Abstract—The article deals with the tool in Matlab GUI form that is designed to analyse a mechatronic system sensitivity and tolerance. In the analysed mechatronic system, a torque is transferred from the drive to the load through a coupling containing flexible elements. Different methods of control system design are used. The classic form of the feedback control is proposed using Naslin method, modulus optimum criterion and inverse dynamics method. The cascade form of the control is proposed based on combination of modulus optimum criterion and symmetric optimum criterion. The sensitivity is analysed on the basis of absolute and relative sensitivity of system function to the change of chosen parameter value of the mechatronic system, as well as the control subsystem. The tolerance is analysed in the form of determining the range of allowed relative changes of selected system parameters in the field of system stability. The tool allows to analyse an influence of torsion stiffness, torsion damping, inertia moments of the motor and the load and controller(s) parameters. The sensitivity and tolerance are monitored in terms of the impact of parameter change on the response in the form of system step response and system frequency-response logarithmic characteristics. The Symbolic Math Toolbox for expression of the final shape of analysed system functions was used. The sensitivity and tolerance are graphically represented as 2D graph of sensitivity or tolerance of the system function and 3D/2D static/interactive graph of step/frequency response.

Keywords—Mechatronic systems, Matlab GUI, sensitivity, tolerance.

I. INTRODUCTION

This contribution deals with designing of an application which enable complex sensitivity and tolerance analysis of mechatronic system based on GUI utilisation created in the Matlab tool. The impulse is ongoing necessity of further advancement of control of electric drives containing flexible elements causing undesirable effects in its behaviour.

II. CONTROLLED SYSTEM

The object of interest is a DC motor (DCM) containing a coupling with flexible elements which was chosen as an analysed mechatronic system (Fig. 1). A special type of drive – torque motor – was analysed. This type of drive is characterized by high torsion moment at relatively low speed.

A parasitic effects caused by resonant frequency and antiresonant frequency in the connection with flexible elements were eliminated by double notch filter. If the inertia of the transmission mechanisms is low compared to the motor and the load, the flexible coupling between the motor and the load can be treated as a two-mass motor/load system [4], [7].

Transfer function of an armature in DCM electrical part depicted in Fig. 2 is:

\[ G_{arm}(s) = \frac{1}{R_a (U_a / R_a) + 1} \]  

(1)

Transfer function of a flexible coupling of two mass depicted in Fig. 3 is:

\[ G_{f}(s) = \frac{J_1 s^2 + b_1 s + d_{12}}{J_M J_1 s^3 + b_1 (J_M + J_1) s^2 + d_{12} (J_M + J_1) s} \]  

(2)
Transfer function of a double notch filter is:

\[ G_F(s) = \frac{1/\omega_n^2 s^2 + \left(2\xi_n/\omega_n\right)s + 1}{1/\omega_n^2 s^2 + \left(2\xi_n/\omega_n\right)s + 1} \tag{3} \]

where resonant frequency and resonance damping are:

\[ \omega_n = \sqrt{\frac{d_{11} J_M + J_L}{J_M J_L}}, \quad \xi_n = \frac{b_{12}}{2\sqrt{d_{11} J_M J_L}}, \]

and antiresonant frequency and antiresonance damping are:

\[ \omega_a = \sqrt{\frac{d_{11}}{J_L}}, \quad \xi_a = \frac{b_{12}}{2\sqrt{d_{11} J_L}}. \]

### III. CONTROL SYSTEM

Different methods for controlling the system design were applied [1], [2], [4]-[7]. The control system design was based on idealised conditions with infinitely rigid connection being considered instead of flexible connection between the actuator and the load.

#### A. Feedback Control of Angular Velocity

Transfer function of system depicted in Fig. 4 is:

\[ G_1(s) = \frac{G_F(s) G_{DCM_{-FJ}}(s)}{1 + G_F(s) G_{DCM_{-FJ}}(s)} \tag{4} \]

where \(G_F(s)\) is controller (6), (7), (9) and transfer function of a DCM with flexible joint with/without filter is:

\[ G_{DCM_{-FJ}}(s) = \frac{G_{arm}(s) \Phi G_{FJ}(s) G_F(s)}{1 + G_{arm}(s) \Phi G_{FJ}(s) G_F(s)} \tag{5} \]

1) PI Controller – Naslin Method

Coefficients for controller of PI type in form (6) calculated based on solution of inequalities derived from system closed control loop are in Table I.

2) PID Controller – Modulus Optimum Method

Coefficients for controller of PID type in form (7) calculated based on solution of equations system in matrix form derived from open control loop frequency response are in Table II.

3) PID Controller – Method of Inverse Dynamics

For possibility to calculate controller coefficients by Method of Inverse Dynamics was the transfer function of an actuator rigidly connected with a load modified to the form:

\[ G_{DCM} = \frac{K}{T_0^2 s^2 + 2\xi T_0 s + 1} \tag{8} \]

The coefficients of PID controller in form (9) calculated based on this method are in Table III.

#### B. Cascade Control of the Mechatronic System

Transfer function of system depicted in Fig. 5 is:

\[ G_2(s) = \frac{G_{R2}(s) G_{CL2}(s) \Phi G_{FJ}(s) G_F(s)}{1 + G_{R2}(s) G_{CL2}(s) \Phi G_{FJ}(s) G_F(s)} \tag{10} \]

where \(G_{CL2}(s)\) is speed controller (13), (14), transfer function of a current closed control loop is:

\[ G_{CL2}(s) = \frac{G_{R1}(s) G_{arm}(s)}{1 + G_{R1}(s) G_{arm}(s)} \tag{11} \]
GR(s) is current controller (12).

Fig. 5 Model of DCM cascade control

1) Current Controller
The tuning method based on the Modulus Optimum optimization criterion was used for current controller design. According to this criterion was specified a PI controller in form:

\[ R_{PI} = k_{PI} \frac{T_{pI}s + 1}{T_{pI}s} \]  

(12)

The coefficients of cascade control subordinate PI current controller designed by Modulus Optimum Method are in Table IV.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>k_{PI}</td>
<td>0.025</td>
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<tr>
<td>T_{pI}</td>
<td>5</td>
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</tbody>
</table>

TABLE IV
PI (MOM) CURRENT CONTROLLER PARAMETERS

2) Speed Controller I
The tuning method based on the Symmetric Optimum Criterion was used for first version of speed controller design. According to this method was specified a PID controller in form:

\[ R_{PID} = k_{PID} + \frac{T_{PID}s + 1}{T_{PID}s} + T_{d} \frac{s}{T_{d}s} \]  

(13)

The coefficients of cascade control master PID speed controller designed by Symmetric Optimum Criterion are in Table V.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
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<td>k_{PID}</td>
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<tr>
<td>T_{PID}</td>
<td>7.2917</td>
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<tr>
<td>T_{d}</td>
<td>4083.3</td>
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</table>

TABLE V
PID (SOC) CURRENT CONTROLLER PARAMETERS

3) Speed Controller II
The tuning method based on the Modulus Optimum criterion was used for second version of speed controller design. According to this method was specified a PD controller in form:

\[ R_{PD} = k_{PD} \left(1 + T_{d} \frac{s}{T_{d}s}\right) \]  

(14)

The coefficients of cascade control master PD speed controller designed by Modulus Optimum Method are in Table VI.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
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<td>k_{PD}</td>
<td>58.3333</td>
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<tr>
<td>T_{d}</td>
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</tbody>
</table>

TABLE VI
PD (MOM) CURRENT CONTROLLER PARAMETERS

IV. SYSTEM SENSITIVITY
The behaviour of the system is described by its characteristic functions, i.e. characteristics, the system sensitivity analysis is based on a sensitivity analysis of its characteristic functions.[3], [8], [9]

If a system is given by the input \( u(\xi) \) and the response \( y(\xi) \), it can be expressed by:

\[ y(\xi) = F\{\xi, \alpha, u_0, u(\xi)\} \]  

(15)

and the relative differential sensitivity expressing the percentage change in the system function for a 1% change in the value of the parameter is given by:

\[ S_\alpha(\xi, \alpha_k) = \frac{\partial \ln F\{\xi, \alpha\}}{\partial \ln \alpha_k} S_\alpha(\xi, \alpha_k) \]  

(17)

V. SYSTEM TOLERANCE
The system tolerance expresses a change in system parameters, which does not alter the behavior of the system beyond the permitted respectively the desired behavior. [3], [8], [9] If the system behaviour is described by (15) and change of system response for defined input \( u_0, u(\xi) \) with the...
change of the $k$th parameter $\alpha_k$ is expressed as:

$$\Delta y(\xi, \alpha_k) = \frac{\partial F(\xi, \alpha_k)}{\partial \alpha_k} \Delta \alpha_k$$  \hspace{1cm} (19)

If the permitted change is denoted as $\Delta y(\xi, \alpha_k)$, then permitted change of parameter $\alpha_k$ tolerance is possible to express (19) as:

$$\Delta \alpha_k = \frac{1}{S(\xi, \alpha_k)} \Delta y(\xi, \alpha_k)$$  \hspace{1cm} (20)

where $S(\xi, \alpha_k)$ according to (16) is absolute differential sensitivity of the system to changes in the parameter $\alpha_k$.

The tolerance of the system response was analysed based on the transfer function of closed-loop control system according to (4) and (10).

The tolerance of the frequency-response characteristic was analysed based on the frequency response of closed-loop control system representing the system function (18).

The Symbolic Math Toolbox [11] was used for expression of system function for sensitivity and tolerance analysis. In designed tool were utilised this toolbox functions:
- syms for symbolic variables definition
- poly2sym for transform polynomial coefficient vector to symbolic polynomial
- subs for symbolic variables substitution
- diff for expression derivation
- eval for expression evaluation
- sym2poly for symbolic-to-numeric polynomial conversion
- numden for numerator and denominator extraction of expression

VII. CONCLUSION

The result of this work is the tool in Matlab GUI form that is designed to analyse a mechatronic system sensitivity and
tolerance. Designed tool allows an analysis of influence of torsion stiffness, torsion damping, inertia moment of the motor, inertia moment of the load and controllers parameters (for different methods of control system design) in terms of the impact of these parameter change on the response in the form of system step response and system frequency-response logarithmic characteristics.

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REFERENCES