Forecast of the Small Wind Turbines Sales with Replacement Purchases and with or without Account of Price Changes

V. Churkin, M. Lopatin

Abstract—The purpose of the paper is to estimate the US small wind turbines market potential and forecast the small wind turbines sales in the US. The forecasting method is based on the application of the Bass model and the generalized Bass model of innovations diffusion under replacement purchases. In the work an exponential distribution is used for modeling of replacement purchases. Only one parameter of such distribution is determined by average lifetime of small wind turbines. The identification of the model parameters is based on nonlinear regression analysis on the basis of the annual sales statistics which has been published by the American Wind Energy Association (AWEA) since 2001 up to 2012. The estimation of the US average market potential of small wind turbines (for adoption purchases) without account of price changes is 57080 (confidence interval from 49294 to 64866 at P = 0.95) under average lifetime of wind turbines 15 years, and 62402 (confidence interval from 54154 to 70648 at P = 0.95) under average lifetime of wind turbines 20 years. In the first case the explained variance is 90.7%, while in the second - 91.8%. The effect of the wind turbines price changes on their sales was estimated using generalized Bass model. This required a price forecast. To do this, the polynomial regression function, which is based on the Berkeley Lab statistics, was used. The estimation of the US average market potential of small wind turbines (for adoption purchases) in that case is 42542 (confidence interval from 32863 to 52221 at P = 0.95) under average lifetime of wind turbines 15 years, and 47426 (confidence interval from 36092 to 58760 at P = 0.95) under average lifetime of wind turbines 20 years. In the first case the explained variance is 95.3%, while in the second - 95.3%.

Keywords—Bass model, generalized Bass model, replacement purchases, sales forecasting of innovations, statistics of sales of small wind turbines in the United States.

I. INTRODUCTION

The present work is devoted to the forecast of the development of small wind energy (power of plants up to 100 kW), a feature of which is that the demand for small wind turbines is formed mainly by households.

The cost of electricity produced by wind turbines has got mainly two components - depreciation and the cost of annual operations and maintenance (O&M). Annual maintenance costs are typically 1% to 2% of installed cost. Lifetimes of wind turbines are from 10 to 20 years and more. As a result of performed R&D the cost of electricity produced by wind turbines is decreasing steadily, approaching the cost of traditional energy sources. It is also important [1] that the alternative energy creates a lot of jobs for highly skilled workers (scientists, designers, engineers, etc.). So the U.S. wind industry employs more than 400,000 people. And the number of new high-skilled workers in these industries is steadily increasing, which is a prerequisite to economic growth acceleration.

An important component of innovation and economic growth is patented activity. Researchers from MIT and nonprofit organization Santa Fe Institute with the help of colleagues from more than 100 countries, found that the new patents in the field of nuclear technology did not appear so often, at the same time there was a modest increase in new technologies for coal, oil and gas [2]. So the number of patents in the field of fossil fuels has grown at an average of 100 per year between 1975 and 2000, to about 300 per year in 2009. The same patents in technologies related to renewable energy, increased from an average of 200 patents per year between 1975 and 2000, to more than 1,000 in 2009. Simultaneously, patents in the field of wind energy rose by an average of 19% per year between 2004 and 2009.

In [3], the author gives the sales forecast of small wind turbines, based on the Bass model of diffusion of innovations (regarding macroeconomic factor) and the USA available statistics for 2001 - 2009. The purpose of the paper is to estimate the US small wind turbines market potential and forecast the small wind turbines sales in the US with replacement purchases and with or without account of price changes.

We use the U.S. statistics (as a representative of the Information Society with available statistics).

II. THE SALES FORECAST FOR THE INNOVATIVE PRODUCTION ON THE BASIS OF THE BASS DIFFUSION MODEL

A. The Basic Bass Model

In his paper Bass [4] considers diffusion of innovations. He describes the process of market adoption of new products through the interaction between those who have already made a purchase and potential buyers. This model is widely used in forecast, especially in predicting the distribution of products and technologies.

The essence of the Bass model (BM) is as follows [4]. Suppose there is some market with a completely new product (goods or services) appears. This product creates a new demand, i.e. there is a certain amount of people wanting to buy this product, and those who have already made a purchase...
In discrete time the BM is as follows [4]:

\[ S(t) = N(t) - N(t-1) = p[m - N(t-1)] + q[N(t-1)/m](m - N(t-1)) \]  

(1)

where \( m \) is the maximum number of potential buyers or potential market; \( N(t) \) - cumulative sales or cumulative number of customers for the time \( t \); \( S(t) \) - increase in the number of customers or sales in the period \( t \); \( p \) is the coefficient of innovation; \( q \) is the coefficient of imitation.

Potential buyers are composed of two groups - innovators and imitators. Thus, purchases in the period \( t \), \( S(t) \), are modeled as the sum of purchases of innovators and imitators. The number of innovators in period \( t \) is proportional to the remaining number of those who have not purchased from the number of potential buyers at the beginning of period \( (t-1) \), i.e. \( p(m - N(t-1)) \), while the number of imitators is in proportion to the number of those who have not purchased, and the share of those who have already purchased, i.e. \( q[N(t-1)/m](m - N(t-1)) \).

When making initial purchases innovators are not guided by the number of previous buyers, and mainly by the massmedia (the so-called external influence) that is incorporated in the coefficient of innovation \( p \). At the same time the number of imitators are influenced by the previous buyers, the effect of word of mouth (the so-called internal influence), as reflected by coefficient of imitation \( q \).

The basic model of Bass in a continuous time is

\[ f(t) = \frac{[p + qF(t)]^\prime}{1 - F(t)} \]  

(2)

where \( F(t) \) is a cumulative share of buyers in time \( t \) (with respect to the market potential, \( F(t) = N(t)/m \); \( f(t) = dF(t)/dt \) is a function of density (intensity) of the purchases at the time \( t \); \( f(t) = dF(t)/dt = S(t)/m = (1/m)dN(t)/dt \).

The differential equation (2) refers to the Riccati equation which solution in this case has the form [4]

\[ F(t) = \frac{(1 - e^{-(p+q)t})}{(1 + qe^{-(p+q)t})/p} \]  

(3)

where \( c \) is a constant determined by the initial condition. For the zero initial condition, we have

\[ F(t) = (1 - e^{-(p+q)t})/((1 + qe^{-(p+q)t})/p) \]  

(4)

\[ N(t) = m(1 - e^{-(p+q)t})/(1 + qe^{-(p+q)t}) \]  

(5)

\[ f(t) = (p + q)e^{-(p+q)t} [1 + q/p]/(1 + qe^{-(p+q)t}/p) \].

If \( p < q \), then the curve \( F(t) \) has an inflection point \( t^* \), in which the derivative of \( f(t) \) is zero, then

\[ t^* = -(1/(p + q)) \ln(p/q) \]  

(6)

This point has the following parameters

\[ N(t^*) = mF(t^*) = (m/2)(1 - (p/q)) \]  

(7)

\[ dN(t)/dt = mf(t^*) = m(p + q)^2/(4q) \]  

(8)

If \( p < q \), then (4) gives the so-called S-shaped (sigmoidal) curve. The graph \( f(t) \) is symmetrical on the interval \([0, 2t^*] \) with respect to the point \( t^* \), i.e. \( f(t^* - t) = f(t^* + t), t \in [0, t^*] \). In the case of \( p \geq q \) the function \( f(t) \) is a monotonically decreasing function of time.

B. Identification of the Parameters

Let us briefly examine procedures for identifying the parameters of Bass model [1]. A series of procedures was proposed to estimate the BM parameters \( p \), \( q \) and \( m \). In the paper [5] the authors provide a comparison of four assessment procedures: the estimates based on ordinary least squares (OLS) [4], estimates based on non-linear least squares method (NLS) [6], estimates based on maximum likelihood (MLE) [7], estimates based on the algebraic method (AE) [8].

For OLS evaluation linear regression analysis is used. The main advantage of OLS estimation procedures is in the ease of its implementation. One of the well-known approaches to obtaining NLS estimates \( p \), \( q \) and \( m \) is described in the work [6]. The authors defining \( x(t) \) as the number of buyers in the period \( t \), use

\[ N(t) - N(t-1) = m[F(t) - F(t-1)] + \varepsilon(t), \]  

(9)

where \( \varepsilon(t) \) is independently distributed error term and \( F(t) \) is the cumulative distribution function of buyers (4). Then using the algorithm of the method of nonlinear least squares (9) and (4) they obtain evaluation parameters \( p \), \( q \), \( m \). NLS estimates is not unbiased, but is only consistent (i.e., asymptotically converges in probability with increasing sample size). As noted, this method calculates the standard errors of the estimated parameters and eliminates the temporal offset inherent in OLS procedure. However, it is sensitive to the initial values of the solution (for a preliminary search of the initial parameter values OLS procedure is recommended). Strictly speaking, NLS estimates can be quite bad and biased when obtained from a data set with a few and noisy observations. In [5], comparison of 4 methods was performed on several datasets. The authors of this study concluded that the NLS provides the best and most effective parameters estimates (in terms of the standard error). On the other hand, the method OLS is the most straightforward to implement. In any case, the authors note [5] that the stable and robust (independent of the shape of the distribution) parameters estimates can be obtained only if the source data have at least ten observations and contain the peak of the curve \( f(t) \). However it can be noted that the expectation of a sufficient number of measurements reduces the usefulness of the forecast.
C. The Generalized Bass Model

In [9], the authors describe the following generalized Bass model (GBM)

\[
\frac{f(t)}{1-F(t)} = \left\{ p + q F(t) \left( 1 + \beta_1 \frac{P'(t)}{Pr(t)} + \beta_2 \frac{A'(t)}{A(t)} \right) \right\}
\]

(10)

where an additional factor on the right side of the equation is designed to simulate the marketing mix variables. Namely, \(P'(t)\) and \(A(t)\) is the price and the cost of advertising at time \(t\), \(P'(t)\) and \(A'(t)\) — the rate of change prices and advertising costs at time \(t\), respectively.

It is clear that the GBM has emerged as a compromise on the one hand; it allows obtaining the solution of (10) in closed form expression, and, on the other hand, in a special way to take into accounting the impact of additional processes on the innovations.

Let us note, that in addition to the marketing mix variables in the model other factors can be taken into account, for example, tax incentives, tariffs, macroeconomic factors, etc. [1]. The following is a solution of (10) for an arbitrary finite number of factors \(A_i\), \(i = 1, ..., n\):

\[
F(t) = \frac{1 - \exp\left\{ -(p + q) t \sum_{i=1}^{n} \alpha_i \ln A_i(t) \right\}}{1 + \frac{q}{p} \exp\left\{ -(p + q) t \sum_{i=1}^{n} \alpha_i \ln A_i(t) \right\}}
\]

(11)

From (11) we obtain the following expression for the density

\[
f(t) = \frac{(p + q)}{p} \left\{ \sum_{i=1}^{n} \alpha_i \frac{A_i(t)}{A(t)} \right\} \cdot \exp\left\{ -(p + q) t \sum_{i=1}^{n} \alpha_i \ln A_i(t) \right\}
\]

\[
\times \frac{1 + \frac{q}{p} \exp\left\{ -(p + q) t \sum_{i=1}^{n} \alpha_i \ln A_i(t) \right\}}{1 - \exp\left\{ -(p + q) t \sum_{i=1}^{n} \alpha_i \ln A_i(t) \right\}}.
\]

(12)

There is an interesting property of GBM: if the percentage change in price and advertising remain the same from period to period (i.e. exponential growth or decrease), then, as we see from (11), we actually come to BM [4].

III. The Sales Forecast of the Small Wind Turbines in the United States on the Basis of the Diffusion Model

A. Input Data

What do we know about statistics of small wind turbines? The U.S. market is the most famous. Nevertheless, there is no sufficient certainty. Industrial production and sale of wind turbines began in 1979. According to [10], the dynamics of sales of small wind turbines in the United States you can see in Fig. 1.

Additional data are given in the DOE report [11] and are shown in Fig. 2.

Thus, in 1978 there were no sales, i.e. \(N(1978) = 0\), or moving the reference time, \(N(0) = 0\). The problem is that there was no further information on the sales until 2000, and

the data from 2001, had been presented only in the form of increments - \(S(t)\).

![Fig. 1 The sale statistics of small wind turbines in the U.S. (10 year included) [10]](image)

![Fig. 2 The sales statistics of small wind turbines in the U.S. (2012 year included) [11]](image)

![Fig. 3 Installed project costs of wind turbines over time [12]](image)

In this paper we analyze the effect of prices of small wind turbines on the dynamics of sales. In [12], there are the averages and the graph of the polynomial trend line of the installed project costs of wind turbines over time.

As [13] states: "... apparently weak relationship between project size and cost", it is believed that Fig. 3 is also valid for the small wind turbines.

B. Results

After the expiration of lifetime the wind turbine is replaced. In addition to purchases of innovators and imitators (adoption purchases), according to the model of Bass, there are purchases associated with the replacement (replacement purchases). In general, there are adoption purchases, replacement purchases, and multiple purchases. If households buy a second product, but continue to use the original product, it is believed that the new product belongs to multiple purchases.

The published statistics do not share these three types of purchases and contain aggregated sales. This complicates the forecast. Although the majority of diffusion models are applied to aggregated sales, [14] noted that they may be acceptable in the early stages of the product life cycle. However, at a later stage, this application leads to an overestimation of external influence in the process of

\[
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\]
diffusion. It is a vast area of research of innovative products markets models. Among the first works [15] assumed that the overall purchases consist of two components: adoption purchases and replacement purchases. In this paper a Rayleigh distribution for modelling the replacement purchases is used. A similar model was proposed in [16], but for modelling replacement purchases they use a truncated normal distribution. In our work, in addition to adoption purchases, consider only the replacement purchases (the probability of buying wind turbines, in addition to working purchases, consider only the replacement purchases (the purchases and replacement purchases. In this paper a Rayleigh period (t91, t] adoption purchases occur. They can also cause the consequence, the number of replacement purchases in the time can write the expression for the average number of average number in the unit interval equals N(t91)λ. Thus, we interval of finite length obeys the Poisson distribution, and the exponential distribution with parameter N(t91)λ. As a distribution function to the replacement of one of them has an immediate replaces them with entirely new ones. Then, if wind turbine, the user remains committed to wind turbines and the product is absent. It is not important for the exponential important because, the initial sales statistics often lack for the annual aggregate purchases during the period t, then

\[ Y(t) = N(t) + r(t) \]  

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As a result of integration we obtain:

\[ Y(t) = N(t) + \lambda \int_{0}^{t} N(x)dx. \]  

Using (4) we get:

\[ Y(t) = N(t) + \lambda \times \left( 1 + \frac{q}{p} \right) \]  

or

\[ Y(t) = N(t) + \lambda \ln[\phi(t) - \phi(t - 1)]. \]  

Next

\[ \phi(t) - \phi(t - 1) = 1 + \frac{q}{p} \]  

To identify the parameters on the basis of annual aggregate sales we use (4), (19), (20):

\[ Y(t) = N(t) + \lambda \times \left( 1 + \frac{q}{p} e^{-(p+q)t} \right) \]  

We will carry out simple sensitivity analysis of the resulting estimates of model parameters and forecasts sales of small wind turbines on their average lifetime (for two values of 15 and 20 years).

The estimation of the US average market potential of small wind turbines (for adoption purchases) without account of price changes is 57080 (confidence interval from 49294 to 64866 at P = 0.95) under average lifetime of wind turbines 15 years (λ=0.067 year), and 62402 (confidence interval from 54155 to 70648 at P = 0.95) under average lifetime of wind turbines 20 years (λ=0.05 year). In the first case the explained variance is 90.7%, while in the second - 91.8%.

The results of the model parameter identification are obtained in the program Statistica, they are shown in Table I.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>RESULTS OF PARAMETER IDENTIFICATION OF BM BASED ON AGGREGATIVE ANNUAL SALES (THOUSANDS UNITS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ = 15 years</td>
<td>λ = 20 years</td>
</tr>
<tr>
<td><strong>Estimate</strong></td>
<td><strong>Standard - error</strong></td>
</tr>
<tr>
<td>m</td>
<td>57.0803146</td>
</tr>
<tr>
<td>p</td>
<td>0.00000008</td>
</tr>
<tr>
<td>q</td>
<td>0.52814101</td>
</tr>
</tbody>
</table>

The results of calculations with (3), (21) and the found parameters are shown in Fig. 4.

Graph of \(Y_i\) in Fig. 4 is not shown because it is virtually identical to \(Y_i\) and \(Y_i\).

Next, we will examine the effect of the small wind turbines price changes on their sales using the generalized Bass model. This requires a price forecast. To do this, on the basis of digitizing the averages from Fig. 3, the polynomial regression function of prices is obtained:

\[
\text{poly_reg}(t) = 7,340592 - 0,722092 \cdot t + 0,028682 \cdot t^2 - 0,000362 \cdot t^3.
\]  

![Fig. 4 The forecast of sales of small wind turbines in the USA: \(Y_1\) - aggregative annual sales; \(Y_1\) and \(Y_1\) - the forecast of aggregative cumulative sales for \(\lambda = 15\) and 20 years correspondingly; \(Y_1\) - the forecast of aggregative annual sales for \(\lambda = 15\) years; \(Y_1\) and \(Y_1\) - the forecast of cumulative adoption purchases for \(\lambda = 15\) and 20 years correspondingly.](image)

Bad property of the GBM solution is that it depends on the factor \(A(t)\) measure scale [1]. For certainty, the scale can be taken so that \(A(0) = 1\) (in the general case of a multitude of factors \(A(t)\), \(t = 1, \ldots, n\)). This scale is realized by dividing the prices shown in tab. 3 on the extrapolation of \(A(0)\) from (22). Logarithm of the values thus obtained is taken so that \(A(t) = 1\) (in the general case of a multitude of factors). For certainty, the scale can be taken so that \(A(t) = 1\) (in the general case of a multitude of factors). For certainty, the scale can be taken so that \(A(t) = 1\) (in the general case of a multitude of factors). For certainty, the scale can be taken so that \(A(t) = 1\) (in the general case of a multitude of factors).

Substituting (23) into (11) and substituting the result of this in (17) and (19), we obtain:

\[
Y(t) - Y(t - 1) = N(t) - N(t - 1) + \lambda m[\varphi_p(t) - \varphi_p(t - 1)],
\]  

where if \(t > 16\)

\[
\varphi_p(t) = \frac{t - 1}{1} + \frac{1}{p + q} \left[ \frac{1}{p} + \frac{1}{q} \right] \frac{1}{1 + 0,1a} \times
\]

\[
1 - \frac{q}{p} e^{-(p+q)(t-1,5)} + \frac{1}{1 + 0,1a} \times
\]

\[
1 + \frac{q}{p} e^{-(p+q)(t-1,5)}.
\]  

When it is considered that, \(p\) is much less than \(q\), then we get a simple relation:

\[
\varphi_p(t) = \frac{t - 1}{1} + \frac{1}{p + q} \left[ \frac{1}{p} + \frac{1}{q} \right] \frac{1}{1 + 0,1a} \times
\]

\[
1 + \frac{q}{p} e^{-(p+q)(t-1,5)}.
\]  

And finally

\[
\varphi_p(t) - \varphi_p(t - 1) = 1 + \frac{1}{q} \left[ 1 - \frac{q}{p} e^{-(p+q)(t-1,5)} \right].
\]  

We use (11) and (24), (27) and \(\ln\regres(t)\) for the identification of the model parameters on the basis of annual aggregate sales and the small wind turbines price changes. Now for the problem of regression analysis two independent variables \(t\) and \(\ln\regres\) are used.

The estimation of the US average market potential of small wind turbines (for adoption purchases) in that case is 42542 (confidence interval from 36092 to 58760 at \(P = 0.95\)) under average lifetime of wind turbines 20 years, and 47426 (confidence interval from 32863 to 52221 at \(P = 0.95\)) under average lifetime of wind turbines 15 years. In the first case the explained variance is 95.3\%, while in the second – 95.3\%. The results of the model parameter identification are shown in Table II.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard Error</th>
<th>Lower Conf - Limit</th>
<th>Upper Conf - Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>42,542019</td>
<td>4,27871064</td>
<td>32,8629031</td>
<td>52,2211349</td>
</tr>
<tr>
<td>(p)</td>
<td>0,023841E-23</td>
<td>0,023841E-23</td>
<td>0,023841E-23</td>
<td>0,023841E-23</td>
</tr>
<tr>
<td>(q)</td>
<td>0,771216622</td>
<td>0,10369208</td>
<td>0,53648824</td>
<td>1,00578442</td>
</tr>
<tr>
<td>(a)</td>
<td>-25,2166288</td>
<td>5,91690208</td>
<td>-38,6015912</td>
<td>-11,8316664</td>
</tr>
<tr>
<td>(m)</td>
<td>47,4262402</td>
<td>5,01020066</td>
<td>36,0923789</td>
<td>58,7601016</td>
</tr>
<tr>
<td>(p)</td>
<td>1,040351E-20</td>
<td>1,040351E-20</td>
<td>1,040351E-20</td>
<td>1,040351E-20</td>
</tr>
<tr>
<td>(q)</td>
<td>0,723619247</td>
<td>0,09803856</td>
<td>0,501840599</td>
<td>0,945397895</td>
</tr>
<tr>
<td>(a)</td>
<td>-23,0491778</td>
<td>6,34681419</td>
<td>-37,406609</td>
<td>-8,69162663</td>
</tr>
</tbody>
</table>

The results of calculations with (11), (24) and the found parameters are shown in Fig. 5.

In Fig. 5 the cumulative curves of adoption purchases have got the stabilization beginning from \(t = 35\) (2013 year). But in Fig. 4 it does not occur yet. This can be explained by the fact that the point of maximum penetration has already been passed and, in addition, according to the forecast of prices (22) the period of high prices extends (according to [12] it began early in the century).
parameters (excluding the coefficient of innovation) and based on GBM (Table II) and the likely scenario of price parameters values for BM and GBM (Tables I, II) one can forecasts of small wind turbines sales showed their weak to a constant (equal to $\lambda \cdot m$) (Figs. 4, 5).

With time the annual adoption purchases converge to zero (in the mean), and the aggregative annual purchases converge to a constant (equal to $\lambda \cdot m$) (Figs. 4, 5).

According to the results given in Tables I and II, the coefficient of innovation has an extremely small value. Analyzing the applications of the Bass model to other innovative products and in particular to photovoltaic systems [1], we can see the following trend: the more expensive the product - the less active the innovators are and the lower coefficient of innovation is. In general, the solution of nonlinear regression problems is associated with the study of a coefficient of innovation is. In general, the solution of nonlinear regression problems is associated with the study of a

...the simple sensitivity analysis of the obtained model parameters (excluding the coefficient of innovation) and forecasts of small wind turbines sales showed their weak sensitivity to average lifetime of wind turbines (Tables I, II), (Figs. 4, 5).

IV. CONCLUSION

The simple sensitivity analysis of the obtained model parameters (excluding the coefficient of innovation) and forecasts of small wind turbines sales showed their weak sensitivity to average lifetime of wind turbines (Tables I, II), (Figs. 4, 5).

With time the annual adoption purchases converge to zero (in the mean), and the aggregative annual purchases converge to a constant (equal to $\lambda \cdot m$) (Figs. 4, 5).

According to the results given in Tables I and II, the coefficient of innovation has an extremely small value. Analyzing the applications of the Bass model to other innovative products and in particular to photovoltaic systems [1], we can see the following trend: the more expensive the product - the less active the innovators are and the lower coefficient of innovation is. In general, the solution of nonlinear regression problems is associated with the study of a number of local minima of the sum of squared errors. This task is complicated by the increasing number of estimated parameters. In particular, this occurs when the parameters are dependent (strongly correlated, redundant).

Nevertheless, in all calculations for different initial values of the parameters and the presence of the convergence, the estimations of the parameters differ very little from the estimations, presented in Tables I and II.

The practical side of this work is that by using the found parameters values for BM and GBM (Tables I, II) one can more accurately assess the market potential of small wind turbines in the USA in view of their lifetime. Furthermore, based on GBM (Table II) and the likely scenario of price changes (similar to forecast prices (22)) corresponding sales forecast of small wind turbines can be obtained.

REFERENCES


