Line Heating Forming: Methodology and Application Using Kriging and Fifth Order Spline Formulations

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Abstract—In this article, a method is presented to effectively estimate the deformed shape of a thick plate due to line heating. The method uses a fifth order spline interpolation, with up to C³ continuity at specific points to compute the shape of the deformed geometry. First and second order derivatives over a surface are the resulting parameters of a given heating line on a plate. These parameters are determined through experiments and/or finite element simulations. Very accurate kriging models are fitted to real or virtual surfaces to build-up a database of maps. Maps of first and second order derivatives are then applied on numerical plate models to evaluate their evolving shapes through a sequence of heating lines. Adding an optimization process to this approach would allow determining the trajectories of heating lines needed to shape complex geometries, such as Francis turbine blades.

Keywords—Deformation, kriging, fifth order spline interpolation, first, second and third order derivatives, C³ continuity, line heating, plate forming, thermal forming.

I. INTRODUCTION

Metal forming by line heating is attractive due to the process flexibility. The design and manufacture of tools in the other processes, such as stamping, pressing and bending take time and are very costly. Since line heating uses the temperature effect to deform the plates without tooling design and external forces, the small production batch becomes less costly. Francis turbine blades are made of high strength steel and are usually manufactured using a punch and die process. Since every turbine is different from one plant to another, a new tooling is required each time when a new project is launched [1]. Therefore, Francis turbine blades that have complex shapes are made in small production batch. Traditionally, the blades are shaped by hot stamping of thick plates, but due to the high tooling cost and low production rate, the manufacturing cost is dramatically high. On the other hand, shipbuilders have used efficiently a thermal gradient method using a fifth order spline interpolation, with up to C³ continuity, to determine the trajectories of heating lines needed to shape complex geometries, such as Francis turbine blades. Adding an optimization process to this approach would allow determining the scanning path for the laser heating lines to automate this forming process but advanced techniques for finding the appropriate trajectories require substantial and complex calculations. Because of the huge dimension of hull, shipbuilders cannot afford these expensive press tooling. They developed an alternative contactless way of shaping plates taking advantage of the gradient of temperature created through the plate thickness by a heating source [2]. The process consists of increasing locally and quickly the temperature of one side of the plate with a heating source, such as a torch, to induce plastic strains in the plate. During cooling, the plate starts to deform such that the material in the heated region bends locally, and results in permanent deformation in the plate. An experienced and skilled worker could manage to shape a flat plate into a hull by repeating the process at specific locations over the surface of the workpiece. The research on metal forming process by line heating started by experiments in the early 1980’s following by numerical modeling based on finite element method. For example, Machida et al. carried experimentally local heating on a plate [3]. McCarthy reported experimental results of the effects of main parameters on the deformation of forming by laser line heating [4]. Furthermore, Arnet and Vollertsen investigated experimentally the effects of main parameters on the bending angle of convex shape by laser forming [5]. Lately, numerical investigations have been carried out. Kyrsanidi et al. developed a three-dimensional model for laser line heating [6]. Yu et al. developed a numerical model based on finite element method for laser line heating that reduced the CPU time by zone remeshing approach [7]. As this process is complex and the deformation of single pass is usually very small, heating lines and number of pass are required to be planned before running the numerical simulations, experiments and production. In addition, the process automation becomes important to increase the productivity and reduce the operator’s labor [8]. Automating the process to achieve a very accurate geometry can be overwhelming in terms of time and costs considering the countless experimental steps needed. Therefore, analytical investigations become important and are increased recently. Liu et al. developed an analytical approach to determine the scanning path for the laser heating lines design from the strain field of the curved plate [9]. Chen and Chu assessed the relationship between the stress and the temperature distributions resulting from a line heating source moving along the axial direction of a cylinder [10]. Son et al. developed an analytical model for determining the plate deformation with the heat input, material properties and plate thickness in metal forming by line heating [11]. Reutzel et al. developed a differential geometry approach based on...
fundamental coefficients to predict deformation induced by multi heat line [12].

One approach, presented in this paper, is to use the known parameters of a given heating line and to apply them at different location on a numerical model. The evolution of the plate shape can then be followed without having to realize physically expensive experiments until a satisfying plate shape can then be followed without having to realize the work of Krige. A huge advantage of the kriging method in comparison with the least squares method is that it allows the construction of a structured grid over a surface [13].

II. METHODOLOGY

A. Dual Kriging Interpolation

This method was initially developed by Krig [14] for mining exploitation in the 1950’s. The kriging method was named after Matheron [15], who conducted a rigorous study of the work of Krige. A huge advantage of the kriging method in comparison with the least squares method is that it allows surfaces to pass through all the data points of a model. In addition, surface equations are continuous and derivable, i.e. the kriging method is a very interesting approach for reverse engineering problems like the one presented in this paper. The coordinates of all points on the surface are firstly recorded in a structured database (see Fig. 1), row by row, and column by column. Each point is in the form of P (s, t) where s and t denote normalized parameters. The parameterization can be discrete or a function of a distance between successive points in Euclidean coordinate system.

\[
X_k(s) = \sum_{i=1}^{M} a_i p_i(s) + \sum_{j=1}^{N} b_j K(|s - s_j|) \tag{2}
\]

and for a row m

\[
X_m(t) = \sum_{i=1}^{k} a_i p_i(t) + \sum_{j=1}^{m} b_j K(|t - t_j|) \tag{3}
\]

where \(k\) (\(k = 1\) to \(JP\)) denotes the number of columns, and \(m\) (\(m = 1\) to \(JP\)) denotes the number of rows. The coefficients \(a_i\), \(b_j\), \(a_i\), and \(b_j\) are obtained by solving the equations \(x_{ik} = x_k(s_{ik}) = x_k(t_{ik})\) at each data point. Finally it can be shown that:

\[
x(s, t) = [x_k(s)]^T X_m(t) \tag{4}
\]

and similar functions are used for \(y(s, t)\) and \(z(s, t)\). These functions allow the construction of a structured grid over a surface [13].

Fig. 2 presents the kriging surface of a finite element (FE) model of a plate deformed by a line heating passing across its mid length. With xyz the nodes of the mesh, it is well noticeable that the kriging model passes exactly by every node of the deformed plate.

![Fig. 1 Kriging interpolation of a 3D surface](image-url)

![Fig. 2 Kriging of a FE model of a line heated plate](image-url)

B. Fifth Order Spline Interpolation

Line heating of plates generated mainly bending distortion,
and usually very small deformation occurs. Consequently, a
large number of line heating are necessary to achieve a
specific geometry. The idea is then to superpose a sequence of
line heating over the work piece to get the intended shape. A
rather simple approach for studying the step by step shape
forming is to apply the same parameters extracted from the
deformed plate after one single line heating pass. These
parameters form a map of the first and second derivatives of
the deformed shape. Applying sequentially, and at different
locations, several line heating passes to an initial flat blank
will progressively shape the flat plate into a 3D surface.

Bending is related to the radius curvature that can be
computed from the second derivative and the first one at each
point. For very small deflection, one can even assume that the
radius of curvature can be approximated with the inverse of
the second derivative. Then, in order to compute the new
shape of a flat geometry after one or several line heating
passes, the slope and the inverse of the radius of curvature
will progressively shape the flat plate into a 3D surface.

The boundary conditions at each point lead to:
\[ x(s = 0) = x_i, \quad x'(s = 0) = x_i', \quad x''(s = 0) = x_i'' (8) \]

\[ x(s = 1) = x_i, \quad x'(s = 1) = x_i', \quad x''(s = 1) = x_i'' (9) \]

Solving for the \( a_i, b_i, \) and \( c_i \) with similar equations for \( y \) and
\( z \), will give the fifth order spline drawn from \( P_i \) to \( P_j \) as shown
in Fig. 3.

A piecewise 5th order spline passing through a set of \( n \) points
implies now that the third order derivatives are equal at
each interior point. Solving the system for the curvatures at
the interior points leads to a \( C^2 \) model. At an interior point \( P_n \) in
between neighbors points \( P_{i-1} \) and \( P_{i+1} \), the rate of curvature is
given by:
\[ x_{i-1}''' = x_i'''(s = 1) \quad \text{for the interval } P_{i-1} \text{ to } P_i \]
\[ x_i''' = x_i'''(s = 0) \quad \text{for the interval } P_i \text{ to } P_{i+1} \]

Equaling (10) to (11) lead to a system of n-2 linear
equations for the unknown curvatures. Typically:
Curvature at the first point \( P_1 \):
\[ 3x''_1 - x''_1 = -x''_{1/3} - 12x_1' - 8x_1 - 20x_1 + 20x_2 \]

Curvature at an interior point \( P_i \):
\[ -x''_{i-1} + 6x''_i - x''_{i+1} = 8x_{i-1}' - 8x_{i+1}' + 20x_{i-1} - 40x_i + 20x_{i+1} \]

Curvature at the end point \( P_n \):
\[ x''_{n-1} - 3x''_n = -x''_{n/3} - 12x_{n-1}' - 20x_{n-1} + 20x_n \]

Assuming that the slopes \( x' \) and the curvatures \( x'' \) at each
data point are known, it is now possible to compute the
cordinates of the points \( P_i \), and then to evaluate the effect of
sequence of heating lines on a geometry. If third order
derivative at end points are unknown, they are simply taken as
zero. Note that at least one coordinate of a data point should
be given to ensure a non-singular system of equations.
III. 1D APPLICATION

A. Computing y Coordinates of a Rolled Plate

First of all, a test has been made with a plate rolled according to different radius of curvature. Since the parametric equation of the rolled shaped is easy to derived, first and second order derivatives can be derived too. The first and second derivatives are then applied for different radius, step by step from an initial flat plate. Results are shown in Fig. 4.

B. Correcting x Coordinates of a Rolled Plate

It is clear that y coordinates are computed correctly but since the plate is roll bent, x coordinates do not represent correctly the deformed shape. Although the plate encounters plastic deformation, its mid plane remains elastic. Then the length of the plate should remain constant, and then the initial distance between two consecutive points should remain constant too. In order to keep constant the distance from one point to another, a parabola is used to fit three consecutive points Pᵢ, Pⱼ, and Pₖ and then the arc length s of the curve along these three points is computed with (15)

\[ s = \int \sqrt{1 + \beta'^2} \, dx = \frac{\ln(\sqrt{\beta'(\beta' + 1)}) + \sqrt{\beta'(\beta' + 1)}}{2a} \]  

(15)

Pᵢ and Pₖ are then moved along the parabola until the arc length between two consecutive points equals the initial distance separating them when the plate was flat. Fig. 5 shows the results with the corrected values for x. The new computed coordinates now match very well with the theoretical ones.

IV. 2D APPLICATION

The algorithms presented for curves are now extended to surfaces. The system is now solved for all data points Pᵢⱼ located at the intersection of a row i (s parameter) and a column j (t parameter). Results are presented in Figs. 6-11 for a typical pillow shape built using bicubic Bezier splines. A second example is shown in Figs. 12-17 for a blade shape surface. For these two cases, the assumption of deformation by bending without membrane stress is kept. Note that corrections have been applied to re-compute the x and y coordinates to satisfy a constant distance from point to point along an s or a t profile.
Figs. 7-10 and Figs. 13-16 display the variation of first and second derivatives all over the surface for a given case, but only for z coordinates. Similar maps are computed for x and y coordinates (not shown here for brevity) and are used to re-
compute the new locations of the data points and consequently the achieved deformed shape. Figs. 11 and 17 show that the rebuilt surface has been adjusted by keeping constant the initial distance in between data points of the flat plate, using a similar algorithm as presented in the previous section.

V. CONCLUSION
A methodology using kriging interpolation and a fifth order piecewise spline, ensuring C⁵ continuity at every data points, has been developed for computing the geometry of a plate submitted to a set of heating lines. The initial parameters of the deformed geometry, which are the maps of first and second derivatives all over the surface, are applied to an initial flat plate and the resulting distorted shape is then computed. An algorithm has been also applied to keep the initial distance in between data points considering the assumption that the plate mainly encounters bending without significant membrane stresses. The algorithm can now be applied to determine how a surface could be distorted, knowing locally the effects of a given heating line on the slopes and the second derivatives. Further work, including experiments, will be developed for optimizing the set of heating line to achieve a targeted shape.

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REFERENCES