Study of Natural Convection Heat Transfer of Plate-Fin Heat Sink in a Closed Enclosure

Han-Taw Chen, Tzu-Hsiang Lin, Chung-Hou Lai

Abstract—The present study applies the inverse method and three-dimensional CFD commercial software in conjunction with the experimental temperature data to investigate the heat transfer and fluid flow characteristics of the plate-fin heat sink in a rectangular closed enclosure. The inverse method with the finite difference method and the experimental temperature data is applied to determine the approximate heat transfer coefficient. Later, based on the obtained results, the zero-equation turbulence model is used to obtain the heat transfer and fluid flow characteristics between two fins. To validate the accuracy of the results obtained, the comparison of the heat transfer coefficient is made. The obtained temperature at selected measurement locations of the fin is also compared with experimental data. The effect of the height of the rectangular enclosure on the obtained results is discussed.

Keywords—Inverse method, FLUENT, Plate-fin heat sink, Heat transfer characteristics.

I. INTRODUCTION

Natural convection heat transfer with heat sink has been widely used in engineering applications. In recent years, with the rapid development of electronic technology, promoting the heat transfer rate under the working process at the desired operating temperature may play an important role to ensure reliable operation of the electronic components. The appropriate design of the heat sink has gradually become attractive for these applications because they provide a more economical solution to the above problem.

A great amount of research has been devoted to determine natural convection heat transfer between two plate-fins. Numerous investigations have been conducted to study heat transfer and empirical correlation of rectangular fins. Bodota and Osterle [1] applied numerical method to study the heat transfer and flow characteristic between two vertical plate-fins. Baskaya [2] applied numerical method to investigate the optimum fin spacing for the plate fins on a horizontal plate. Harahap and Setio [3] obtained empirical correlation for the plate fins on the horizontal and vertical plate by the experiment. For many engineering applications, the heat sink was often placed in a closed enclosure. This has forced many researchers to study the natural convection heat transfer and fluid flow in a closed enclosure. Nada [4] obtained empirical relationship of natural convection heat transfer in the horizontal and vertical closed narrow enclosure with the fins on a heated rectangular plate by the experiment. Yalcin [5] applied the finite volume method to investigate the optimum fin spacing and fin height in limited enclosure. Tari and Mehtash [6], [7] applied ANSYS FLUENT solver with the zero-equation turbulence model to investigate the natural convection heat transfer from the inclined plate-fin heat sink. The test aluminum fins with two different lengths, three different heights and the thickness of 3 mm are used. A non-conformal mesh structure with a very fine grid around the cooling assembly and a coarse grid for the rest of the room is employed. Grid independence is achieved by examining three different grid densities. Then the medium density mesh with 2,834,264 cells is selected to obtain the numerical results. To handle the radiative heat transfer, the emissivity of the aluminum heat sink is 0.2. The results show that their proposed empirical formulas are in good agreement with literature data. The optimum fin spacing values are within 8.8-9.9 mm and 12.3-13.9 mm ranges for the downward and upward facing horizontal cases, respectively. It can be found as [3], [7] that the calculated temperature is not compared with the experimental data at the selected measurement locations on the fin. Recently, Chen [8] used the inverse method, ANSYS FLUENT solver with the standard k-ε turbulence model and the experimental data to investigate the natural convection heat transfer from the upward horizontal plate-fin heat sink in a closed enclosure. The length, width and height of the rectangular enclosure are 0.39 m, 0.16 m and 0.39 m, respectively. The test fins with 0.1 mm in length and 0.001 m in thickness is made of AISI 304 stainless material. It can be found as [8] that, in order to obtain more accurate numerical results, the comparison between the experimental data and the calculated temperature can be required at the selected measurement locations of the fin.

In order to investigate the effect of the flow model on the numerical results obtained, the present study applies the inverse method and commercial software FLUENT [9] with two different turbulence models and the experimental temperature measurements to determine the heat transfer and flow characteristics between two fins mounted on a heated horizontal plate in a closed enclosure. The zero-equation turbulence model is introduced. In order to validate the accuracy of the results obtained, a comparison of the average heat transfer coefficient between the present results and those obtained from the correlation will be made. The calculated temperature is also compared with the experimental data at the selected measurement locations of the middle fin.

II. MATHEMATICAL FORMULATION OF INVERSE SCHEME

The two-dimensional inverse heat conduction problem is first introduced to estimate the unknown heat transfer coefficient on the middle fin of the upward horizontal plate-fin.
heat sink for various values of the fin spacing. The temperature of the fin at the selected measurement locations and the ambient air temperature are measured from the experimental apparatus constructed in a closed rectangular enclosure. The inverse method in conjunction with the finite difference method, the experimental temperature data and the least squares method is used to predict the heat transfer coefficient on the vertical fins mounted on a heated horizontal plate. Due to the assumption of non-uniform distribution of heat transfer coefficient, the entire fin is divided into several sub-fin regions before performing an inverse method. The heat transfer coefficient in each sub-fin region is assumed to be an unknown constant. The measurement locations and sub-fin regions is shown in Fig. 1.

![Fig. 1 Physical geometry of the measurement locations and sub-fin regions](image)

The total surface area of the edge of the fin relative to the total surface area of the fin is small enough in the present study. This implies that the actual heat transfer rate dissipated through the fin tip is much smaller than the total heat transfer rate drawn from the fin base. Thus, the boundary conditions at the edge surface of the fin may be assumed to be insulated [10], [11]. Under the assumption of steady state and constant thermal properties, the heat conduction equation for a thin fin can be expressed as:

\[
\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{2h(x,y)}{k_f}(T - T_0) , \quad 0 < x < L , \quad 0 < y < H
\]  

where \(x\) and \(y\) are the Cartesian coordinates. \(L\), \(H\) and \(t\) denote the length, height and thickness of the rectangular fins, respectively. \(h(x,y)\) is the unknown heat transfer coefficient. \(k_f\) is the thermal conductivity of the fin. \(T_0\) and \(T_c\) represent the fin base temperature and the average temperature of the top surface temperature of the enclosure respectively.

The unknown heat transfer coefficient in each sub-fin region is assumed to a constant. Thus the finite difference form as (1) in the \(k_{th}\) sub-fin region can be expressed as:

\[
\frac{(T_{i+1,j} - 2T_{i,j} + T_{i-1,j})}{L_x^2} + \frac{(T_{i,j+1} - 2T_{i,j} + T_{i,j-1})}{L_y^2} = \frac{2}{k_f} \frac{\bar{h}_{T_{i,j}}}{L_xL_y}
\]

for \(i = 1, 2, ..., N_x, j = 1, 2, ..., N_y, k = 1, 2, ..., N\) (5)

where \(N_x\) and \(N_y\) are, respectively, the number of nodes in the \(x\)- and \(y\)-directions. \(\ell_x\) and \(\ell_y\) are defined as \(\ell_x = L_x/(N_x - 1)\) and \(\ell_y = L_y/(N_y - 1)\). \(N\) is the number of sub-fin region.

The finite difference form of the boundary conditions (2)-(4) can be written as:

\[
T_{x, j} = T_{x,j} \quad \text{and} \quad T_{x,j-1} = T_{x,j-1} \quad \text{for} \quad j = 1, 2, ..., N_y
\]

and

\[
T_{i, y} = T_{i,y} \quad \text{and} \quad T_{i,N_y-1} = T_{i,N_y} \quad \text{for} \quad i = 1, 2, ..., N_x
\]

The difference equations for the nodes at the interface between two neighboring sub-fin regions and the intersection of four neighboring sub-fin regions are similar to those shown as [11], [12]. To avoid repetition, they are not shown in this manuscript. 

The rearrangement as (5) in conjunction with its corresponding difference equation yields the following matrix equation as:

\[
[K][T] = [F]
\]

where \([K]\) is the global conduction matrix. \([T]\) is the matrix representing the nodal temperatures. \([F]\) is the force matrix. The fin temperature at the selected measurement locations can be obtained from (8) using the Gauss elimination algorithm.

In order to estimate the unknown heat transfer coefficient \(\bar{h}_j\) in the \(j_{th}\) sub-fin region, additional information of the measured fin temperature at \(N\) measurement locations can be required. The more the number of the analysis sub-fin region are, the more accurate the estimate of the unknown heat transfer coefficient may be. However, it might be difficult to measure the temperature distribution on the middle fin of the present problem using the infrared thermography. Excessive thermocouples in the fin may also significantly interrupt the flow and heat transfer behaviors between two fins. Thus, the thermocouples may be applied to measure the fin temperatures at the selected measurement locations. The heat transfer rate dissipate from the \(j_{th}\) sub-fin region \(q_j\) can be written as:
\[ q_j = 2T_i \int A_j (T - T_i) dA, \text{ for } j = 1, 2, ..., N \] (9)

The average heat transfer coefficient on the fin \( h \) can be defined as:

\[ h = \left\{ \frac{\sum_{j=1}^{N_j} h_j A_j}{\int A_j (T - T_i) dA} \right\} \] (10)

and

\[ h = \left( \frac{N_j h}{\sum_{j=1}^{N_j} A_j} \right) \] (11)

Due to the limitations of the number of thermocouples, (10)-(11) may not be suitable for the present study. Thus, the \( h \) value may be approximated as [12]:

\[ h \approx \sum_{j=1}^{N_j} \frac{h_j A_j}{A_j} \] (12)

and

\[ h \approx \sum_{j=1}^{N_j} \frac{h_j}{N_j} \] (13)

where \( h_j \) denotes the heat transfer coefficient at the \( j_{th} \) grid point. \( N_j \) represents the total number of grid points on the lateral surface. The average heat transfer coefficient \( h \) on the fin is obtained from (12) for the inverse method and from (13) for the commercial software FLUENT. Due to the assumption of the constant heat transfer coefficient in each sub-fin region for inverse method, the \( h \) value obtained from commercial software FLUENT can have better accuracy than that from it.

The actual total heat transfer rate dissipated from the fin to the ambient \( Q \) can be expressed as:

\[ Q = \sum_{j=1}^{N} q_j \] (14)

The heat transfer coefficient based on the fin base temperature can be defined as:

\[ h_{iso} = \frac{Q}{2A_j (T_i - T_j)} \] (15)

The details of the unknown values \( h_j \) estimated using a least squares minimization technique can be found as [10], [11]. In order to avoid repetition, they are not shown in this manuscript. Once the \( h_j \) values for \( j = 1, 2, ..., N \) are determined, \( h \) and \( h_{iso} \) can be obtained from (12)-(15).

### III. THREE-DIMENSIONAL NUMERICAL ANALYSIS

A half section of a fin is selected for the computation of the commercial software. Under the assumptions of steady and constant thermal properties, the three-dimensional fin heat conduction equation can be expressed as:

\[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0, \quad 0 < x < L, \quad 0 < y < H, \quad 0 < z < \frac{t}{2} \] (16)

The boundary conditions at \( z = 0 \) and \( z = t/2 \) can be expressed as:

\[ -k \frac{\partial T}{\partial z} = h(T - T_i), \text{ at } z = \frac{t}{2} \] (17)

and

\[ \frac{\partial T}{\partial z} = 0, \text{ at } z = 0 \] (18)

where \( x, y \) and \( z \) are the Cartesian coordinates.

Understanding the details of the local heat transfer and fluid flow distributions between the two fins may be very important in the design of the heat sink. At the same time, in order to obtain more accurate numerical results, the computational fluid dynamics commercial software FLUENT [9] in conjunction with the appropriate flow model and the experimental temperature data is also applied to determine the heat transfer coefficient and the temperature of the fin. The ambient air with constant properties can be assumed to be incompressible. The flow between two fins of the heat sink may be assumed to be three-dimensional, symmetrical, turbulent, steady and no viscous dissipation. Due to the fin base temperature higher than the ambient temperature, the buoyancy effect and the radiation heat loss from the fins to the surrounding is considered. The zero-equation turbulence model is applied to simulate the flow between two fins.

Many researchers used various numerical methods to investigate the heat transfer characteristics of the plate-fin heat sink. However, the independence of the grid is usually assumed in the open literature. The effect of the grid points on the numerical results obtained is rarely studied. Therefore, the present study applies computational fluid dynamics commercial software FLUENT [9] with the zero-equation turbulence model to determine the heat transfer and fluid flow characteristics between two fins of the plate-fin heat sink on an upward horizontal plate.

The flow between two fins is modeled by a zero-equation turbulence model. The continuity, momentum and energy equations in the ambient air region can be expressed in tensor form as:

\[ \frac{\partial u_i}{\partial x_i} = 0 \] (19)

\[ \rho \frac{\partial u_i}{\partial x_i} + v_{eff} \frac{\partial^2 u_i}{\partial x_j^2} + g_i \beta \frac{\partial T}{\partial x_i} (T_s - T_i) \] (20)

and
where $x_i, i = 1,2,3$ respectively indicate $x, y$ and $z$. $u, p, g_i$ and $T_e$ are the component of velocity, pressure, acceleration of gravity in the $x$ direction and the air temperature, respectively. $T_e$ denotes the average temperature of the top surface of the enclosure. $\beta$ is the volumetric thermal expansion coefficient. $\delta_{ij}$ is the Kronecker delta. $\rho$, $\nu_{eff}$, $C_p$ and $k_a$ are, respectively, the density, effective kinematic viscosity, specific heat and thermal conductivity of the air. They are all assumed to be constant. $u_{off}$ and $k_a$ are defined as $u_{off} = u_i + \nu$ and $k_a = \rho \nu_i C_p / Pr_t$. $\nu$ and $v_i$ are the laminar and turbulent kinematic viscosities respectively.

The turbulent Prandtl number $Pr_t$ is given as $Pr_t \approx 0.9$ when the air flows on a plane wall surface. The turbulent kinematic viscosity $v_t$ is defined as:

$$v_t = \rho \ell_a^{-2} S$$

where the mixing length $\ell_a$ and $S$ are defined as:

$$\ell_a = \min(0.419d, 0.09W)$$

and

$$S = 2S_y S_y \frac{1}{2}$$

where $d$ is the distance from the wall, $W$ is the distance from the maximum computational domain. The mean strain rate tensor $S_y$ is defined as:

$$S_y = (\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}) \frac{1}{2}$$

A. Boundary Conditions

The matching condition of the temperature and heat flux at the fin-fluid interface can be written as:

$$T = T_0$$

and

$$\frac{k}{\delta_{ij}} \frac{\partial T}{\partial x_j} = k_a \frac{\partial T}{\partial x_j}$$

At the solid surface, the no-slip condition is specified. The fin base temperature $T_0$ is constant. The fin temperature at the selected measurement locations, $T_i$ and $T_{off}$, are measured. Other walls are insulated.

IV. EXPERIMENTAL APPARATUS

The present experiment is conducted in a closed rectangular enclosure. The length and width of the rectangular enclosure in this study are the same as [8]. In other words, its length $L_x$ and width $W_x$ are $L_x = 0.39 m$ and $W_x = 0.16 m$. The height of the rectangular enclosure is 0.16m in this study. The test fins with 0.1m in length, 0.04m in height and 0.001m in thickness is made of AISI304 stainless material. The ambient air temperature and the fin temperature are measured using the T-type thermocouple. The diameter of the T-type thermocouple is about 0.13mm. The limit of its error is $\pm 0.4%$ for $0^\circ C \leq T \leq 350^\circ C$. The schematic diagram of three parallel rectangular fins mounted on the top surface of a heated horizontal plate is shown in Fig. 2. In order to heat three parallel fins, a square heater with 0.08m in length is fixed on the bottom of this plate using the adhesive tapes (Nitto Denko Co., Ltd).

The test fins and horizontal plate enclosed with an insulating material are heated to about 7600 seconds using a 40W heater. The four thermocouples placed in a gap between the middle fin and the horizontal plate are fixed to $(L/5, 0), (2L/5, 0), (3L/5, 0)$ and $(4L/5, 0)$. The average of the four measured temperatures is taken as the fin base temperature $T_0$. The contact thermal resistance between the fin and the horizontal plate is assumed to be negligible.

V. RESULTS AND DISCUSSION

All physical properties are evaluated at the average value of the fin base temperature and ambient air temperature. All computations are performed with $H=0.04 m$, $t/L=0.01$, $S=0.02m$ and $k_r=14.9W/mK$. The measured emissivity of the fin using FT-IR spectrum 100 (Perkin Elmer Corp., Ltd) is 0.1. $N_x=21$ and $N_y=17$ are performed for the inverse method and commercial software FLUENT. Obviously, the fin height is higher than that given by [6]. The commercial software FLUENT [9] is applied to determine the fluid flow and heat transfer characteristics between two fins of the upward horizontal plate-fin heat sink. An unstructured grid system containing a non-uniform distribution of the grid points is used to determine all of the numerical results. The high density of grid points between two fins and coarse grid points in the rest of the closed enclosure are used. The fin dimensions in x-, y- and z-directions are 100mm, 40mm and 1mm, respectively. The grid system used in the present study is shown in Fig. 2.

The number of grid points in the z-direction of the fin is taken as 7 and the total number of grid points in the x-y plane is 2430.

Thus, the total number of grid points in the fin is 2,499. The total number of grid points in the closed enclosure is 110,820. The initial guess of the unknown heat transfer coefficients $h_i$ is...
taken as unity for the inverse method. The rectangular fin is divided into eight regions, i.e., \( N = 8 \). The eight thermocouples are fixed at the selected locations of the fin, as shown in Fig. 1. \( T_j^{iso} \) represents the fin temperature obtained using commercial software FLUENT [9] at the \( j \)th measurement location.

Various heat transfer correlations can be proposed for the natural convection heat transfer from the plate-fin heat sink. Among these heat transfer correlations, the correlation proposed by Nada [4] may not be suitable for this study because the value of \( S/H_e \) is not less than 0.8. \( T_0 \) compare the present results with those obtained from the correlation as [5], [6], [12], the ambient temperature is defined as \( T_\infty = (T_0 + T_c)/2 \). The hydraulic diameter \( D \) and the clearance parameter \( C \) are defined as:

\[
D = \frac{2H_e S}{(H + S/2)}
\]

(27)

and

\[
C = \frac{(H - H)}{H_\epsilon}
\]

(28)

The correlation as [5], [6], [12] are:

\[
Nu_{\epsilon} = 1.5846 \times 10^{-3} Gr_1^{0.646} \left( \frac{H_\epsilon}{H_e} \right)^{0.016} C^{0.016}
\]

(29)

\[
Nu_{\epsilon} = 0.0915 \times (Gr \cdot Pr)^{0.416} \text{ for } 250 < Gr \cdot Pr < 10^6
\]

(30)

where \( 200 \leq Ra_S \leq 6 \times 10^5 \), \( 0.016 \leq S / L \leq 0.2 \). The Rayleigh number \( Ra_S \), new modified Grashof number \( Gr \) and Nusselt numbers \( Nu \) and \( Nu_{\epsilon} \) are defined as:

\[
Ra_S = \frac{g \beta(T_e - T_\infty) S^3}{\alpha v}
\]

(32)

\[
Gr = \frac{g \beta(T_e - T_\infty) D^4}{v^2} \text{ and } Gr = Ra_S \left( \frac{H_\epsilon}{H_e} \right)^{0.016} \left( \frac{S}{H_e} \right)^{0.416}
\]

(33)

and

\[
Nu = \frac{T_e - T_\infty}{S/k_s} \text{ and } Nu_{\epsilon} = \frac{hS}{k_s}
\]

(34)

A comparison of \( T_e \), \( \bar{h} \) and \( \bar{h}^{iso} \) is shown in Table I for \( S = 0.02 \) m, \( H = 0.04 \) m and two different values of \( H_\epsilon \). It can be found that the maximum value of the clearance parameter \( C \) shown as [5] is \( 2/7 \). The present results of \( \bar{h}^{iso} \) obtained from the inverse method and commercial software FLUENT [9] agree with those obtained in the open space [5], [6], [12] for \( C = \frac{3}{4} \).

However, the value of \( \bar{h}^{iso} \) presented by [5] slightly deviates from that given by [8] for \( C = \frac{35}{39} \) because the value of \( C \) is greater than \( 2/7 \). This implies that the correlation shown as [5] may not be suitable for a larger clearance parameter. In order to determine a more accurate numerical result, the appropriate grid points are selected such that the numerical results of \( \bar{h} \) and \( \bar{h}^{iso} \) are as close as possible to the inverse results. Table I shows that the numerical results of \( \bar{h} \) and \( \bar{h}^{iso} \) obtained are in a good agreement with the inverse results. The temperatures of the fin obtained at the selected measurement locations also agree with the experimental data. It is obvious that the temperatures of the fin obtained are closer to the experimental data than that shown as [8]. This implies that the present results have better accuracy than those obtained using the standard \( k-\epsilon \) turbulence model [8]. In addition, the obtained results of \( \bar{h} \) and \( \bar{h}^{iso} \) decrease with increasing clearance parameter. This phenomenon may result from the effect of the flow model.

<table>
<thead>
<tr>
<th>Present results</th>
<th>Ref. [8]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_\epsilon = 0.16 ) m</td>
<td>( H_\epsilon = 0.39 ) m</td>
</tr>
<tr>
<td>( T_0 = 351.74 ) K</td>
<td>( T_0 = 348.32 ) K</td>
</tr>
<tr>
<td>( T_1 = 305.45 ) K</td>
<td>( T_1 = 311.92 ) K</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \bar{h} ) (W/m²-K)</th>
<th>Inverse</th>
<th>FLUENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{h}^{iso} ) (W/m²-K)</td>
<td>Inverse</td>
<td>Ref. [6]</td>
</tr>
</tbody>
</table>

\[
Ra_S \times 10^5 \quad 11.24 \quad 2.21
\]

\[
N_s \quad 110820 \quad 412587
\]

**VI. CONCLUSION**

The present study proposes the inverse method and the computational fluid dynamics commercial software FLUENT in conjunction with the experimental temperature data to determine the fluid flow and heat transfer characteristics between two fins of the plate-fin heat sink. The results show that the results of \( \bar{h} \) and \( \bar{h}^{iso} \) using a zero-equation turbulence model have better accuracy than those using the standard \( k-\epsilon \) turbulence model. The inverse and numerical results of \( \bar{h}^{iso} \) obtained are also close to those obtained from the correlation as [12]. This implies that the present results are reliable. The commercial software FLUENT in conjunction with inverse results or experimental data can be applied to obtain more accurate fluid flow and heat transfer characteristics of plate-fin.
heat sink with an appropriate flow model and grid points. This study is useful in electronics cooling applications.

ACKNOWLEDGMENT

The authors gratefully acknowledge the financial support provided by the National Science Council of the Republic of China under Grant No. NSC 102 - 2221 - E - 006 - 177-MY3. We would also like to thank Professor Chin-Hsiang Cheng at National Cheng Kung University to give us the computational fluid dynamics commercial software FLUENT.

REFERENCES