
K. Sandjak, B. Tiliouine

Abstract—Structural analysis of flexible pavements has been and still is currently performed using multi-layer elastic theory. However, for thinly surfaced pavements subjected to low to medium volumes of traffics, the importance of non-linear stress-strain behavior of unbound granular materials (UGM) requires the use of more sophisticated numerical models for structural design and performance of such pavements. In the present work, nonlinear unbound aggregates constitutive model is implemented within an axisymmetric finite element code developed to simulate the nonlinear behavior of pavement structures including two local aggregates of different mineralogical nature, typically used in Algerian pavements. The performance of the mechanical model is examined about its capability of representing adequately, under various conditions, the granular material non-linearity in pavement analysis. In addition, deflection data collected by Falling Weight Deflectometer (FWD) are incorporated into the analysis in order to assess the sensitivity of critical pavement design criteria and pavement design life to the constitutive model. Finally, conclusions of engineering significance are formulated.

Keywords—Nonlinear resilient behavior, unbound granular materials, RLT test results, FWD backcalculations, finite element simulations, pavement response and performance.

I. INTRODUCTION

Low to moderate volume roads in Algeria cover more than 70% of the total road network of the country [1]. These road structures typically consist of thin asphalt layers and structurally significant unbound base and sub-base granular layers over subgrade to carry the traffic load. It follows that accurate modelling of granular layer behaviour is of crucial importance for the analysis and prediction of pavement structural performance [2], [3].

The conventional way of designing a flexible pavement structure in road pavement engineering is to assume a constant stiffness over the granular layer thickness or to derive empirically this stiffness from a rigidity ratio depending on the mechanical properties of the subgrade soil [2], [4]. However, experimental evidence [5]-[7] shows clearly that the resilient modulus of an unbound granular layer is a non-linear function of the stress levels acting at the various points of the aggregates.

Because of this material non-linearity and the sensitivity of the main pavement design criteria to the likely variations of the resilient modulus of the granular layers, numerical simulation of the non-linear mechanical behaviour of unbound granular materials (UGM) need be developed [3], [8], [9]. Various constitutive models have been developed to this end [7], [9]. These resilient models can be categorized into two main classes; the resilient modulus models [6], [8], [10], [11] and the shear-volumetric strain models [3]-[5], [12], frequently used for UGM nonlinear characterization in pavement engineering.

The main objective of this paper is to examine the effect of nonlinearity in unbound materials on the response and the performance of a granular pavement. The UGM nonlinear resilient model adopted by the AASHTO manual design guide is used [8]. This model is then implemented within an axisymmetric FEM code developed herein to evaluate its influence on nonlinear analysis and structural performance of road pavements. The finite element simulation results are validated using FWD backcalculated moduli to predict critical pavement response and design life. In addition, the sensitivity of pavement design criteria and estimated pavement design life to the likely variations of unbound granular material mineralogical nature is assessed and conclusions of engineering significance are formulated.

II. ANALYSED PAVEMENT STRUCTURE

In order to study the Effect of the behaviour nonlinearity in granular layers on pavement response and performance, a typical pavement structure has been selected. Details of the analysed pavement structure, with 5 m of subgrade soil over a rigid bottom under two thicknesses of unbound granular layers and a relatively thin asphalt layer, are summarized in Table I.

<table>
<thead>
<tr>
<th>LAYER THICKNESSES OF STUDIED PAVEMENT STRUCTURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asphalt (m)</td>
</tr>
<tr>
<td>0.05</td>
</tr>
</tbody>
</table>

The structure represents, practically, a flexible pavement with structurally significant unbound granular layers commonly used as pavements subjected to low to medium traffic volumes. The pavement structure was analysed by varying the mechanical properties of two unbound granular materials with different mineralogical nature while keeping all other parameters unchanged. The subgrade soil was not considered as a stress dependent material; thus, only the stiffness-stress dependency in the unbound granular material was investigated.
III. MATERIALS AND MECHANICAL PROPERTIES

A frequently used non-destructive in-situ device commonly used to estimate the elastic moduli of the constitutive layers of a pavement structure is the Falling Weight Deflectometer (FWD). The analysis of deflection data collected from FWD in situ tests provides a relatively rapid and reliable procedure to characterize the stiffness properties of constitutive layers of an existing pavement structure. In the present study, based on the FWD measured load and deflections, issued from an experimental pavement section built as part of an intensive project on local unbound granular materials [13], the elastic moduli of the analyzed pavement structure have been backcalculated using the ELMOD software.

Various studies have been recently performed [1], [13], [14], to estimate laboratory measured resilient moduli of subgrade and base materials from FWD backcalculated moduli. The AASHTO suggests multiplying the backcalculated moduli for UGM and subgrade soils by appropriate adjustment factors to determine the Mr values used for pavement design, in accordance with the values reported in Table II [15].

| TABLE II |

<table>
<thead>
<tr>
<th>Layer type</th>
<th>Location</th>
<th>Mr/EFWD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregates</td>
<td>Between a stabilized and HMA layer</td>
<td>1.43</td>
</tr>
<tr>
<td></td>
<td>Below a PCC layer</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td>Below an HMA layer</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>Below a stabilized subgrade</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>Below an HMA or PCC layer</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>Below an unbound aggregate base</td>
<td>0.35</td>
</tr>
</tbody>
</table>

A. Asphalt

The experimental results issued from the FWD test show that the FWD backcalculated modulus of the asphalt layer and Poisson’s ratio are found to be equal to 5042 MPa and 0.35 respectively. Note that this value of asphalt layer elastic modulus is 26% greater than the average value prescribed by the Algerian manual for pavement design [1].

B. Unbound Granular Materials

The resilient modulus of pavement granular materials is a key input parameter in the analysis of flexible pavement structures [8], [14]. It is of paramount importance in UGM characterization and pavement structural performance prediction [16].

Many techniques including laboratory testing [7], [17], non-destructive in-situ investigations and correlations with empirical parameters [6], [13] were proposed to measure the resilient modulus of unbound granular materials. However, laboratory determination, in the form of Repeated Load Triaxial tests, has been regarded as the most accurate method of obtaining the resilient modulus property.

The resilient modulus (Mr) is defined as the ratio of the stress deviator (σd) to the resilient axial strain (εr).

\[ M_r = \frac{\sigma_d}{\varepsilon_r} \]

where, σd = Major principal stress, σ3 = Minor principal stress (confining pressure), \( \varepsilon_r \) = Recoverable strain

\[ \sigma_d = \sigma_1 - \sigma_3 \]

Several Models [6], [9], [18] have been developed over the years that combine applied stresses and material characteristics to describe the nonlinear behaviour of granular materials under traffic loading.

The K-θ model has been the most famous for characterizing the resilient response of granular bases and sub-base materials [21]. The resilient modulus (Mr) is given as:

\[ M_r = k_1 \theta k_2 \]

where θ is the first invariant of stress tensor given as follows:

\[ \theta = \sigma_1 + \sigma_2 + \sigma_3 = \sigma_1 + 2 \sigma_3 \]

where, \( \sigma_2 \) = Intermediate principal stress, \( k_1, k_2 \) = Material constants.

Uzan [11] observed that the K-θ model did not summarize measured data well when shear stresses were significant, and proposed a three-parameter model. This model is given as:

\[ M_r = k_1 \theta k_2 \sigma_d k_3 \]

Witczak and Uzan [19] proposed a modification to the Uzan model by replacing the deviator stress term in (2) by an octahedral shear stress term. This octahedral shear stress model also considers the dilation effect that takes place when a pavement element is subjected to a large principal stress ratio \( \sigma_1 / \sigma_3 \).

This model is called Universal Model and is given as:

\[ M_r = k_1 \times \theta k_2 \times \tau_{oct} k_3 \]

where \( \tau_{oct} \) is the octahedral shear stress given as:

\[ \tau_{oct} = \frac{1}{3} \sqrt{\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_1 - \sigma_3\right)^2} \]

The parameters \( k_1, k_2 \), and \( k_3 \) are multiple regression constants evaluated from resilient modulus test data.

The Expanded universal model [8], [15] introduced in the AASHTO Mechanistic Empirical Pavement Design Guide (MEPDG), estimates the resilient modulus using a generalized constitutive model for Level 1 analysis for the nonlinear stress-dependent modeling of both the unbound aggregates and fine-grained soils. The resilient modulus used in the 2008 MEPDG design procedure, is as:

\[ M_r = k_1 P_a \left( \frac{\theta}{P_a} \right)^{k_2} \left( \frac{\tau_{oct}}{P_a} + 1 \right)^{k_3} \]

where \( P_a \) is the atmospheric pressure.
In this paper, the expanded universal model is used to assess the effect of nonlinearity in granular layers on pavement response and performance.

1. RLT Test Results of the Two Aggregates Considered in the Study

Aggregates used in this study were provided respectively from Cap-Djenet and Bordj-Bouarreridj deposits located in Northern Algeria. These two deposits have great potential for high production levels of granular materials [7]. Aggregates issued from Cap-Djenet (CAP) deposit are of basaltic origin. They were produced from volcanic eruptions. Basalt is a rough stone, fairly light in weight and grey in colour.

Aggregates issued from Bordj-Bouarreridj (BBA) deposit are limestones, produced from crushing sedimentary rocks composed mainly of calcium carbonate (CaCO3). Most limestones are hard and durable. They typically produce strong aggregates with low water absorption and are suitable for road stones.

The main results of the RLT tests for the two local unbound granular materials are summarised in Table II. Non-linear regression analysis is carried out to determine model parameters of the expanded universal model for both UGM BBA and CAP. The adequacy of resilient modulus prediction was assessed in this study using the coefficient of determination, R2, which represents the proportion of variation in the predicted variable that is accounted for by the regression model and takes on values ranging from zero (i.e. no correlation) to one (i.e. perfect correlation).

The main results of the regression analysis including the coefficient of determination (R2) are presented in Table III.

![Table III: MODEL PARAMETERS FOR THE STUDIED UGM](image)

<table>
<thead>
<tr>
<th>UGM</th>
<th>Expanded universal model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K1(MPa)</td>
</tr>
<tr>
<td>BBA</td>
<td>3.14</td>
</tr>
<tr>
<td>CAP</td>
<td>2.58</td>
</tr>
</tbody>
</table>

It is shown that the expanded universal model gives high values of the coefficient of determination for both studied UGM.

Based on the model regression parameters reported in Table III, the considered constitutive model may be used at a later stage for the nonlinear finite element analysis of flexible pavements utilizing the two local tested aggregates. Moreover, this model may be advantageously used to determine more realistic average values of characteristic resilient modulus Mrc than those issued from empirical classification of unbound granular materials [5], [7].

It is to be noted that the in-situ FWD tests were performed on the pavement test section utilizing UGM BBA as base and sub-base granular materials. Thus, the value of the field backcalculated modulus EFWD must be multiplied by an adjustment factor equal to 0.62 as indicated in Table II [15].

C. Subgrade Soil

The mechanical properties of the subgrade soil were not considered variable in the present study; the value of 63 MPa for the resilient modulus of the subgrade soil is estimated on the basis of the FWD backcalculated subgrade soil modulus (using ELMOD software) converted to an equivalent laboratory Mr value using an adjustment factor equal to 0.35 (see Table II). Poisson's ratio is taken equal to 0.35.

Note that the above value of the subgrade resilient modulus is rather close to the minimum value of subgrade modulus corresponding to subgrade soil type S2 defined in the Algerian manual for pavement design.

IV. NUMERICAL MODELING OF PAVEMENT STRUCTURES

The continuous trend towards improved computing facilities coupled with increasing knowledge of the mechanical properties of materials has enhanced the crucial need for the development of finite element codes for the non-linear analysis and design of pavements, both in mainframe and personal computers.

A. Finite Element Modeling

The finite element method is particularly efficient for modelling the non-linear behaviour of pavement structures as it can easily accommodate variability in material properties, changes in pavement geometry and modifications in applied loading. In pavement engineering, pavement structures are often modelled as axi-symmetric systems. The finite element domain is modelled using 8-node rectangular ring elements; each node having two degrees of freedom associated with the nodal displacement components in the vertical and the radial directions [20], [21]. The elements each contain four Gaussian points at which stresses and strains are calculated. The mesh is automatically made for a structure by superposed layers of elements, the material parameters being constant for each layer. The heights of these horizontal layers are constant or in geometrical progression in some groups, and the computer code does the same by columns of elements. The structure is meshed as shown in Tables IV and V. The letters R and P respectively stand for a regular mesh and a mesh in geometrical progression.

![Table IV: VERTICAL MESH](image)

<table>
<thead>
<tr>
<th>Layers</th>
<th>Number of elements</th>
<th>Mesh type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asphalt</td>
<td>3</td>
<td>R</td>
</tr>
<tr>
<td>Base</td>
<td>5</td>
<td>R</td>
</tr>
<tr>
<td>Sub-base</td>
<td>5</td>
<td>R</td>
</tr>
<tr>
<td>Subgrade</td>
<td>6</td>
<td>P</td>
</tr>
</tbody>
</table>

Near the load where the stress and strain gradients are large, the mesh should be fine whereas at greater distances from the applied load, the mesh can be coarser. Because of axi-symmetry, both geometrical and material, only the region to one side of the load centreline was considered.

The mesh is fixed at the bottom allowing no lateral movement and rollers on the sides allow vertical displacement
to take place. For illustration purposes, a schematic representation of a F.E model used for pavement analyses is presented in Fig. 1 (not to scale).

<table>
<thead>
<tr>
<th>Zones</th>
<th>Number of elements</th>
<th>Mesh type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>R</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>R</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>R</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>R</td>
</tr>
</tbody>
</table>

Fig. 1 Schematic representation of a F.E model (not to scale)

The pavement structure was subjected to a circular load which has radius of 17.5 cm and uniform pressure of 676 kPa.

Table VI summarises constitutive material properties of pavement structure layers including BBA granular base and sub-base materials for the three UGM nonlinear models considered in present investigation.

<table>
<thead>
<tr>
<th>Layers</th>
<th>Adjusted FWD modulus (MPa)</th>
<th>Linear model modulus (MPa)</th>
<th>Constitutive models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asphalt</td>
<td>5042</td>
<td>5042</td>
<td>Linear elastic</td>
</tr>
<tr>
<td>Base</td>
<td>285</td>
<td>600</td>
<td>Linear elastic</td>
</tr>
<tr>
<td>Sub-base</td>
<td>285</td>
<td>189</td>
<td>Nonlinear (4)</td>
</tr>
<tr>
<td>Subgrade</td>
<td>63</td>
<td>63</td>
<td>Linear elastic</td>
</tr>
</tbody>
</table>

B. Method of Resolution

The application of the finite element method [21] to the analysis and design of flexible pavement structures characterised by nonlinear elastic behaviour may be briefly summarised as:

\[
\{e\} = [B] \{\varepsilon\} \\
\{\sigma\} = [D] \{e\} 
\]

where, \{\varepsilon\}: Resilient strain tensor, \{\sigma\}: Resilient stress tensor, \{U\}: Nodal displacement vector, \{F\}: Strain displacement matrix, \{D\}: Elasticity matrix.

\[
\{F\} = \sum_i \{b\}_i^T \{u\}_i \, dv 
\]

and:

\[
\{F\} = [K]^T \{U\} 
\]

with:

\[
\]

In (7)-(9), \([K]^T\) is the secant stiffness matrix of the structure, which depends on the stresses, \(\{F\}\) is the nodal force vector, which is given by the load due to the half-axle of a vehicle, to calculate \(\{U\}\), the direct iteration method [4], [20], [21] was used as:

\[
\{U^n\} = [K_{n-1}^{-1}] \{F\} 
\]

and:

\[
\{p^n\} = [D_{n-1}^{-1}] [B] \{U^n\} 
\]

This method is illustrated in Fig. 2. The initial values are initial stresses due to self-weight. The program continues to iterate until sequential displacement computations agree with some specified tolerance.

V. RESULTS OF NONLINEAR ANALYSIS AND DISCUSSION

The main results of the numerical analyses were summarised in terms of values of the design criteria generally used in pavement engineering and the number of load repetitions to failure obtained from either the fatigue cracking or the rutting distress models.

A. Design Criteria

- The deflection at the surface, which is, to some extent, an indication of the structure ability to bear repeated traffic loads.
- The horizontal tensile strain at the bottom of the bituminous layer usually related to risks of asphalt layer cracking by tensile fatigue failure.
- The vertical strain at the top of the soil usually related to risks of rutting of pavement.

**B. Number of Load Repetitions to Failure**

Radial strain at the bottom of Asphalt layer and vertical strain at the top of Subgrade are two critical responses, which are also used to control fatigue and rutting of flexible pavements respectively [1], [2]. Error in computation of these two responses results in erroneous prediction of the pavement performance [22], [23].

Several fatigue and rutting models have been developed to relate the asphalt modulus and the measured strains to the number of load repetitions to pavement failure [2], [24]. Most of the fatigue cracking and the rutting failure models usually take the following forms:

\[ N_c = f_1 \times \varepsilon_h \times \frac{1}{E_1} \]

\[ N_r = f_4 \times \varepsilon_v \]

where, \( N_c \): Allowable load repetitions to prevent the cracking fatigue of the asphalt layer, \( N_r \): Allowable load repetitions to prevent the rutting at the top of subgrade soil due to accumulated pavement deformation, \( \varepsilon_h \): Maximum horizontal tensile strain at the asphalt layer, \( \varepsilon_v \): Compressive vertical strain at the top of subgrade soil, \( E_1 \): The elastic modulus of the asphalt layer

The design life of a flexible pavement is the minimum number of load repetitions required to cause either fatigue or rutting failure. The performance model considered in this study is the model proposed by Asphalt Institute [2], [25] with the following \( f_i \) values for the regression coefficients:

\( f_1 = 0.0796, f_2 = -3.291, f_3 = -0.854, f_4 = 1.365E-09, f_5 = -4.477 \)

**C. Sensitivity of Design Parameters to UGM Constitutive Models**

The variations within the tested pavement section of the surface deflection, the horizontal strain and the vertical strain are plotted in Figs. 3, 4, 5 respectively.

From Figs. 3-5, compared to FWD results, using nonlinear model to simulate unbound aggregate behaviour gives more realistic results than the assumption of linear behaviour.

The values of the three design criteria and design life (for fatigue cracking and rutting distress models) reported in Tables VII and VIII were computed using the linear and the nonlinear expanded universal model for granular base and sub-base layers of the analysed pavement structure. Validation by FWD backcalculations, of finite element simulation results using the UGM BBA has been performed and corresponding results are presented in Table VII. In addition, for the sake of clarity, variation of design criteria and design life for the linear and nonlinear models based on UGM CAP are summarized in Table VIII. Only the mineralogical nature and hence the stiffness of the unbound granular materials was varied. All the other geometrical and mechanical parameters of the pavement layers were kept unchanged.
Furthermore, it is noted from Tables VII and VIII that allowable load repetitions computed based on Asphalt Institute transfer functions for predicting fatigue cracking and rutting design life, are notably different when using linear and nonlinear models of granular layers (especially for the fatigue cracking failure mode). It follows that conventional design methods usually based on multi-layer linear elastic theory significantly overestimate the pavement design life. This clearly illustrates the need of using proper constitutive modeling for characterization of non-linear aggregate behaviour and prediction of pavement response on the one hand, and the crucial importance of developing well-calibrated distress models especially for predicting fatigue cracking design life on the other hand.

### TABLE VII

**VALIDATION BY FWD BACK CALCULATIONS OF F.E. SIMULATION RESULTS (UGM BBA)**

<table>
<thead>
<tr>
<th>Models</th>
<th>FWD</th>
<th>Linear</th>
<th>Nonlinear</th>
</tr>
</thead>
<tbody>
<tr>
<td>W (mm)</td>
<td>0.870</td>
<td>0.725 (0.83)</td>
<td>0.792 (0.91)</td>
</tr>
<tr>
<td>εh (10^-6)</td>
<td>270</td>
<td>136.20 (0.50)</td>
<td>183 (0.68)</td>
</tr>
<tr>
<td>εv (10^-6)</td>
<td>842</td>
<td>707.20 (0.84)</td>
<td>824 (0.97)</td>
</tr>
<tr>
<td>Nc</td>
<td>433 332</td>
<td>4 119 670 (9.50)</td>
<td>1 558 521 (3.59)</td>
</tr>
<tr>
<td>Nv</td>
<td>79 526</td>
<td>173 673 (2.18)</td>
<td>87 604 (1.10)</td>
</tr>
<tr>
<td>Design life</td>
<td>79 526</td>
<td>173 673</td>
<td>87 604</td>
</tr>
</tbody>
</table>

Values in parentheses represent ratios of F.E simulation results to corresponding FWD backcalculated values.

### TABLE VIII

**VARIATION OF DESIGN CRITERIA AND DESIGN LIFE BASED ON UGM CAP**

<table>
<thead>
<tr>
<th>Models</th>
<th>Linear</th>
<th>Nonlinear</th>
</tr>
</thead>
<tbody>
<tr>
<td>W (mm)</td>
<td>0.831</td>
<td>0.892 (1.07)</td>
</tr>
<tr>
<td>εh (10^-6)</td>
<td>188</td>
<td>238.60 (1.26)</td>
</tr>
<tr>
<td>εv (10^-6)</td>
<td>821.60</td>
<td>912 (1.11)</td>
</tr>
<tr>
<td>Nc</td>
<td>1,426,217</td>
<td>650,917 (0.46)</td>
</tr>
<tr>
<td>Nv</td>
<td>88,756</td>
<td>55,620 (0.63)</td>
</tr>
<tr>
<td>Design life</td>
<td>88,756</td>
<td>55,620</td>
</tr>
</tbody>
</table>

Values in parentheses represent ratios of F.E nonlinear simulation results to corresponding linear values.

### D. Effect of UGM Mineralogical Nature on Design Criteria and Structural Performance

It is observed from the results reported in Tables VII and VIII that the three design criteria and pavement design life are very sensitive to changes in UGM mineralogical nature. In particular, the horizontal strain at the bottom of the asphalt layer is the most sensitive design parameter to variations in the mechanical characteristics of the unbound granular materials. In addition, it is clearly seen that the use of stiffer aggregates (UGM BBA) can reduce significantly the tensile strain at the bottom of the asphalt layer and hence increase the pavement fatigue life.

### VI. SUMMARY AND CONCLUSIONS

In the present study, the expanded universal nonlinear resilient model for unbound aggregates has been implemented in a numerical code based on axi-symmetric finite elements to study the nonlinear resilient behaviour of unbound granular materials and the structural performance of road pavements. Numerical investigations have been carried out to assess the influence of the nonlinear resilient model on the behaviour of two local unbound aggregates of different mineralogical nature typically used for road construction in Algeria.

Simulation results show that computed critical pavement design criteria and predicted pavement design life using linear and nonlinear analysis are found to be substantially different. This clearly demonstrates the importance of nonlinear characterization of base and sub-base granular aggregates for accurate pavement design and the need to develop well calibrated transfer functions for performance prediction especially for predicting fatigue cracking design life.

The simulation results also show that the use of stiffer aggregates can reduce significantly the tensile strain at the bottom of the asphalt layer and hence increase the pavement fatigue life, which could reduce the construction cost of road pavements. This is especially important in flexible road pavements subjected to low to moderate volumes of traffic.

### REFERENCES


