Abstract—The 6th version of Universal modeling method for centrifugal compressor stage calculation is described. Identification of the new mathematical model was made. As a result of identification the uniform set of empirical coefficients is received. The efficiency definition error is 0.86 % at a design point. The efficiency definition error at five flow rate points (except a point of the maximum flow rate) is 1.22 %. Several variants of the stage with 3D impellers designed by 6th version program and quasi three-dimensional calculation programs were compared by their gas dynamic performances CFD (NUMECA FINE TURBO). Performance comparison demonstrated general principles of design validity and leads to some design recommendations.

Keywords—Compressor design, loss model, performance prediction, test data, model stages, flow rate coefficient, work coefficient.

NOMENCLATURE

\( \phi \) flow rate coefficient
\( \psi_T \) work coefficient
\( \zeta \) losses efficiency
\( \epsilon_m \) compressibility coefficient

Subscripts
1 impeller inlet
2 impeller exit
bl blade
des design
m meridional
p pressure side
s suction side
thr throat

I. GOAL AND OBJECTS OF MODELING

TOTAL power of centrifugal compressors installed is measured by dozens and dozens MWt in industrial countries. The simplest of centrifugal compressors for pipe line industry is shown in Fig. 1.

These machines have power in range 4-32 MWt, exit pressure to 12,5 MPa, number of stages up to 8 in one body. Compressors for other industries are more complicated usually.

Power consumption depends on many factors but the sound gas dynamic design is the first of them. Design procedure consisted of more or less proven rules to establish main flow path dimensions at pre-PC era. Numerous model tests at special test rigs were obligatory before new compressor fabrication.

The modern gas dynamic design becomes easier and more reliable. i.e. given compressor pressure ratio is guaranteed at given mass flow rate with good efficiency. Experimental check and improvement followed are not obligatory in many cases [14], [15]. Three-step numerical procedure is applied usually. The first step of design consists of some mean line field type calculations to find the best flow path main dimensions.

The TU SPb R&D “Laboratory of compressor problems” was very active in 1960 – 1980th. The TU SPb compressor school founded by Prof. K. Seleznev is one of the leading Russian R & D centers [13]. Flow behavior study has lead to the physical model formulation and its mathematical description. The design procedure scientific background and practical application are well presented [1]-[6], [8]-[10]. Fig. 2 shows that measured and calculated velocity diagrams at impeller blades are rather similar at design flow rate – but the exit area where the wake is formatted [12].
This similarity has proved Q3D non viscous calculation effective application. A powder paint inserted in a flow path sticks to surfaces where shear stress are close to zero. This way wake and separation zones formation can be visualized. Fig. 3 demonstrates flow behavior in the 2D impeller with work coefficient $\Psi_T \approx 0.65$.

3D wake zone at the second part of the suction side of the blade is visible. Flow separation never occurs at hub/shroud surfaces and impeller blade pressure sides, etc. Flow visualization at Fig. 4 demonstrates wake zone and secondary flows on a suction side of an impeller at close to surge flow rate and flow separation in a vane diffuser at a design flow rate.

II. UNIVERSAL MODELING BASIC FEATURES AND ACHIEVEMENTS

Based on the physical model the field type Universal modeling method is well presented to specialists ([7]-[10], [14], [15]) so the basic information is presented below only.

The math model identification is based on gas dynamic performances of model stages. Loss of a head is calculated on each surface of a flow path and is summarized. Shear stress force coefficient and mixing loss coefficients are presented as functions of flow velocity gradients along surfaces and along normal direction. A velocity diagram of non viscous flow is described schematically. Inlet and exit velocities at two sides of a blade are used for calculations (are marked with dots at Fig. 5).
A friction force coefficient is presented as a sample for a suction side:

\[ c_{fr} = c_f \left[ 1 - X_f \left( \frac{w_i / u_s}{R_{in} / D_2} \right)^{X_i} + X_f \left( \frac{w_{sl}}{w_{si}} \right)^{X_i} \right] \]  

Two members of (1) present normal \( \left( \frac{w_i / u_s}{R_{in} / D_2} \right) \) and tangential \( \left( \frac{w_{sl}}{w_{si}} \right) \) velocity gradients. The basic value of the drag force coefficient \( c_f \) is the coefficient of a thin plate [11]:

\[ c_f = X_{f_{0,307}} = \frac{0.0307}{R_{in}^0} \text{ - hydraulically smooth surface,} \]

\[ c_f = X_{f_{1,89}} = \frac{1}{(1.89 + 1.62 \cdot \lg \frac{1}{R_{in} / \lambda s})^{X_i}} \text{ - rough surface.} \]

Empirical coefficients gradients \( X_i \) are subjects of model identification by balance of calculation and measured performance curves. Their numbers \( i \) are individual and show coefficient’s position in the equations. Equations (4), (5) connect drag force coefficients on both blade surfaces with the friction loss coefficient of impeller blades and the last one with an efficiency loss:

\[ \zeta_{fr} = \frac{2 \pi \bar{w}_e}{\bar{w}_e} \left[ c_{fr} \left( \frac{w_i}{w_{sl,1}} \right)^2 \bar{w}_i + c_{fr} \left( \frac{w_p}{w_{sh,1}} \right)^2 \bar{w}_p \right] \]  

\[ \Delta \eta_{prof} = \frac{\zeta_{fr, prof}}{\zeta_f} \left( \frac{\pi_2}{2} \right) \]  

Friction loss coefficients for hub and shroud surfaces are calculated on the same principle. The physical model is based on the absence of flow separation everywhere in an impeller but a suction side of a blade. The separation point is calculated by (6):

\[ \dot{w}_s = \frac{w_s}{w_{si}} = X_f \left[ 1 + X_f \left( \frac{w_i / u_s}{R_{in} / D_2} \right)^{X_i} \right] \]  

The normalized normal velocity gradient presents in (6) and reflects Rossby number influence on a boundary layer condition. Mixing loss coefficient corresponds to scheme of sudden expansion:

\[ \zeta_{mix} = X_f \left( \frac{w_i / u_s}{w_i / w_{sl}} \right) \sin \beta_1 = \frac{c_{fr}}{w_{sl}} \left( \dot{w}_s \right)^2. \]

The empirical equations and the proper coefficients \( X_i \) serve for off-design regimes calculation and take into account 3-D flow character and compressibility influence. The equations for stator elements of a stage also present in the math model. The 4th version of computer programs was widely used in academic and design practice. Several dozens of compressor with delivery pressure up to 12.5 MPa, number of stages 1-8, power up to 25 MWt were designed for some Russian and foreign manufacturers. Amount of compressor installed exceeds 400 pieces with total power close to 5 000 000 KWh. In all cases the design parameters were achieved without model tests. But one problem existed for a designer.

The math model basic principle is that empirical coefficients \( X_i \) are independent of a stage parameters and similarity criteria. It means that a single set of \( X_i \) must describe gas dynamic performances of any stage at any condition with accuracy of experiments. The 4th version accuracy for design point efficiency is inside 2% that is not sufficient for design practice. There are different sets of \( X_i \) for different types of model stages participated in identification process. A designer had to choose proper sets for the designed objects. This disadvantage also pointed out that the physical and math models do not reflect all important factors.

III. New Version of Math Model Identification and Verification Results

The situation was revised and several important novels were introduced in 5th version [7]-[10], [14], [15] and in the next, 6th one too [10]. The flow path main dimensions input are more precise and complete – for 3-D impellers especially.
Velocity diagram schematization (Fig. 5) is made on the base of numerical experiment by Q-3-D calculations [7], [8]. Leakage in labyrinth seals is taken into account, etc. The main novel in the math model is connected with mixing losses calculation. The empirical coefficient $X'_i$ in (6) is presented as a function of two gas dynamic parameters of an impeller:

$$X'_{mix} = X'_i \left(1 + X'_i \cdot \psi_{T,des}^{X_i} + X'_i \left(1 - \frac{w_2}{w_1}\right)^{X_i}\right). \quad (8)$$

The new model identification was made with the use of 37 tests of 8 basic model stages with parameters $\Phi_{des} = 0,028 - 0,075$, $\psi_{T,des} = 0,42 - 0,75$ tested at $M_u = 0,60 - 0,86$. Basic stages have variants with different hub ratio, vaneless diffuser width, number of diffuser vanes, impeller blade trailing edge configuration.

The identification was made for all tests and a single set of the empirical coefficients was created. The average accuracy of efficiency calculation for all 37 tests is 1,09% - all flow rate range but the stall regime. The last is not important for industrial compressors. The average accuracy for design flow rate is 0,88%. Fig. 6 presents graphic information about matching of measured and calculated performances.

Three compressors were chosen for the new model verification. The 4-stage booster pipe line compressor – Fig. 7 – with optimal specific speed of stages $K_{n,des} = \Phi_{des}^{0.5} / \psi_{T,des}^{0.75}$ [13] was designed by the Universal modeling method.

Good correlation of measured and calculated performances was expected – Fig 8.

The two – stage booster compressor with parameters close to the compressor above was quite unusual as the higher pressure rise was achieved in two stages only. As result specific speed was very far from optimal range. The first stage parameters were $\Phi_{des} = 0,028$, $\psi_{T,des} = 0,82$. As result – very narrow channels with high level of friction losses and impeller blades with exit angle $\beta_{0/2} = 104^\circ$ – Fig. 9.

The second example compressor was designed by the Universal modeling method too (4th version), but its stages had no analogs. Anyway, the compressor demonstrated design parameters at the plant test and at the installation. The most satisfactory performance modeling result is shown at Fig. 10.

The third sample compressor was designed by its manufacturer on the base of model stages designed decades ago. Fig. 11 shows unusual now a meridian shape of impeller channels with $h_2 - h_1$. It leads to excessive flow deceleration $w_2 / w_1 < 0,5$ (recommendation [7] $w_2 / w_1 \geq 0,60 - 0,65$).
High mixing losses lead to low efficiency. Anyway, the modeling result is satisfactory in this case too - Fig. 12.

One more object of calculations was the tested stage with 3D impeller and vaneless diffuser. Gas dynamic performances are compared at Fig. 13.

IV. SAMPLE OF OPTIMIZATION PRACTICE

Results of identification and verification of the 6th version of the model demonstrate its validity. It gives the possibility to optimize main flow path dimensions by variants’ comparison.

Several important aspects of design cannot be solved by the simplified approach of Universal modeling though. The attractive way is to apply CFD calculations to the problem. The test results shown at Fig. 14 are compared with NUMECA FINE TURBO calculated performances – Fig. 15.

The result of the calculation is quite typical. CFD calculations overestimate work coefficient and underestimated incidence losses at negative incidence angle (Φ > Φdes). The same results were obtained with ANSYS CFX application and with other types of stages. The common positive result is that efficiency prediction is quite satisfactory at Φ ≈ Φdes.

The stage with Φdes = 0.105, Φf,des = 0.56 was designed on principles [13] and optimized by 6th version computer program. Several approaches of 3D impeller configuration choice were compared by calculations of efficiency at Φ ≈ Φdes. The important problem is 3D configuration choice. The “geometry” principle is quite simple. Blade angles are linear functions of meridian length. The alternative is to choose blades’ configuration by velocity diagram optimization. The compared functions βc = f (l) are presented at Fig. 16. CFD calculation has shown that ηdes is
0.4% higher in the second case and the advantage is higher at $\Phi > \Phi_{des}$.

There is the opinion that increase of the work coefficient along trailing edge from hub to shroud direction in 3D impellers can improve flow uniformity and diminish head losses in diffusers. The impeller variant with $\psi_{T_{des}} = f(b_r) = const$ was compared with the variant with $\psi_{T_{des}} = 1.03\psi_{T_{des}}$.

It appeared that the flow field uniformity is inferior in case of $\psi_{T_{des}} = f(b_r) = var$ but the stage efficiency is higher at 0.2% anyway.

VI. CONCLUSION

Several other problems were studied by variants’ comparison. It could be concluded that general rules well proven in previous designs are valid for high flow rate stages with 3D impellers too. Some details of design are constantly improved by variants’ comparison and more precise design recommendations are formulated as result.


