effect of submerged flexible vegetation on flow structure and resistance," *J.Hydrol*., vol.307, pp: 233-241, 2005.
[10] D.C.M.Augustijn, F. Huthoff and E.H. Van Velzen," Comparison of vegetation roughness descriptions,"*in Conf. 2008. Int. Conf. on Fluvial Hydraulics, River flow*, Çeşme. Izmir, Turkey, pp.3-8.
[11] D. Klopstra, H. J. Barneveld, J. M. van Noortwijk, and E. H. Van Velzen,"Analytical Model for Hydraulic Roughness of Submerged Vegetation". *In Proc. Managing Water: Coping with Scarcity and Abundance. 27 th IAHR Congress*, San Fransisco, New York, 1997, PP. 775-780.
[12] M.B. Stone, and H.T. Shen, "Hydraulic Resistance of Flow in Channels with Cylindrical Roughness," *J. Hydraul. Eng*., vol.128, no.5, pp.500-506, 2002.
[13] E.H. Van Velzen, P. Jesse, P. Cornelissen and H. Coops, "Stromingsweerstandvegetatieinuiterwaarden,"; *RIZA Reports*, Arnhem, Netherlands, 2003.
[14] M.J. Baptist, V. Babovic, J.R.Uthurburu, M.Keijzer, R.E. Uittenbogaard, A.Mynett, A.Verwey,"On inducing equations for vegetation resistance,".*J. Hydraul. Res*.,vol.45, no. 4, pp.435-45, 2007.
[15] F. Huthoff, D.C.M. Augustijn, and S. J. M.Hulscher,."Analytical solution of the depth-averaged flow velocity in case of submerged rigid cylindrical vegetation. Water Resour. Res.,vol. 43, pp.1-10, 2007.
[16] W. Yang, and S. Choi, "A Two-layer approach for depth-limited open-channel flows with submerged vegetation," J. Hydraul. Res., vol.48, no. 4, pp. 466–475, 2010.

[17] W.X. Huai, Y.H. Zeng, Z.G. Xu, Z.H. Yang, "Three-Layer Model for vertical velocity distribution in open channel flow with submerged rigid vegetation," *Adv. Water Resour.*, vol.32, no.4, pp.487–492, January 2009.

[18] J. M. V. Samani, and M.Mazaheri, "An analytical model for velocity distribution in transition zone for channel flows over inflexible submerged vegetation," *J. Agr. Sci. Tech.*, vol.11, pp.573-584, 2009.

[19] H. A. Einstein, and R. B. Banks, "Fluid resistance of composite roughness," *Transactions of the Amer. Geoph. Union*. vol.31, no.4, pp. 603-610, 1950.

[20] F. López, and M. H. García, "Open-channel flow through simulated vegetation: Suspended sediment transport modeling," *Water Resour. Res*, vol.34, no.9, pp. 2341-2352, 1998

[21] D. Poggi, A, Porporato, L. Ridolfi, J.D. Albertson, and G. G. Katul, "The effect of vegetation density on canopy sub-layer turbulence," *Bound.-Layer Meteor*, vol.111, no.3, pp. 565-587, 2004.

[22] E. Murphy, M. Ghisalberti and H. Nepf, "Model and laboratory study of dispersion in flows with submerged vegetation," *Water Resour. Res*, vol .43, no.5, pp.1-12, 2007.

[23] T. Tsujimoto, and T. Kitamura, "Velocity profile of flow in vegetated-bed channel," *Progressive Report*. Hydr. Lab., Kanazawa University, Japon, pp. 43-55, 1990.

[24] J. Jarvela, "Effect of submerged flexible vegetation on flow structure and resistance," J.*Hyd*, vol. 307, pp. 233-24, 2005.