Realization of Soliton Phase Characteristics in 10 Gbps, Single Channel, Uncompensated Telecommunication System

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Abstract—In this paper, the dependence of soliton pulses with respect to phase in a 10Gbps, single channel, dispersion uncompensated telecommunication system was studied. The characteristic feature of periodic soliton interaction was noted at the Interaction point (I=6202.5Km) in one collision length of L=12405.1 Km. The interaction point is located for 10Gbps system with an initial relative spacing (\(q_0\)) of soliton as 5.28 using Perturbation theory. It is shown that, when two in-phase solitons are launched, they interact at the point I=6202.5 Km, but the interaction could be restricted with introduction of different phase initially. When the phase of the input solitons increases, the deviation of soliton pulses at the ‘I’ also increases. We have successfully demonstrated this effect in a telecommunication set-up in terms of Quality factor (Q), where the Q=0 for in-phase soliton. The Q was noted to be 125.9, 38.63, 47.53, 59.60, 161.37, and 78.04 for different phases such as 10\(^\circ\), 20\(^\circ\), 30\(^\circ\), 45\(^\circ\), 60\(^\circ\) and 90\(^\circ\) degrees respectively at Interaction point (I).

Keywords—Soliton interaction, Initial relative spacing, phase, Perturbation theory and telecommunication system.

I. INTRODUCTION

Solitons are self-trapped wave packets which can travel long distance with clear balance between dispersion and nonlinearity [1]. Solitons exists in various media like sound waves, liquid waves, matter waves, charge density waves and electromagnetic waves [2]. Solitons in fiber was demonstrated theoretically by [3], later the effect was noted experimentally by [4]. Although, the fascinating nature of soliton travelling dispersion less for long distance transmission, it suffers from collision at particular distance with respect to initial spacing between them and phase at the initial level. This behavior of solitons mimicking the particle nature was been a major hurdle in transforming soliton to practical perspective [5].

Aitchison et al. gives the picture of soliton interaction with respect to in-phase and out of phase of launched pulse experimentally [6]. He demonstrates that the solitons interaction is avoided at interaction point with respect to introduction of phase. Sonja Zentner et al. simulated the idea of soliton interaction with a sequence of three pulses and noted the impact of particle behavior with an input pulse of T\(_p\)=2 ps for propagation length of 20L\(_0\) [7]. Branimir Jakšić et al. clearly demonstrated the two soliton pulse response with respect to the phase and also amplitude [8]. He also shows the avoidance of soliton interaction by imparting different amplitude. Various authors have demonstrated the interaction pattern theoretically where, Konar et al. gives the intra-channel interaction in Kerr nonlinearity [9]. Recently Mitschke et al. has suggested the possibility of soliton beyond binary level, which could help in providing high bitrate to the system [10]. So, proper investigations on phase characteristics of soliton are needed to ensure the future phase shift keyed soliton transmission.

We have demonstrated the soliton interaction with various telecom fibers like SMF, LEAF, TERALIGHT and TRUEWAVE+ in our previous work with in-phase soliton in 100 Gbps system [11] to note the controllability of soliton interaction with respect to nonlinear co-efficient and dispersion following [12]. Following above said authors, we now demonstrate the soliton characteristics and realized the effect in a basic single channel telecommunication system. The results are characterized in terms of Q-Factor.

The paper is organized as follows, in second section, the basic concept of soliton and interaction is presented. In third section, the values used for theoretical analysis and simulation are shown. Following, in fourth section, the analysis of soliton interaction with respect to phase and amplitude is noted by Perturbation theory. In fifth section, the results are discussed and finally the conclusion part is presented.

II. BASIC THEORY OF SOLITON PROPAGATION AND THEIR INTERACTION NATURE

Solitons are self-trapped beams of finite cross section travel without divergence associated with the freely diffracting beams. This is only possible, if the medium response in such a way that it acts as a self-focusing mechanism for the light. The self-trapping is a consequence of strong interaction between the electromagnetic wave and the medium, with the wave modifying the medium locally and in response being modified.

It could be noted from Fig. 1, where, the very narrow beam travelling in a medium that does not respond to the light, or that do not affect the medium, gets diffracted and broadens with the distance. In case of nonlinear material, the light modifies the medium (varying like refractive index, absorption, conversion to other frequencies etc.) such that the refractive index resembles accordance to the intensity of the light. So, the refractive index becomes high for the peak of the pulse comparatively to the tail resulting in formation of optical lens. Because of this lens effect, the beam becomes narrower and narrower as it travels along the length which forms a precursor to the soliton formation. When the self-focusing is
balanced by the diffraction, then the concept of soliton emerges [13].

Among the soliton properties, interaction due to the peculiar nature as resembling the massive particle can be coherent or incoherent. Coherent response occurs when the nonlinear medium responds instantaneously to the interference effect between the overlapping beams normally happens in Kerr or a quadratic nonlinear medium. An in-phase beam constructively builds the intensity which increases the refractive index which is balanced by the diffraction, then the concept of soliton emerges [13].

So, interaction occurs. For out-of phase solitons, they overlap destructively, resulting in deviation of the centroid due to the decrease in the refracting index induced in the waveguide (Fig. 2 (b)).

![Fig. 1 Basic characteristics of light exhibiting various physical mechanisms in linear and nonlinear media](image1)

![Fig. 2 Characteristic feature of in-phase (a) and out-of phase (b) soliton in interaction and repulsion](image2)

Similarly, incoherent interaction occurs when the relative phase between the beams varies much faster than the response time of the medium which is not a case in coherent soliton interaction to be discussed.

### III. SYSTEM SET-UP AND DESIGN

In this section, the simulation parameters and design flows are discussed. The system is operated with the bitrate of 10Gbps, where the bitslot can be noted as inverse of bitrate. But the Full Width Half Maximum (FWHM) of the pulse is related to the initial pulse width as,

\[ T_{FWHM} = 2 \ln(1 + \sqrt{2}) T_0 \approx 1.763 T_0 \]  

From (1) the initial pulse width is calculated. In order to launch soliton the Dispersion length \( L_D \) and Nonlinear length \( L_{NL} \) must be equal. The Dispersion length and Nonlinear length can be given as \( L_D = \frac{T_0^2}{|\beta_2|} \) and \( L_{NL} = 1/\gamma P_0 \) respectively, where \( \beta_2 \) is the dispersion parameter, \( \gamma \) is the nonlinear coefficient, \( \gamma = n_2 \alpha_0 / c A_{eff} \) where, \( n_2, \alpha_0, c, \) and \( A_{eff} \) represents to non-linear refractive index, frequency, speed of the light and effective area of fiber respectively. The power required to launch soliton can be calculated from (2), where for fundamental soliton \( N=1, \)

\[ N^2 \geq \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} \]  

It is to be noted, that the soliton pulse experience their originality with constant phase and intensity for a period called Soliton period \( (T_0) \) which is given in (3)

\[ T_0 = \frac{n T_0^2}{2 |\beta_2|} \approx \frac{T_{FWHM}}{|\beta_2|} \]  

The collision period is the distance at which the soliton interacts and retains their initial state after collision. Since the soliton interaction is periodic and depends on initial relative spacing \( (q_0) \) between them and phase, they can be derived by means of Perturbation theory as, (see Section IV)

\[ L_{coll} = \frac{n}{2} L_D \exp(q_0) \]  

The values fixed for the fiber is given in Table I, while the simulation parameters calculated from (1)-(4) are noted in Table II, and the simulation set-up is shown in Fig. 3. The mode-locked laser is responsible for creating the soliton pulses which are hyperbolic secant pulses. The bits from the Psuedo Random Binary Sequence (PRBS) generator are coded into pulses by Line coder. Here, we use Return-to-zero (RZ) format to, where only half of the bitslot is utilized. The Mach-Zehnder type modulator is used as optical modulator here, where the optical pulses are generated by the concept of constructive or destructive phase. The voltages to the plates of modulator are controlled by the line coder such that the laser is modulated according to the coded electrical pulses such that the optical pulses are born and injected into the fiber. The optical pulses are converted back to electrical sequence by PIN type photo detector with the responsivity of 1 A/W. The electrical pulses are filtered by a low pass Bessel filter with an order of 4. The cut-off frequency of the filter is 0.75xbitrate. Then the 3R generator is used to acquire the pulses with good quality where the reshaping, retiming and re-amplification are done. Finally, the electrical pulses are analyzed for their qualitative transmission by a Bit-error rate analyzer and Time domain analyzer. The bit sequence length and number of
samples per bit are 16 and 64 such that the number of samples per second is 1024.

\[
\text{PRBS} \rightarrow \text{LINE CODER} \rightarrow \text{MODE LOCKED LASER} \rightarrow \text{AMPLIFIER} \rightarrow \text{SMF} \rightarrow \text{FILTER} \rightarrow \text{PHOTODETECTOR} \rightarrow \text{BER ANALYZER}
\]

Fig. 3 Simulation set-up for 10 GbpsRZ keyed, single channel, uncompensated telecommunication system

### Table I

<table>
<thead>
<tr>
<th>S.no</th>
<th>Parameters</th>
<th>Values</th>
<th>Units</th>
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<tr>
<td>1</td>
<td>Reference wavelength</td>
<td>1550</td>
<td>nm</td>
</tr>
<tr>
<td>2</td>
<td>Dispersion Coefficient</td>
<td>-20</td>
<td>ps²/Km</td>
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<tr>
<td>3</td>
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<td>ps/nm/Km</td>
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<td>4</td>
<td>Attenuation</td>
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<td>dB/Km</td>
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<td>5</td>
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<td>6</td>
<td>Non-linear index</td>
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<td>m²/W</td>
</tr>
<tr>
<td>7</td>
<td>Mode field diameter</td>
<td>9.2-10</td>
<td>µm</td>
</tr>
</tbody>
</table>

The pulse tracing inside the nonlinear fiber can be given by Nonlinear Schrödinger’s equation as follows \[14\],

\[
\frac{\partial A(x,t)}{\partial x} = \frac{-i}{2} \frac{\partial^2 A(x,t)}{\partial t^2} + \beta_2 \frac{\partial^4 A(x,t)}{\partial t^4} + \beta_3 \frac{\partial^6 A(x,t)}{\partial t^6} - \gamma |A(x,t)|^2 A(x,t) \tag{5}
\]

where \(\alpha\) represents the attenuation or loss in the fiber, \(\beta_2\) represents the second order dispersion which corresponds to parameter Dispersion co-efficient (D) by \(\beta_2 = -\lambda^2 D / 2 \pi c\), \(\beta_3\) represents third order dispersion which corresponds to Dispersion Slope (S) by \(\beta_3 = -\lambda^4 S / (2 \pi c)^2\) and \(\gamma = -\eta \omega_0^2 / c \alpha_{eff}\) represents the non-linearity in the fiber. From (1), the attenuation and third order dispersion is neglected for the soliton formation, so the equation becomes,

\[
\frac{\partial A}{\partial t} + \frac{\partial^2 A}{\partial t^2} - \gamma |A|^2 A = 0 \tag{6}
\]

The inverse of bit spacing \((1 / T_B)\) in a system represents the total bit rate. The importance of collision comes to the effect on estimating how far the distance between two bits is spaced. Interaction of soliton can be studied by considering two fields such that the total field is given as \(u = u_1 + u_2\), where

\[
u_i(\xi, \tau) = \eta_j \left[ \eta_j (\tau - q_j) \right] \exp(i \phi_j - i \delta_j t) \tag{7}
\]

It is to be noted that \(u\) satisfies NLS equation rather than individually by \(u_1\) and \(u_2\). The NLS equation for the two overlapping fields can be given as,

\[
i \frac{\partial u_1}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u_1}{\partial \tau^2} + |u_1|^2 u_1 = -2 |u_1|^2 u_2 - u_1^2 u_2^* \tag{8}
\]

\[
i \frac{\partial u_2}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u_2}{\partial \tau^2} + |u_2|^2 u_2 = -2 |u_2|^2 u_1 - u_1^2 u_1^* \tag{9}
\]

In (8) and (9) the terms present in the right hand side of the equation acts as the perturbation and are responsible for non-linear interaction between the two solitons. Now, by introducing soliton parameters such as \(\eta, \delta, \phi\) and \(q\) and performing algebraic calculations, we form two equations as,

\[
\frac{\partial^2 q}{\partial \xi^2} = -4 e^{-2 i \phi} \cos(2 \psi) \tag{10}
\]

\[
\frac{\partial^2 \psi}{\partial \xi^2} = -4 e^{-2 i \phi} \sin(2 \psi) \tag{11}
\]

Equations (10) and (11) are solved mathematically and considered for the two solitons are in same frequency and amplitude,

\[
q(\xi) = q_o + \frac{1}{2} \ln[ \cos h^2 (2 \xi e^{-\phi} \sin \psi_o) + \cos^2 (2 \xi e^{-\phi} \cos \psi_o)] - 1 \tag{12}
\]

In (12), \(q_o\) and \(\psi_o\) represents the initial spacing and phase respectively. It is seen that for a certain value of \(\psi_o\), the q becomes zero which is known as collision. On considering in-phase soliton \((\phi=0)\), the relative spacing \((q)\) changes periodically during the propagation as,

\[
q(\xi) = q_o + \ln[ \cos (2 \xi e^{-\phi})] \tag{13}
\]

For the in-phase soliton, q becomes zero at the time of
collision which can be formulated as,
\[ \xi = \frac{1}{2} e^{\psi_0 (cos) - 1} (e^{-\psi_0}) = \frac{\alpha}{4} \exp(q_o) \]  \hspace{1cm} (14)

The above approximate value is valid for \( q_o > 5 \). Due to the periodic nature of \( q(\xi) \) in (13), the solitons collide and separate each other periodically, thus forming oscillatory motion. Thereby, this oscillatory period is called as collision length \( (L_{coll}) \). It is given as follows,
\[ L_{coll} = \frac{\pi}{2} L_D \exp(q_o) \equiv z_o \exp(q_o) \]  \hspace{1cm} (15)

where, \( z_o \) is the soliton period and can be given as,
\[ z_o = \frac{\pi}{2} L_D = \frac{\pi}{2} \left[ \frac{1}{|\psi_0^2|} \right] = \frac{T_{WHM}}{2|\beta_0|} \]  \hspace{1cm} (16)

Equation (15) gives an approximate value of collision distance, but a more accurate model is given by using inverse scattering theory as in [15],
\[ \frac{L_{coll}}{L_D} = \frac{\pi \sinh(2q_o) \cosh(q_o)}{2q_o \sinh(2q_o)} \]  \hspace{1cm} (17)

The above equation finally gives the clear picture on one collision length of solitons.

V. RESULTS AND DISCUSSIONS

In this section, firstly, the effect of soliton tracing is simulated for one soliton period of \( z_o = 63.2 \) Km with an input power of \( P_o = 17.53 \) mW and in second part the soliton interaction is studied.

A. Soliton in one Soliton Period

Fig. 5 shows the soliton tracing in 10 Gbps system. It could be seen that the peak power is constant along the distance and undisturbed due to the dispersion in the fiber.

Fig. 5 (a) Formation of soliton pulse for a distance of one soliton period \( z_o = 63.2 \) Km with an input power of \( P_o = 17.5 \) mW in 10 Gbps system (\( T_o = 28.4 \) ps)

Fig. 5 (b) Contour plot of Fig. 5 (a)

Fig. 4 Numerical analysis of two solitons with different phase launched with an initial relative spacing of \( q_o = 5.28 \)

Fig. 4 gives the analysis on phase characteristics of soliton. The solitons were separated initially at \( q_o = 5.28 \) and they get attracted or repelled depending upon the phase for a normalized distance (\( \xi \)). From the figure, we could clearly visualize the soliton interacted at the interaction point 6202.5 Km for 10Gbps system, which is also demonstrated [11]. When the launched solitons are with phase of 10°, 15°, 45° and 90°, the interaction was avoided at \( I_o \) and the deviation increases with respect to increase in phase.
The pulse normally travelling in the fiber with anomalous regime ($\beta_2 < 0$ or $D > 0$) has blue frequencies in the leading edge and red frequencies in the trailing edge resulting in broadening of pulse. But with an introduction of power that generates more red frequencies in the leading edge can prevent this broadening. The Self-phase modulation is responsible for this generation of red frequencies helping in the cancellation of chirp induced by dispersion. Fig. 6 (a) shows the unchirped soliton launched and Fig. 6 (b) shows that the instantaneous frequency is constant across the pulse. This proves that the soliton pulse has sustained by cancelling the positive (SPM) and negative chirps (GVD).

Fig. 7 gives the phase characteristics of soliton where the pulse with zero initial phases has sustained even after the depicted length. The soliton, we simulated perfectly respect (5) without considering the attenuation and third order dispersion. It must be noted that in the simulation set-up in Fig. 3, we use the amplifier only to overcome the loss and spontaneous emission (ASE) noise of the amplifier is not considered such that the fiber can be treated lossless. Since the third order disturbs the soliton stability, its effect also neglected [16].

**B. Soliton Phase Dependence in One Collision Period**

In this section, the soliton pairs are noted for one collision length of $L_{coll} = 12405.1$ Km with an interaction point of $I_p = 6202.5$ Km. Fig. 8 shows the phase dependence of solitons in interaction and repelling each other. Fig. 8 (a) clearly shows the two in-phase soliton interacted at $I_p$ we calculated using Perturbation theory in Section IV, such that the interaction point was successfully located. When the solitons are introduced with phase shifts the interaction was avoided with respect to value of phase. Figs. 8 (b)-(f) show the increase in deviation of pulse with respect to phase angles of 10°, 20°, 30°, 45°, 60° and 90° respectively.
Fig. 8 (a) Soliton pairs tracing the fiber of length 12405.1 Km with an initial phase of 0° separated initially $q_0 = 5.28$ in 10 Gbps system (Left column shows the 3D view while in right shows the contour plot).

Fig. 8 (b) Soliton pairs tracing the fiber of length 12405.1 Km with an initial phase of 10° separated initially $q_0 = 5.28$ in 10 Gbps system. (Left column shows the 3D view while in right shows the contour plot).

Fig. 8 (c) Soliton pairs tracing the fiber of length 12405.1 Km with an initial phase of 20° separated initially $q_0 = 5.28$ in 10 Gbps system. (Left column shows the 3D view while in right shows the contour plot).
Fig. 8 (d) Soliton pairs tracing the fiber of length 12405.1 Km with an initial phase 30° separated initially $q_o=5.28$ in 10 Gbps system. (Left column shows the 3D view while in right shows the contour plot)

Fig. 8 (e) Soliton pairs tracing the fiber of length 12405.1 Km with an initial phase of 45° separated initially $q_o=5.28$ in 10 Gbps system. (Left column shows the 3D view while in right shows the contour plot)

Fig. 8 (f) Soliton pairs tracing the fiber of length 12405.1 Km with an initial phase of 60° separated initially $q_o=5.28$ in 10 Gbps system. (Left column shows the 3D view while in right shows the contour plot)
Fig. 8 (g) Soliton pairs tracing the fiber of length 12405.1 Km with an initial phase of 90° separated initially \(q_0=5.28\) in 10 Gbps system. (Left column shows the 3D view while in right shows the contour plot)

Fig. 9 (a) Time domain analyzer view of soliton pairs tracing the fiber of length 12405.1 Km with an initial phase of 0° separated initially \(q_0=5.28\) in 10 Gbps system

Fig. 9 (b) Time domain analyzer view of soliton pairs tracing the fiber of length 12405.1 Km with an initial phase 10° separated initially \(q_0=5.28\) in 10 Gbps system

Fig. 9 (c) Time domain analyzer view of soliton pairs tracing the fiber of length 12405.1 Km with an initial phase 20° separated initially \(q_0=5.28\) in 10 Gbps system

Fig. 9 (d) Time domain analyzer view of soliton pairs tracing the fiber of length 12405.1 Km with an initial phase 30° separated initially \(q_0=5.28\) in 10 Gbps system
The interaction and repulsion is mainly concerned with the coherent beam that is constructively or destructively builds. The coherent solitons highly interferes when they have same phase resulting in doubling of initial amplitude. Fig. 9 (a) gives the two soliton interacted pulse where the power is double the initial power of 17.53 mW. But the mismatch in the phase results in respective destructive interference leading to restrict interaction. From Figs. 9 (b)-(g) it could be seen that the power level of the soliton pair is not equal which says there were no clear separation at interaction point but this could not affect the system performance more. It must be noted that from Fig. 9 (h), that at 90° there is a clear separation which demonstrates there was more deviation of pulse at this interaction.

Now the effect of phase is analyzed at the output by BER analyzer. Fig. 10 shows the performance in terms of Q factor. For in-phase soliton the Q value is perfectly zero at L_{coll}=6202.4 Km depicting there exist bit error due to overlapping of pulse. As the soliton interaction is periodic, after interaction, they return to their initial state with Q~278 at the end of fiber length of one collision period (L_{coll}= 12405.1 Km). From the analysis it could be seen that for phases 10°, 20°, 30° and 45°, we have fair Q-value all along the distance where they yielded Q of 125.9, 47.5, 38.6, and 59.6 respectively. But when soliton launched with 60° and 90°, there was bit error at the transmission distance. It can be well identified that these high phase values apart from avoiding the interaction they deviate more such that they interact with the soliton pulses in the
adjacent bit slot. The soliton pulses experiences bit error at less distance for 90° than 60° which states the pulses for 90° deviates more comparatively resulting in interaction with the adjacent soliton pulses.

VI. CONCLUSION

The effect of soliton phase in 10 Gbps, single channel telecommunication system was demonstrated. It could be shown that the introduction of phase can be a fascinating technique in avoiding the peculiar nature of soliton interaction but care must be taken in imparting phase to the soliton pulses. The transmission of soliton pulses to long distance is possible by technique in avoiding the peculiar nature of soliton interaction shown that the introduction of phase can be a fascinating telecommunication system was demonstrated. It could be

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