Heat and Mass Transfer in MHD Flow of Nanofluids through a Porous Media Due to a Permeable Stretching Sheet with Viscous Dissipation and Chemical Reaction Effects

Yohannes Yirga, Daniel Tesfay

Abstract—The convective heat and mass transfer in nanofluid flow through a porous media due to a permeable stretching sheet with magnetic field, viscous dissipation, chemical reaction and Soret effects are numerically investigated. Two types of nanofluids, namely Cu-water and Ag-water were studied. The governing boundary layer equations are formulated and reduced to a set of ordinary differential equations using similarity transformations and then solved numerically using the Keller box method. Numerical results are obtained for the skin friction coefficient, Nusselt number and Sherwood number as well as for the velocity, temperature and concentration profiles for selected values of the governing parameters. Excellent validation of the present numerical results has been achieved with the earlier linearly stretching sheet problems in the literature.

Keywords—Heat and mass transfer, magnetohydrodynamics, nanofluid.

I. INTRODUCTION

THE study of the boundary layer flow of an electrically conducting fluid through a porous media has many applications in manufacturing and natural process which include cooling of electronic devices by fans, cooling of nuclear reactors during emergency shutdown, cooling of an infinite metallic plate in a cooling bath, textile and paper industries, glass-fiber production, manufacture of plastic and rubber sheets, the utilization of geothermal energy, the boundary layer control in the field of aerodynamics, food processing, plasma studies and in the flow of biological fluids. The concept of stretching sheet was first used by [1] to study the flow of an incompressible viscous fluid with a linearly varying surface velocity. Later, various authors extended this work to various aspects of flow and heat transfer over a stretching surface [2]-[5].

Magnetohydrodynamics (MHD) is the study of the flow of electrically conducting fluids in a magnetic field. Many experimental and theoretical studies on conventional electrically conducting fluids indicate that magnetic field markedly changes their transport and heat transfer characteristics. The study of magnetohydrodynamics has many important applications, and may be used to deal with problems such as cooling of nuclear reactors by liquid sodium and induction flow meter, which depends on the potential difference in the fluid in the direction perpendicular to the motion and to the magnetic field [6]. Recently, the application of magnetohydrodynamics in the polymer industry and metallurgy has attracted the attention of many researchers. Several researches investigated the MHD flow [7]-[12].

Dissipation is the process of converting mechanical energy of downward-flowing water into thermal and acoustical energy. Viscous dissipation is of interest for many applications: significant temperature rises are observed in polymer processing flows such as injection modeling or extrusion at high rates. Aerodynamic heating in the thin boundary layer around high speed aircraft raises the temperature of the skin. In a completely different application, the dissipation function is used to define the viscosity of dilute suspensions Einstein [13]: Viscous dissipation for a fluid with suspended particles is equated to the viscous dissipation in a pure Newtonian fluid, both being in the same flow (same macroscopic velocity gradient). Vajravelu and Hadjinicolou [14] analyzed the heat transfer characteristics over a stretching surface with viscous dissipation in the presence of internal heat generation or absorption. The effects of thermal radiation and viscous dissipation on boundary layer flow of nanofluids over a permeable moving flat plate are numerically studied by [15]. Gebhart [16] and Gebhart and Mollendorf [17] studied the effect of viscous dissipation in natural convection processes. They observed that the effect of viscous dissipation is significant in vigorous natural convection and mixed convection processes. They also showed the existence of a similarity solution for the external flow over an infinite vertical surface with an exponential variation of surface temperature. Javed and Sina [18] studied the viscous flow over nonlinearly stretching sheet with effects of viscous dissipation. They found that for large Prandtl numbers, the temperature profile decreases. Habibi et al. [19] studied the mixed convection MHD flow of nanofluid over a non-linear stretching sheet with effects of viscous dissipation and variable magnetic field.

The presence of porous media in a boundary layer flow can significantly change the flow field and, as a consequence, affect the heat transfer rate at the surface. Porous media are generally modelled using the classical Darcy formulation,
which implies that the mean filter velocity is proportional to the summation of the pressure gradient and the gravitational force. The model is empirical and cannot be derived analytically via a momentum on a small element of porous medium. The Darcian model has been widely used non-Newtonian porous media heat transfer flows. For regimes where higher velocities may occur, the Darcy linear model is inadequate and refinements have to be employed. The Darcy–Forchheimer (DF) model is probably the most popular modification to Darcian flow utilized in simulating inertial effects. It has been used extensively in chemical engineering analysis and also in materials processing simulations. Recently, the flow through porous media has attracted the interest of different researchers due to its diverse application in biotechnology, chemical and petroleum industries, in geophysical fluid dynamics such as beach sand, wood, sandstone, limestone, the human lung and in small blood vessels [20], [21].

In the studying of boundary layer flows the phenomenon of suction or injection is very important. The boundary layer thickness is controlled by introducing suction at the surface of the moving body. The effect of the suction/injection on the boundary layer flow has been studied by [22]-[26].

Heat and mass transfer problems with a chemical reaction have received a considerable amount of attention in recent years. In processes such as drying, evaporation, energy transfer in a cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Natural convection processes involving the combined mechanisms are also encountered in many natural processes and industrial applications, such as in the curing of plastics, the cleaning and chemical processing of materials and the manufacture of pulp and insulated cables. Chamka [27] studied the MHD flow over a uniformly stretched vertical permeable surface subject to a chemical reaction. Afifi [28] analyzed the MHD free convective flow and mass transfer over a stretching sheet with a homogeneous chemical reaction of order n (where n was taken to be 0, 1, 2 or 3). The influence of a chemical reaction on heat and mass transfer due to natural convection from vertical surfaces in porous media subject to Soret and Dufour effects was studied by [29]. He showed that the thickness of the concentration boundary layer decreases as the Lewis number increases, a phenomenon also evident when a chemical reaction is absent. Kandasamy and Palanimani [30] carried out an analysis of the effects of chemical reactions on heat and mass transfer on a magnetohydrodynamic boundary layer flow over a wedge with ohmic heating and viscous dissipation in a porous medium.

Fluid heating and cooling are important in many industries such as power, manufacturing, transportation, and electronics. Effective cooling techniques are greatly needed for cooling any sort of high-energy device. Common heat transfer fluids such as water, ethylene glycol, and engine oil have limited/poor heat transfer capabilities due to their low heat transfer properties. In contrast, metals have thermal conductivities up to three times higher than these fluids, so it is natural that it would be desired to combine the two substances to produce a heat transfer medium that behaves like a fluid, but has the thermal conductivity of a metal. A lot of experimental and theoretical researches have been made to improve the thermal conductivity of these fluids. In 1993, during an investigation of new coolants and cooling technologies at Argonne national laboratory in U.S, Choi invented a new type of fluid called Nanofluid [31]. Nanofluids are fluids that contain small volumetric quantities of nanometer-sized particles, called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid [32]. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes. Common base fluids include water, ethylene glycol and oil. Nanofluids commonly contain up to a 5% volume fraction of nanoparticles to see effective heat transfer enhancements. Nanofluids are studied because of their heat transfer properties: they enhance the thermal conductivity and convective properties over the properties of the base fluid. Moreover, the presence of the nanoparticles enhance the electrical conductivity property of the nanofluids, hence are more susceptible to the influence of magnetic field than the conventional base fluids. Typical thermal conductivity enhancements are in the range of 15-40% over the base fluid and heat transfer coefficient enhancements have been found up to 40% [33]. Thermophysical properties of nanofluids such as thermal conductivity, diffusivity and viscosity have been studied, among others, [34]-[36].

After the pioneer investigation of [31], thriving experimental and theoretical researches were undertaken to discover and understand the mechanisms of heat transfer in nanofluids. The knowledge of the physical mechanisms of heat transfer in nanofluids is of vital importance as it will enable the exploitation of their full heat transfer potential. Masuda et al. [37] observed the characteristic feature of nanofluid is thermal conductivity enhancement. This observation suggests the possibility of using nanofluids in advanced nuclear systems [38]. A comprehensive survey of convective transport in nanofluids was made by [39], which states that a satisfactory explanation for the abnormal increase of the thermal conductivity and viscosity is yet to be found. He focused on further heat transfer enhancement observed in convective situations. Khan and Pop [40] presented a similarity solution for the free convection boundary layer flow past a horizontal flat plate embedded in a porous medium filled with a nanofluid. Makinde and Aziz [41] studied MHD mixed convection from a vertical plate embedded in a porous medium with a convective boundary condition.

Majority of the above studies are restricted to boundary layer flow and heat transfer in Newtonian fluids. However, due to the increasing importance of nanofluids, in recent years a great attention has been given to the study of convective transport of nanofluids. In addition to this the flow of nanofluids through a porous media with effects of chemical reaction and viscous dissipation has been given less attention. Therefore, the aim of the present paper is to study the magnetohydrodynamic flow and heat transfer of nanofluids through a porous media due to a permeable stretching sheet.
with viscous dissipation and chemical reaction effects. The combined effect of all the above mentioned parameters has not been reported so far in the literature.

The governing highly nonlinear partial differential equation of momentum, energy and concentration fields has been simplified by using suitable similarity transformations and then solved numerically with the help of a powerful, easy to use method called the Keller box method. This method has already been successfully applied to several non linear problems corresponding to parabolic partial differential equations. As discussed in [42], the exact discrete calculus associated with the Keller-Box Scheme is shown to be fundamentally different from all other mimic numerical methods. The box-scheme of Keller is basically a mixed finite volume method, which consists in taking the average of a conservation law and of the associated constitutive law at the level of the same mesh cell.

The paper is organized as follows: the mathematical formulation of the problem is presented in Section II. Section III outlines the numerical procedure whilst results and discussions are presented in Section IV. Section V concludes the paper.

II. MATHEMATICAL FORMULATION

Consider the two-dimensional steady laminar flow of an incompressible nanofluid over a stretching sheet. If we consider the Cartesian coordinate system with the origin fixed in such a way that, the x-axis is taken along the direction of the continuous stretching surface and the y-axis is measured normal to the surface of the sheet. Two equal but opposite forces are applied along the sheet so that the wall is stretched, keeping the position of the origin unaltered. The fluid is electrically conducting under the influence of an applied magnetic field. The fluid is a water based nanofluid containing nanoparticles. It is assumed that the base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermophysical properties of the nanofluid are given in Table I (see [43]). With the above assumptions, the boundary layer equations governing the nanofluid flow, the heat and the concentration fields can be written in dimensional form as [44], [45].

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, 
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_f}{\rho_f} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2 y}{\rho_f n_f} u - \frac{\mu_f}{\rho_f} \frac{1}{x} u, 
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \sigma \frac{\partial^2 u}{\partial y^2} + \frac{\mu_f}{\rho_f n_f} \left( \frac{\partial u}{\partial y} \right)^2, 
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = D \frac{\partial^2 c}{\partial y^2} + D_1 \frac{\partial^2 T}{\partial y^2} - K_0 (C_w - C_c), 
\]

where \( u \) and \( v \) are the velocity components in the x and y directions respectively, \( T \) is the temperature, \( C \) is the concentration of the nanofluid, \( C_0 \) is the concentration of the nanofluid far from the sheet \( B_0 \) is the uniform magnetic field strength, \( \sigma \) is the electrical conductivity, \( c_p \) is the specific heat at constant pressure, \( D \) is the species diffusivity, \( D_1 \) is the coefficient that signifies the contribution to mass flux through temperature gradient, \( K \) is the permeability of the porous medium, \( K_0 \) is a chemical reaction parameter; and \( \mu_f, \rho_f, \alpha_f, \) and \( (\rho c_p)_f \) are the dynamic viscosity, density, thermal conductivity and heat capacitance of the nanofluid, respectively, which are given as \([46],[47]\).
with boundary condition

\[ f(0) = f_w, f'(0) = 1, g(0) = 1, h(0) = 1, f(\eta) \rightarrow 0, g(\eta) \rightarrow 0, h(\eta) \rightarrow 0, \text{as} \; \eta \rightarrow \infty \]  

(12)

where \( f(\eta), g(\eta), \) and \( h(\eta) \) are the dimensionless velocity, temperature and nanoparticle concentration, respectively, primes denote differentiation with respect to the similarity variable \( \eta \), and \( M = \frac{g_d}{\beta u_f} \) (Magnetic parameter), \( K_t = \frac{\nu}{\alpha \omega} \) (Porous medium parameter), \( Pr = \frac{\nu}{\alpha_f} \) (Prandtl number), \( Ec = \frac{u_d^2}{(\gamma_0)(T_w - T_c)} \) (Eckert number), \( Sc = \frac{\nu}{\rho D} \) (Schmidt number), \( \gamma = \frac{K_0}{K_1} \) (Scaled chemical reaction parameter), and \( Sr = \frac{D_t}{(T_w - T_c)} \) (Soret number).

The permeability or suction/injection parameter \( f_w = -\frac{v_w}{\sqrt{Bv_f}} \) which shows the strength of the mass transfer at the sheet is negative for mass injection, positive in the presence of suction along the surface and it has a zero value for an impermeable surface. The physical quantities of interest in this problem are the local skin friction coefficient \( C_f \), the Nusselt number \( Nu_x \), which represents the rate of heat transfer at the surface of the plate and the local Sherwood number \( Sh_x \), which represents the rate of mass transfer at the surface of the plate, which are defined as

\[ C_f = \frac{2\tau_w}{\rho u_f^2}, \quad Nu_x = \frac{x_{fw}}{k_f(T_w - T_c)}, \quad Sh_x = \frac{x_{fw}}{D(\rho C_w - \rho C_d)} \]  

(13)

where \( \tau_w \) is the skin friction, \( q_{fw} \) is the heat flux and \( J_{fw} \) is the mass flux through the plate, which are given by

\[ \tau_w = -\mu_q \left( \frac{\nu}{\alpha_f} \right)_{\eta = 0}, \quad q_{fw} = -k_q \left( \frac{\nu}{\alpha_f} \right)_{\eta = 0}, \quad J_{fw} = -D \left( \frac{\rho C_w}{\rho C_d} \right)_{\eta = 0} \]  

(14)

Making use of (8) and (5) in (13), the dimensionless skin friction coefficient, wall heat and mass transfer rates are obtained as

\[ C_f(1 - \phi)^2 \sqrt{Re_x} = -2f''(0), \quad Nu_x = \frac{x_{fw}}{k_f(T_w - T_c)}, \quad Sh_x = \frac{x_{fw}}{D(\rho C_w - \rho C_d)} = -h(0) \]  

(15)

The skin friction coefficients for different values of the magnetic parameter \( M \) and nanoparticle volume fraction \( \phi \) are shown in Table III for different Prandtl number \( Pr \). The comparisons are found to be in an excellent agreement (see Tables II and III).

The skin friction coefficients for different values of the magnetic parameter \( M \) and nanoparticle volume fraction \( \phi \) are shown in Table II. Increasing the value of \( M \) results in a considerable increase in the skin friction coefficient; this is due to the opposition to the flow caused by the induced Lorenz force. The results show a good agreement with [48].

The heat transfer coefficients are shown in Table III for different Prandtl number \( Pr \). It is clear that the heat transfer coefficient increases with Prandtl numbers. The present results are in good agreement with the earlier findings by [3].
### TABLE I

<table>
<thead>
<tr>
<th>Physical Properties</th>
<th>Fluid Phase</th>
<th>Cu</th>
<th>Ag</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ (kg/m$^3$)</td>
<td>997.1</td>
<td>8933</td>
<td>10500</td>
</tr>
<tr>
<td>$C_p$ (J/kgK)</td>
<td>4179</td>
<td>385</td>
<td>235</td>
</tr>
<tr>
<td>$k$ (W/mK)</td>
<td>0.613</td>
<td>401</td>
<td>429</td>
</tr>
</tbody>
</table>

### TABLE II

Comparison of the skin friction coefficient $f' (0)$ for different values of magnetic parameter $M$ and nanoparticle volume fraction $\phi$ when Prandtl number = 6.2 with [48]

<table>
<thead>
<tr>
<th>$M$</th>
<th>$\phi$</th>
<th>Cu-water</th>
<th>Ag-water</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.05</td>
<td>1.10892</td>
<td>1.1089</td>
</tr>
<tr>
<td>0.1</td>
<td>1.17475</td>
<td>1.1747</td>
<td>1.12250</td>
</tr>
<tr>
<td>0.15</td>
<td>1.20886</td>
<td>1.2089</td>
<td>1.27215</td>
</tr>
<tr>
<td>0.2</td>
<td>1.21804</td>
<td>1.2180</td>
<td>1.28979</td>
</tr>
<tr>
<td>0.05</td>
<td>1.32825</td>
<td>1.3282</td>
<td>1.37296</td>
</tr>
<tr>
<td>0.1</td>
<td>1.33955</td>
<td>1.3396</td>
<td>1.39694</td>
</tr>
<tr>
<td>0.2</td>
<td>1.33036</td>
<td>1.3304</td>
<td>1.39634</td>
</tr>
<tr>
<td>1</td>
<td>0.05</td>
<td>1.45236</td>
<td>1.4524</td>
</tr>
<tr>
<td>0.1</td>
<td>1.46576</td>
<td>1.4658</td>
<td>1.50640</td>
</tr>
<tr>
<td>0.15</td>
<td>1.45858</td>
<td>1.4586</td>
<td>1.51145</td>
</tr>
<tr>
<td>0.2</td>
<td>1.43390</td>
<td>1.4339</td>
<td>1.49532</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>1.72887</td>
<td>1.7289</td>
</tr>
<tr>
<td>0.1</td>
<td>1.70789</td>
<td>1.7079</td>
<td>1.74289</td>
</tr>
<tr>
<td>0.15</td>
<td>1.67140</td>
<td>1.6714</td>
<td>1.71773</td>
</tr>
<tr>
<td>0.2</td>
<td>1.62126</td>
<td>1.6213</td>
<td>1.67583</td>
</tr>
</tbody>
</table>

### TABLE III

Comparison of the wall heat transfer rate $-G'(0)$ for various values of Prandtl number $Pr$, when $\phi = M = K_1 = Ec = f_0 = 0$

<table>
<thead>
<tr>
<th>Pr</th>
<th>[40]</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.72</td>
<td>1.0885</td>
<td>1.3333</td>
</tr>
<tr>
<td>1</td>
<td>2.5997</td>
<td>4.7969</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>4.7969</td>
<td>15.7120</td>
<td></td>
</tr>
<tr>
<td>0.72</td>
<td>1.0886</td>
<td>1.3333</td>
</tr>
<tr>
<td>5</td>
<td>2.5997</td>
<td>4.7970</td>
</tr>
<tr>
<td>10</td>
<td>15.7198</td>
<td></td>
</tr>
</tbody>
</table>

### Fig. 1

Effects of $M$ on the velocity distribution $f'(\eta)$, when $\phi = Sr = 0.2, Pr = 6.2, K_1 = Ec = Sc = 1, f_0 = 0$ in case of Cu-water and Ag-water nanofluids.

### Fig. 2

Effects of $\phi$ on the temperature profile $G(\eta)$, when $M = 0, K_1 = 1, Pr = 6.2, Ec = 1, Sc = 0, \gamma = 0.08, Sr = 0.2, f_0 = 0$ in case of Cu-water and Ag-water nanofluids.

### Fig. 3

Effects of $Ec$ on the temperature profile $g(\eta)$, when $\phi = 0.2, M = 0, K_1 = 1, Pr = 6.2, \gamma = 0.08, Sr = 0.2, f_0 = 0$ in case of Cu-water and Ag-water nanofluids.

The effect of the viscous dissipation parameter $Ec$ on the temperature profile in the case of Cu-water nanofluid is shown in Fig. 3. It is clear that the temperature distribution increases with an increase in the viscous dissipation parameter $Ec$. The combined effect of the magnetic field and the viscous dissipation parameter (see Figs. 3 and 9) is to generate more heat in the boundary layer region and hence to reduce the wall heat transfer rate.
temperature distributions decrease with an increase in the suction/injection parameter \( f_w \).

Fig. 4 shows, the effect of porosity parameter \( K_1 \) on the velocity distribution of the Cu-water nanofluid. It is observed that the velocity distribution decreases with increasing the porosity parameter. This is because the presence of a porous medium increases the resistance to the flow causing a decrease in the fluid velocity.

The effects of various parameters on the wall skin friction, heat and mass transfer rates are shown in Figs. 8-10. Fig. 8 illustrates the effects of the porosity parameter \( K_1 \) and nanoparticle volume fraction \( \phi \) on the wall skin friction. It is observed that the magnitude of the skin friction coefficient \(-f''(0)\) increases with increasing in the porosity parameter and nanoparticle volume fraction for both the Cu-water and Ag-water nanofluids. The wall skin friction is higher in the Ag-water nanofluid compared to the Cu-water nanofluid.
Thus, the Ag-water nanofluid gives a higher drag force in opposition to the flow as compared to the Cu-water nanofluid. The effects of magnetic field parameter $M$ and porous media parameter $K_2$ on the wall heat transfer rate $-g'(0)$ are shown in Fig. 9 in the case of Cu-water and Ag-water nanofluids. The influence of magnetic field is to reduce the wall heat transfer rates. The porous media effect reduces the wall heat transfer rate.

Moreover, the rate of heat transfer at the wall is less in case of the Ag-water nanofluid compared to the Cu-water nanofluid.

The combined effects of the Soret number and Schmidt number on the wall mass transfer rate are shown in Fig. 10 in case of Cu-water and Ag-water nanofluids. It is observed that, the rate of mass transfer in the wall $-h'(0)$ is an increasing function of Soret number and Schmidt number.

**V. CONCLUSIONS**

This paper presents the problem MHD flow, heat and mass transfer of nanofluid through a porous medium due to stretching sheet in the presence of viscous dissipation, chemical reaction and Soret effects. The governing nonlinear partial differential equations were transformed into ordinary differential equations using the similarity approach and solved numerically using the Keller box method. Two types of nanofluids were considered, Cu-water and Ag-water, and our results revealed, among others, the following.

1. Cu-water shows a thicker velocity boundary than Ag-water nanofluid. The velocity boundary layer thickness decreases with increasing the nanoparticle volume fraction and porous medium parameter.

2. Ag-water nanofluid shows a slightly thicker concentration boundary layer than Cu-water nanofluid. The concentration boundary layer thickness increases with increasing the values of $\phi$ and $Ec$.

3. Ag-water nanofluid shows a slightly thicker concentration boundary layer than Cu-water nanofluid. The concentration boundary layer thickness increases with increasing the values of $\phi$ while it decreases with increasing the values of the $Sc$ and $\gamma$.

4. The velocity distribution and temperature profiles of the Cu-water nanofluid decrease with an increase in the suction/injection parameter $f_w$.

5. The skin friction increases with increasing the nanoparticle volume fraction and porous medium parameter; the Ag-water nanofluid shows a higher skin friction than Cu-water nanofluid.

6. The heat transfer rate at the plate surface decreases with increasing the nanoparticle volume fraction, magnetic field parameter, porous medium parameter and Eckert number. The Cu-water nanofluid has a higher rate of heat transfer than the Ag-water nanofluid.

7. The mass transfer rate at the plate surface increases with increasing the nanoparticle volume fraction, Schmidt number, Chemical reaction parameter and Soret number. The Ag-water nanofluid has a higher rate of mass transfer than the Cu-water nanofluid.

A comparison with previously reported data is made and an excellent agreement is noted.

**REFERENCES**


Dr. Yohannes Yirga has worked as an assistant professor in the department of Mathematics, Mekelle University giving different mathematics courses to the post graduate students in Mathematics and Engineering fields. He has worked as a lecturer from 2005- 2010 in the department of Mathematics, Mekelle University, Mekelle, Ethiopia. In 2009, he obtained M.Sc. degree from Addis Ababa University, Addis Ababa, Ethiopia. In 2014, he obtained Ph. D. degree in Mathematics, from Osmania University, Hyderabad, India. His main works in his institute are Applied Mathematics, Physical and Mathematical modeling particularly related to fluid mechanics fields. Since 2005, he has given lectures of Applied Mathematics, Calculus and Geometry Subjects to under-graduate students of science and Engineering in Mekkelle University. He also has given lectures of Numerical and Analytic Methods, Fluid Mechanics to research scholars of Mathematics in Osmania University. He has produced many papers in the fields of Mathematics and Fluid mechanics. He has more than 5 publications.

Daniel Tesfay has worked as a lecturer in the department of Mathematics, Mekelle University giving different mathematics courses to undergraduate students in Mathematics and Engineering fields. He has worked as a lecturer from 2005 to date in the department of Mathematics, Mekelle University, Mekelle, Ethiopia. In 2005, he obtained M.Sc. degree from Addis Ababa University, Addis Ababa, Ethiopia. His main works in his institute are Applied Mathematics, Linear and Nonlinear Optimization, Operations Research and Mathematical modeling. Since 2005, He has given lectures of Applied Mathematics, Analysis, OR, Linear and Nonlinear and Mathematical Modeling courses to undergraduate students of science and Engineering in Mekelle University. Currently he is conducting research on the field of fluid mechanics.