Study of Adaptive Filtering Algorithms and the Equalization of Radio Mobile Channel

Said Elkassimi, Said Safi, B. Manaut

Abstract—This paper presented a study of three algorithms, the equalization algorithm to equalize the transmission channel with ZF and MMSE criteria, application of channel Bran A, and adaptive filtering algorithms LMS and RLS to estimate the parameters of the equalizer filter, i.e. move to the channel estimation and therefore reflect the temporal variations of the channel, and reduce the error in the transmitted signal. So far the performance of the algorithm equalizer with ZF and MMSE criteria both in the case without noise, a comparison of performance of the LMS and RLS algorithm.

Keywords—Adaptive filtering second equalizer, LMS, RLS Bran A, Proakis (B) MMSE, ZF.

I. INTRODUCTION

ADAPTIVE filtering [6] is based on finding optimal parameters by minimizing a performance criterion.

Frequently, this minimization is done by seeking the least squares. The performances of digital transmission system [3], [9] are expressed in terms of reliability. This may be achieved by:

- The coding of channel, or correct coding of error,
- Equalization, which allows to make the most the pass band of the channel offsetting receipt [8] the distortions introduced by the transmission medium, electronic equipment, etc...
- There are two approaches:
  - The adaptive approach to switch to the channel estimation [10] and therefore take into account the temporal variations of the channel,
  - A suboptimal approach called LEVELS.

In this paper, we study the adaptive filtering algorithms such as LMS and RLS algorithms to estimate the coefficients of the FIR filter \( h_E \) in the noisy cases [4], and the equalization algorithm based with ZF and MMSE criteria [7].

II. ADAPTIVE EQUALIZATION

The equalization approach has some drawbacks related to the need for accurate channel estimation and calculation of the correlation matrix of the received data and its inverse [5]. On the other hand, if the channel varies in time, this approach does not allow adjusting the coefficients of the equalizer [1]. In fact, the transversal equalizer on the MSE criterion is based on minimizing the function:

\[
J(h_E) = E[(a_k - z_k)^2] \tag{1}
\]

It is therefore necessary to calculate the gradient as:

\[
\nabla J(h_E) = 2(R_yh_E - R_{ay}) = 0 \tag{2}
\]

This leads to a complexity in costly analytical solution:

\[
H_E = R^{-1}_y R_{ay} \tag{3}
\]

In the adaptive approach, one can dispense with the channel estimation and therefore take into account the temporal variations of the channel [10]:

A. LMS «Least Mean Square» Algorithm

In the implementation of the MSE criterion, an alternative to avoid reverse of \( R_y \) is to apply an iterative method to calculate the coefficients that minimize the cost function: \( J(h_E) \).

From the values of \( h_E(k-1) \) the values can be calculated from \( h_E(k) \) using the algorithm of the gradient:

\[
h_E(k) = h_E(k-1) + \mu (R_{ay} - R_y h_E(k-1)) \tag{4}
\]

With \( \mu \) positive constant called the coefficient adaptation (replacing \( R_y^{-1} \)) for controlling the convergence. However, the calculation of \( \nabla J(h_E(k)) \) always requires knowledge \( R_y \) and \( R_{ay} \) by using a training sequence.

It then modifies the algorithm by replacing the gradient by its estimated (LMS is a gradient algorithm called "stochastic" and not deterministic). Is replaced at each step \( R_y \) and \( R_{ay} \) estimated by \( y_k \cdot y_k^T \) and \( a_k \cdot y_k \). The equation becomes:

\[
h_E(k) = h_E(k-1) + \mu (a_k - y_k^T h_E(k-1)) y_k = h_E(k-1) + \mu (a_k - z_k) y_k = h_E(k-1) + \mu a_k y_k \tag{5}
\]

The error signal \( e_k \) represents the desired difference between the data at time \( k \) and the actual output \( z(kT) \).

The LMS allows every moment to "update" the equalizer filter coefficients in proportion to the estimation error \( e_k \).

In case of variations of the channel, the equalizer will be able to adapt more rapidly than the constant \( \mu \) is greater. It can be assumed that an adequate value \( \mu \) to ensure convergence in the case of channels with slow variations is: \( \mu = \frac{0.2}{(2N+1)^{2N+1}} \) with \( (2N+1) \) number of coefficients of the equalizer, \( P_s \) signal power and \( P_n \) noise power. We can summarize the LMS algorithm in the following diagram:
or $\bar{w}_k$ is a sample of additive Gaussian noise centered (AWGN) of variance $\sigma^2 = E(\bar{w}_k^2)$.

The general idea is to apply an equalizer filter $H_e(k)$ to the samples $y_k$ compensate for the equivalent channel $G(k)$ (Fig.3).

The problem is: what criteria to choose $H_e(k)$?

Consider a transverse filter for $(2N+1)$ coefficients, transverse equalizers [12] are the easiest to implement. Indeed, this is simply to use a digital finite impulse response filter [9] for which the methods of calculation and implementation are well known.

$$z(k) = \sum_{n=-N}^{N} y(-n)h_{e,n}$$  \hspace{1cm} (10)

$k$ represents the time flowing from -2N à 2N for $(2N+1)$ input samples. We can write the relation of convolution matrix form: $Z = YH_e$. With:

$$Z = \begin{pmatrix} z(-2N) \\ z(-2N+1) \\ \vdots \\ z(0) \\ \vdots \\ z(2N) \end{pmatrix}$$  \hspace{1cm} (11)

column vector of dimension (4N+1)

$$H_e = \begin{pmatrix} h_{e,-N} \\ h_{e,-N+1} \\ \vdots \\ h_{e,0} \\ h_{e,1} \\ \vdots \\ h_{e,N} \end{pmatrix}$$  \hspace{1cm} (12)

column vector of dimension (2N+1)

$$Y = \begin{pmatrix} y(-N) & 0 & 0 & \ldots & \ldots & \ldots & 0 \\ y(-N+1) & y(-N) & 0 & \ldots & \ldots & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ y(N) & y(N-1) & y(N-2) & \ldots & \ldots & \ldots & y(-N-1) \\ 0 & 0 & 0 & \ldots & \ldots & \ldots & y(N-1) \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \ldots & \ldots & y(N) \end{pmatrix}$$  \hspace{1cm} (13)

Is a matrix of dimension $(4N+1) \times (2N+1)$ the purpose of the equalization algorithm [7] is to determine the coefficients $[h_{e,n}]$ to minimize the error probability $P_e$, and remove the IES, this algorithm is based on the criteria «Zéro-Forcing» (ZF) and «Minimum Mean Square Error» (MMSE).

IV. SIMULATION AND COMPARISON

A. Performance of the Algorithm of the Equalizer

1. The Equalization Algorithm Based on ZF Criterion

ZF criterion is applied with equalization, for comparing the output of the equalizer, with the channel Bran A [3] response, in the environment noise [10 and 30 dB]
From results obtained it can be seen that the algorithm of the equalizer with ZF criterion gives a satisfactory equalization Bran A channel, consequently, it reduces the effect of noise.

**B. The Equalization Algorithm Based On MMSE Criterion**

We test the performance of the algorithm equalizer with MMSE criterion, with and without noise, to the Bran A channel; values of the SNR by 10 and 30 dB. The algorithm equalization with the MMSE criteria; gives a good equalization of Bran A channel; then the criterion of Mean Square Error (MMSE) criterion is a more robust with respect to noise. It enables a compromise between reducing noise and the interference between the symbols (IES) (Fig. 8).
V. COMPARISON OF RESULTS

From the simulation results, we see that the equalizer obtained by the criterion of MMSE is better than that provided by the criterion ZF, due to the effective inclusion of noise.

VI. PERFORMANCE OF THE LMS AND RLS ALGORITHM

In this section we will make a comparison between the two algorithms of the LMS adaptive equalization and RLS are studied previously for that. Consider the channel Proakis (B) [2], and a modulation amplitude states 4 (4-ASK), with equalization coefficients 9.

It was found by applying the algorithm of the equalizer coefficient values for SNR=50 dB and the two ZF and MMSE criteria:

<table>
<thead>
<tr>
<th>Table I</th>
<th>Coefficients calculated by the algorithm of the equalizer with the MMSE criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_k$</td>
<td>Coefficients $EQM$ : $h_k$</td>
</tr>
<tr>
<td>0.0652</td>
<td>-0.1480 0.2814 -0.4215 1.4793</td>
</tr>
<tr>
<td>-0.4228</td>
<td>0.2832 -0.1524 0.0705</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table II</th>
<th>Coefficients calculated by the algorithm of the equalizer with ZF criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_k$</td>
<td>Coefficients $ZF$ : $h_k$</td>
</tr>
<tr>
<td>0.0816</td>
<td>-0.1640 0.2958 -0.4346 1.4921</td>
</tr>
<tr>
<td>-0.4365</td>
<td>0.2986 -0.1698 0.0883</td>
</tr>
</tbody>
</table>

A. Performance of the LMS Algorithm

The values of the coefficients $h_k$ calculated by the LMS algorithm at the last iteration are:

<table>
<thead>
<tr>
<th>Table III</th>
<th>The coefficients calculated by the LMS adaptation algorithm with $\mu=0.0053$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Coefficients $LMS$ : $\alpha$</td>
</tr>
<tr>
<td>0.0606</td>
<td>-0.1416 0.2679 0.2679 1.4707</td>
</tr>
<tr>
<td>-0.4099</td>
<td>0.2667 -0.1381 0.0603</td>
</tr>
</tbody>
</table>

From Fig. 11 we see that the error signal $e_k$ is low when the number of iterations is important ($M=7000$). And from Figs. 12 and 13 we notice that for a low pitch results in slow convergence. A strong will not lead to closer than results.

Fig. 9 Comparison of the channel response Bran A and the sortie equalizer with the MMSE criterion in the SNR=30 dB cases

Fig. 10 Comparison of the channel response Bran A, and the sortie Equalizer with the MMSE and ZF criterion in the SNR=10 dB cases

Fig. 11 The variation of the error based on iteration number $M=7000$

Fig. 12 The convergence of the equalizer filter coefficients with no convergence of the LMS, $\mu=0.0053$
obtained by the algorithm equalization with criterion MMSE (Tables I and III). The LMS allows every moment to "update" the equalizer filter coefficients in proportion to the estimation error $e_k$. In case of variations of the channel, the equalizer will be able to adapt more rapidly than the constant $u$ is large.

![Convergence of the Equalizer Filter Coefficients](image1)

**Fig. 13** The convergence of the equalizer filter coefficients with no convergence of the LMS, $u = 0.002$

**B. Performance of the RLS Algorithm**

The values of the coefficients $h_k$ calculated by the RLS algorithm adaptation at the last iteration are:

<table>
<thead>
<tr>
<th>TABLE IV</th>
<th>THE COEFFICIENTS CALCULATED BY THE ADAPTATION ALGORITHM WITH RLS, $u = 0.0053$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.0634$</td>
<td>$-0.1536$</td>
</tr>
<tr>
<td>$0.3318$</td>
<td>$-0.6546$</td>
</tr>
<tr>
<td>$1.2883$</td>
<td></td>
</tr>
<tr>
<td>$-0.0626$</td>
<td>$0.0028$</td>
</tr>
<tr>
<td>$-0.0042$</td>
<td>$0.0055$</td>
</tr>
</tbody>
</table>

![Error Against the Number of Iterations](image2)

**Fig. 14** The variation of the error $e_k$ against the number of iterations $M=7000$ with $u = 0.002$

Figs. 14 and 15 show different results with $u = 0.0053$ and $u = 0.002$, we note that the estimate of the error $e_k$ is tends to rapidly to low values when the number of iterations $M$ and $u$ are stronger. Then filter the RLS algorithm is performed correctly, it means that all influences of the noise were suppressed.

![Convergence of the Equalizer Filter Coefficients](image3)

**Fig. 15** The variation of the error $e_k$ against the number of iterations $M=7000$ with $u = 0.002$

![Convergence of the Equalizer Filter Coefficients](image4)

**Fig. 16** The convergence of the equalizer filter coefficients with $u = 0.0053$

![Convergence of the Equalizer Filter Coefficients](image5)

**Fig. 17** The convergence of the equalizer filter coefficients with $u = 0.002$
The curves in Figs. 16 and 17 show the variation of the filter coefficients depending on numbers of iterations, we find that for a low pitch, slow convergence is obtained.

C. Comparison between the LMS and RLS

From simulation results, we see that the LMS converges quickly compared to the RLS algorithm because only one control parameter (the $\mu$ adaptation) and will lead to results closer to that obtained by the algorithm of the equalizer with the MMSE criterion (Tables I, III, and IV). There is another difference between the LMS and RLS is in the term $P(k)$, which allows you to update various coefficients and gives the superiority of the RLS algorithm in terms of speed of convergence but time is running slower.

VII. CONCLUSION

In this paper we presented three algorithms, the first algorithm to equalize the channel Bran A; with the two criteria ZF and MMSE, and the other two algorithms for estimating the parameters of the equalizer filter adjust the channel and reduce the error signal.

Simulation results show that the algorithm of the equalizer is able to equalize the channel Bran A with the MMSE criterion, due to the effective inclusion of noise. Thus the adaptive LMS filter algorithm converges quickly with respect to the RLS algorithm because of the adaptation step, another difference between the LMS and RLS is within the term $P(k)$, which gives a superiority of RLS algorithm in terms of speed of convergence but time is running slower.

REFERENCES


Said Elkassimi obtained the B.Sc. degree in Physics and M.Sc. degree in computer science from polydisciplinary faculty, in 2009 and from Faculty of Science and Technics Beni Mellal, Morocco, in 2012, respectively. Now he is Ph.D student and his research interests include communications, wide-band wireless communication systems, traitement du signal, et l'identification du système.

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