Application of Intuitionistic Fuzzy Cross Entropy Measure in Decision Making for Medical Diagnosis

Shikha Maheshwari, Amit Srivastava

Abstract—In medical investigations, uncertainty is a major challenging problem in making decision for doctors/experts to identify the diseases with a common set of symptoms and also has been extensively increasing in medical diagnosis problems. The theory of cross entropy for intuitionistic fuzzy sets (IFS) is an effective approach in coping uncertainty in decision making for medical diagnosis problem. The main focus of this paper is to propose a new intuitionistic fuzzy cross entropy measure (IFCEM), which aid in reducing the uncertainty and doctors/experts will take their decision easily in context of patient’s disease. It is shown that the proposed measure has some elegant properties, which demonstrates its potency. Further, it is also exemplified in detail the efficiency and utility of the proposed measure by using a real life case study of diagnosis the disease in medical science.

Keywords—Intuitionistic fuzzy cross entropy (IFCEM), intuitionistic fuzzy set (IFS), medical diagnosis, uncertainty.

I. INTRODUCTION

The complexity generally arises from uncertainty in the form of ambiguity is ubiquitous. So, the theory of intuitionistic fuzzy set (IFS) is an excellent mathematical tool originated by [1], which is a forwarded concept of Zadeh’s fuzzy sets theory [2], earning extensive attention from numerous researchers due to its effectiveness in dealing with uncertain situations.

In medical investigations, there are various types of diseases occur that has been associated with some common symptoms such as headache, cough, chest pain, stomach pain etc. To identify the actual disease on the basis of same set of symptoms in first analysis is a complicated task for doctors/experts. So, it is tough to analyze the disease of the patient with respect to the symptoms by the doctors/experts in case of high level of uncertainty.

Various different measures such as similarity, cross entropy, distance, entropy etc. for fuzzy/intuitionistic fuzzy sets have been studied by [3]–[11] and applied to the medical problems. Among them the concept of cross entropy measure for IFS is cynosure for measuring the discrimination information between the pairs of IFS and widely used in different domains [11]–[16]. Recently, [17] introduced an axiomatic definition of divergence for IFSs, and suggested a method for building divergence measures between IFS.

In this paper, an effort has been made to handle the problem by exploiting the proposed measure, which enable the doctors/experts aid in analyzing the correct disease, so that patient can get the correct treatment and obtain the hale and hearty life. In the last few years, measure of intuitionistic fuzzy cross entropy plays an imperative role in reducing uncertainty and aid to the doctors/experts in making the decision about the patient’s disease. This will in turn diminish uncertainty in cases where limited information is available to the experts, as it provides very rapid method of diagnosis the disease with accuracy and less uncertainty. Vlachos and Sergiadis [11] firstly introduced cross entropy measure under intuitionistic fuzzy phenomena and shown its application in different disciplines. However, [14] pointed out the downside of [11] measure and gave the modified measure.

The present article work on decision making in diagnosis the disease of the patient based on symptoms by exploiting the proposed IFCEM. The work is distributed in the manner as follows. Section II presents the some basic definitions. Section III introduces a new intuitionistic fuzzy cross entropy measure (IFCEM) and also states its properties with proof. The propose IFCEM impose on medical diagnosis problem is presented in Section IV. Conclusion is given in Section V.

II. PRELIMINARIES

A. Intuitionistic Fuzzy Sets

An intuitionistic fuzzy set $A$ is defined on $X$ introduced by [1], given by

$$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\},$$  \hspace{1cm} (1)

where $\mu_A(x), \nu_A(x)$ indicate membership degree and non-membership degree to $A$ respectively and the functions $\mu_A(x), \nu_A(x) : X \rightarrow [0,1]$ such that for every $x \in X$

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$  \hspace{1cm} (2)

Further, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ denote the hesitation degree to $A$ such that $\pi_A(x) \in [0,1]$ for all $x \in X$. For convenience, we abbreviate the family of all Atanassov’s intuitionistic fuzzy sets in the universe $X$ by $A_{IFS}(X)$.

Let $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$, $B = \{(x, \mu_B(x), \nu_B(x)) \mid x \in X\} \in A_{IFS}(X)$, then some set operations can be represented as:

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• Complement of $A$

$A^C = \{ (x, v_A(x_i), \mu_A(x_i)) \mid x \in X \}$

• Union of $A$ and $B$

$A \cup B = \{ \left\{ x, \max \{ \mu_A(x_i), \mu_B(x_i) \} \right\} \mid \left\{ x_i \right\} \in X \}$

• Intersection of $A$ and $B$

$A \cap B = \{ \left\{ x, \min \{ \mu_A(x_i), \mu_B(x_i) \} \right\} \mid \left\{ x_i \right\} \in X \}$.

B. Intuitionistic Fuzzy Cross Entropy Measure

Let $A, B \in A_{IFS}(X)$, then a mapping $D: A_{IFS}(X) \times A_{IFS}(X) \rightarrow [0, 1]$ is a divergence measure for $A_{IFS}$, if it satisfies the following axioms:

A1. $D(A \parallel B) \geq 0$.

A2. $D(A \parallel A) = 0$ if and only if $A = B$.

A3. $D(A \parallel B) = D(B \parallel A)$.

Then the measure $D(A \parallel B)$ is called intuitionistic fuzzy cross entropy measure between two IFSs.

III. NEW INTUITIONISTIC FUZZY-CROSS ENTROPY MEASURE

Let us consider $A = \{ (x, \mu_A(x_i)) \mid x_i \in X \}, B = \{ (x, \mu_B(x_i)) \mid x_i \in X \} \in A_{IFS}(X)$ in a universe of discourse $X = \{ x_1, x_2, \ldots, x_n \}$. We define a new measure of intuitionistic fuzzy cross entropy given by

$$D_{IFS}(A \parallel B) = \sum_{i=1}^{n} \left[ \frac{(\mu(x_i))^2 + (\mu(x_i))^2}{2} - \frac{(\mu(x_i) + \mu(x_i))^2}{2} \right] + \left[ \frac{(v(x_i))^2 + (v(x_i))^2}{2} - \frac{(v(x_i) + v(x_i))^2}{2} \right]$$

which estimates the degree of discrimination of uncertain information between the pairs of intuitionistic fuzzy sets (IFSs) $A$ and $B$. For $A, B \in A_{IFS}(X)$ the measure $D_{IFS}(A \parallel B)$ satisfies the following properties:

P1. $D_{IFS}(A \parallel B) \geq 0$ with equality if and only if $A = B$.

Proof. Since $\frac{x^2 + y^2}{2} \geq \left( \frac{x+y}{2} \right)^2$ for $x, y \in [0, 1]$ if and only if $x = y$, therefore $D_{IFS}(A \parallel B) \geq 0$ if and only if $A = B$.

P2. $D_{IFS}(A \parallel B) = D_{IFS}(A' \parallel B') = D_{IFS}(B \parallel A')$.

Proof. Let

$$D_{IFS}(A \parallel B) = \sum_{i=1}^{n} \left[ \frac{(\mu(x_i))^2 + (\mu(x_i))^2}{2} - \frac{(\mu(x_i) + \mu(x_i))^2}{2} \right] + \left[ \frac{(v(x_i))^2 + (v(x_i))^2}{2} - \frac{(v(x_i) + v(x_i))^2}{2} \right] = 0$$

This is possible only if and only if $\mu_A(x_i) = v_A(x_i)$.

P3. $D_{IFS}(A \parallel A') = 0$ if and only if $\mu_A(x_i) = v_A(x_i)$ for all $x_i \in X$.

Proof. Let

$$D_{IFS}(A \parallel A') = \sum_{i=1}^{n} \left[ \frac{(\mu(x_i))^2 + (\mu(x_i))^2}{2} - \frac{(\mu(x_i) + \mu(x_i))^2}{2} \right] + \left[ \frac{(v(x_i))^2 + (v(x_i))^2}{2} - \frac{(v(x_i) + v(x_i))^2}{2} \right] = 0$$

P4. $D_{IFS}(A \parallel B^c) = D_{IFS}(A^c \parallel B)$. 

Proof.
\[
D_{\alpha^*} (A \parallel \bigcup B) = \sum_{i=1}^{n} \left( \sqrt{\frac{(\mu_i(x_i))^2 + (\max(\mu_i(x_i))_i)^2}{2}} - \sqrt{\frac{(\min(\mu_i(x_i), \mu_B(x_i)))^2 + (\mu_B(x_i))^2}{2}} \right) + \sqrt{\frac{(\nu_i(x_i))^2 + (\min(\nu_i(x_i)))^2}{2}} - \sqrt{\frac{(\max(\nu_i(x_i)))^2 + (\nu_i(x_i))^2}{2}} \bigg)
\]

\[
= D(A^C \parallel B).
\]

**P5.** \( D_{\alpha^*} (A \parallel A \cup B) = D_{\alpha^*} (A \cap B \parallel B) \) for \( A \subseteq B \) and \( B \subseteq A \).

**Proof.**

\[
D_{\alpha^*} (A \parallel A \cup B) = \sum_{i=1}^{n} \left( \sqrt{\frac{(\mu_i(x_i))^2 + (\max(\mu_i(x_i), \mu_B(x_i)))^2}{2}} - \sqrt{\frac{(\min(\mu_i(x_i), \mu_B(x_i)))^2 + (\mu_B(x_i))^2}{2}} \bigg)
\]

For \( A \subseteq B \), from equations (4) and (5), we get

\[
D_{\alpha^*} (A \parallel A \cup B) = D_{\alpha^*} (A \cap B \parallel B) = D_{\alpha^*} (A \parallel B). \]  

(6)

Considering equations (4) and (5) for \( B \subseteq A \), we have

\[
D_{\alpha^*} (A \parallel A \cup B) = D_{\alpha^*} (A \cap B \parallel B) = 0. \]  

(7)

From (6) and (7), we obtain the result

\[
D_{\alpha^*} (A \parallel A \cup B) = D_{\alpha^*} (A \cap B \parallel B). \]

**P6.** \( D_{\alpha^*} (A \cap B \parallel A \cup B) = D_{\alpha^*} (A \parallel B). \)

**Proof.**
IV. APPLICATIONS IN DECISION MAKING FOR MEDICAL DIAGNOSIS

The data of the example is taken from [3]. Let there be set of four patients represented by $P = \{\text{Sam, Ben, Joy, Tom}\}$. Their diagnosis and set of symptoms are represented by $D = \{\text{viral fever, malaria, stomach problem, chest pain}\}$, $S = \{\text{temperature, headache, stomach pain, chest pain}\}$. Tables I and II represent the relation between $S \rightarrow D$ and $P \rightarrow S$ respectively. Each element of the tables is specified in the form of a pair of numbers corresponding to the membership, non-membership and hesitation values, respectively. In order to achieve a proper diagnosis, we compute the symmetric discrimination information measure between all patients and diagnosis in view of symptoms observed. This procedure is done for each and every diagnosis. Using (3), the minimum symmetric discrimination information measure pointing to a proper diagnosis is assigned. The evaluated results are mentioned in Table III.

From Table III, we can say that Sam suffers from Malaria, Ben suffers from Stomach problem, Joy and Tom suffer from viral fever. The comparison of the results is mentioned in Table IV.

This completes the proof.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>SYMPTOMS VERSUS DIAGNOSIS CONSIDERED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viral Fever</td>
<td>Malaria</td>
</tr>
<tr>
<td>Temperature</td>
<td>(0.4, 0.0, 0.6)</td>
</tr>
<tr>
<td>Headache</td>
<td>(0.3, 0.5, 0.2)</td>
</tr>
<tr>
<td>Stomach pain</td>
<td>(0.1, 0.7, 0.2)</td>
</tr>
<tr>
<td>Cough</td>
<td>(0.4, 0.3, 0.3)</td>
</tr>
<tr>
<td>Chest pain</td>
<td>(0.1, 0.7, 0.2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>SYMPTOMS VERSUS PATIENT CONSIDERED</th>
</tr>
</thead>
<tbody>
<tr>
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The result has shown the properties. We have also exemplified that the proposed approach to the inter-valued intuitionistic fuzzy cross-entropy and entropic measure (IFCEM) and aid in recognize the correct disease. In future, research may extend the proposed approach to the inter-valued intuitionistic fuzzy cross entropy and open up some new real life applications in other domains.

**V. CONCLUSION**

This paper has defined a new intuitionistic fuzzy cross entropy measure (IFCEM) and also studied its eminent properties. We have also exemplified that the proposed measure has successfully applied to the problem of medical diagnosis with the help of example. The result has shown the efficacy of the proposed measure (IFCEM) and aid in recognize the correct disease. In future, research may extend the proposed approach to the inter-valued intuitionistic fuzzy cross entropy and open up some new real life applications in other domains.

**REFERENCES**


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