Auto-Calibration and Optimization of Large-Scale Water Resources Systems


Abstract—Water resource systems modeling has constantly been a challenge through history for human beings. As the innovative methodological development is evolving alongside computer sciences on one hand, researches are likely to confront more complex and larger water resources systems due to new challenges regarding increased water demands, climate change and human interventions, socio-economic concerns, and environment protection and sustainability. In this research, an automatic calibration scheme has been applied on the Gilan’s large-scale water resource model using mathematical programming. The water resource model’s calibration is developed in order to attain unknown water return flows from demand sites in the complex Sefidroud irrigation network and other related areas. The calibration procedure is validated by comparing several gauged river outflows from the system in the past with model results. The calibration results are pleasantly reasonable presenting a rational insight of the system. Subsequently, the unknown optimized parameters were used in a basin-scale linear optimization model with the ability to evaluate the system’s performance against a reduced inflow scenario in future. Results showed an acceptable match between predicted and observed outflows from the system at selected hydrometric stations. Moreover, an efficient operating policy was determined for Sefidroud dam leading to a minimum water shortage in the reduced inflow scenario.

Keywords—Auto-calibration, Gilan, Large-Scale Water Resources, Simulation.

I. INTRODUCTION

WATER resources systems planning and management has been extensively taken under consideration as its importance has become more noticeable through history. Water shortage and the excessive dispersed growth of populations have constantly been challenged. Assessments have been applied among the variation of global water shortage over the past two millennia by comparing population historical data and outputs of a climate model [1]. The fascinating improvement within computational sciences has encouraged researchers for further concentration on developing more complete, fashionable methodologies to enhance modeling procedures and dare to face more complicated problems. An introduction has been taken into effort to describe an efficient algorithm to manipulate data entry through large municipal and industrial water management problems with the aim of satisfying demands and minimizing costs [2]. Recently, researchers are most likely to impose an optimization procedure upon the simulation outlines to increase the accuracy and efficiency of mathematical models used for modeling complex water resource systems. Until the past four decades, model optimization was not perfectly applied in real world and mostly, it was practiced theoretically in academic institutes [3]. But within the last decade, optimization has become a prominent fragment of water industries and management [4]-[6]. There are a number of studies which discuss the probable issues of the gap between theory and real world implementation of optimization modeling [7]. Some studies have also introduced scenario development and classification approaches which leads to more efficient assessments and decision making targets [8], [9]. Several works of simulation-optimization have been applied to water resource systems analysis. For instance, large non-convex water resources systems can be dealt with the aid of mathematical programming [10]-[12]. Also, decomposition algorithms are utilized to enhance the modeling performance alongside the advances in computer sciences [13]. Moreover, Evolutionary algorithms, such as genetic algorithms, have been integrated with classical mathematical programming approaches, building hybrid models to be able to solve large-scale water resources systems [14]. Large-scale water resources systems calibration can be notified as one of the most prominent factors resulting in a reasonable validity framework of the model. The uncertainty and inaccuracy of a model increases as the number of variables and their connections increase [15], [10], [16]-[34]. Although several systematic approaches have been implemented to provide a flawless modeling institute by the aid of mathematical programming, there is still a lack of confidence recognized to confront mega-scale water resources systems [35]-[43].

In this research, we present a mathematical programming approach in order to calibrate a large-scale water resource assessment model automatically. The process has been put into practice on Gilan’s Large-Scale Water Resources Systems (GLSWRS), Gilan Province, Iran. Alongside this research, many other progresses have been taken into account for Gilan Province [44]-[46].
II. STUDY AREA AND FEATURES

Gilan Province is located at north of Iran, aliened beneath the Caspian Sea, which typically covers a sensible fraction of the most problematical water resources systems in the country (Fig. 1). The greatest water supply to Gilan’s water resources system is Sefidroud’s River inflow, which initiates from the convergence of Shahroud and Ghezel-Ozan Rivers. The Sefidroud Reservoir located at its upstream is one of the most important elements of the basin to result in satisfaction for demands and water requirements. Other constructed elements are diversion channels, which are mainly applied on Sefidroud’s River reaches and other adjacent rivers to distribute and cover demands over the basin. Fig. 2 shows a node-link structure of Sefidroud River.

![Fig. 1 Gilan’s Basin location, Iran](image1)

![Fig. 2 Node-link schematic view of Sefidroud River system](image2)

The dense erratic arrangement of numerous natural river flows and intricate underground water displacements alongside the uncontrollable natural water drainage channels have structured an unpredictable behavior of this water resource system (Fig. 3). Although many proposals have been put into practice to compose a model in order to illustrate a comprehensive overview of Gilan’s water resources system, the results would hardly define the desirable correspondent to practical instances. Since Gilan’s province is expressively rich in water recourses, it is desirable to evaluate the possibility to share more than 30 percent of Sefidroud’s water inflow to other arid and semi-arid central regions of the country. The consequences of this water diversion are to be evaluated by a comprehensive supply-demand simulation model that can be used for assessing future water management policies. The policies are generally accompanied with maximum water demand satisfactions.

![Fig. 3 Gilan’s River network](image3)

III. MODELING FRAMEWORK

In this research, we present a non-linear mathematical program as an optimization approach to calibrate GLSWRS. The model is developed in General Algebraic Modeling System (GAMS). The most complicated part of the system is the central part of Gilan, which mostly is exposed with disordered natural drainage channels. These channels are generally gathered densely in the middle of the system, which ends to Anzali Lagoon. The complex drainage channels and their interconnections and irregular arrangements alongside the extreme water transfer potentials are highly enough to affect the whole water allocation decision making and policies. On the other hand, the temporal flow transfer through the drainage channels in Gilan’s Basin is too sophisticated to monitor. As to encompass the mentioned complicacies within the modeling procedure, an optimization approach has been mounted to calibrate a water balance model of the system. The model is settled through 648 months of historical time-series data which includes both input data and variables to perform. The calibration approach is pointedly developed upon water transfer intensity by applying percentage coefficients to the conveyed water through the natural drainage system. In order to validate the calibration procedure, an objective function has been defined to minimize the difference between monitored river outflows and predicted-by-model ones. Subsequently, the pre-calibrated coefficients have been applied to a large-scale linear program representation of the system in order to evaluate the system’s performance while the system faces a significant water reduction shock in future. It can provide the optimal water management policy to satisfy demands and environmental requirement. Table I shows an overview of the
modeling framework elements. The mathematical model has been advanced in accordance to basins physical characteristics and water distribution interconnections. The calibration and simulation contain 145181 input parameters and 211248 variables, which are mainly related to time-series data.

### TABLE I

<table>
<thead>
<tr>
<th>Objects</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rivers</td>
<td>61</td>
</tr>
<tr>
<td>Demand</td>
<td>63</td>
</tr>
<tr>
<td>Underground water</td>
<td>14</td>
</tr>
<tr>
<td>Return Flows (Natural Drainage Channels)</td>
<td>28</td>
</tr>
<tr>
<td>Artificial Channels</td>
<td>27</td>
</tr>
<tr>
<td>Environment Control Gauges</td>
<td>15</td>
</tr>
<tr>
<td>Total</td>
<td>208</td>
</tr>
</tbody>
</table>

### IV. MATHEMATICAL MODELING

Non-linear and linear mathematical programs have been used to model GLSWRS. Most of the constraints utilized in the problem are known to be attributed to water balance equations, supplies and demands, environmental requirements, and physical constraints due to limitations in storage and water transmit structures. The formulation of a mathematical program in GAMS is illustrated in (1)-(3):

\[
Z = \min \left\{ f(x) \right\} \\
\text{s.t. } g(x) \geq b_j, \quad j \in \text{int} \\
\quad x_i \in s, \quad j \in \text{int}
\]

where \( Z \) is the objective value, \( x \) is the vector of decision variables, \( f(x) \) is the objective function, \( g(x) \) is the left hand side function correspondent to \( i \)th constraint, \( b_j \) is the right hand side parameter assigned to \( i \)th constraint, and \( s \) is the set of feasible values of variables.

#### A. Non-Linear Calibration

A minimization optimization problem was formed in case of evaluating the accuracy of outflows determined by the model against the measured ones. Equation (4) describes the objective function by presenting a non-linear equation which calculates the squared error of the estimated outflow from seven main rivers in the basin.

\[
Z = \min \left\{ \sum_{j=1}^{m} (\text{outflow}_{\text{cal},j} - \text{Outflow}_{\text{obs},j})^2 \right\}
\]

where \( m \) is the number of rivers, \( \text{outflow}_{\text{cal},j} \) and \( \text{outflow}_{\text{obs},j} \) are, respectively, the \( m \) th river’s calculated and observed outflow values. The objective function guides the model’s feasible solutions towards those minimizing the objective function. The model’s feasible space is confined with the constraints. These constraints are defined as equations or inequalities representing physical water balance requirements at river, reservoir, groundwater, and channel nodes. Moreover, various types of water demands including urban, agricultural, industrial and environmental demands are to be satisfied. Correspondingly, water return-flows which are considered to be the most effective and complicated unknown parameters to estimate are used in other constraints. Equations (5)-(8) demonstrate all the constraints of GLSWRS’ optimization model.

\[
S_i + I_j - \text{Ev}_{t,j} - R_j - \sum_{i=1}^{m} \text{Supply}_{i,j} = S_{i,t} \quad \forall t \tag{5}
\]

\[
\text{Min}_S \leq S_i \leq \text{Max}_S \quad \forall t \tag{6}
\]

\[
R_t \leq \text{Max}_R \quad \forall t \tag{7}
\]

\[
S_i \geq S_i \quad \forall \text{Variables} \geq 0 \tag{8}
\]

where \( t \) is the corresponding monthly time-step, \( T \) is the last time period, \( S_i \) is the storage volume of reservoirs in GLSWRS, \( I_j \) is the amount of water inflow to the reservoirs, \( \text{Ev}_{t,j} \) is the volumetric quantity of evaporation from the reservoirs in month \( t \), \( R_j \) is the water release volume from the reservoirs, \( m \) is the number of demand sites in requisition and \( \text{Supply}_{i,j} \) is the \( i \) th supplied volume of water in month \( t \) to demand spots. All water balance equations appear exclusively linear in the model, but some terms such as evaporation are intrinsically non-linear because they are dependent to volume-area curves of the reservoirs. In (5), \( \text{Ev}_{t,j} \) is a linearized function, which is estimated from a non-linear volume-area curve. Groundwater balance equations are correspondingly the same as those of surface reservoirs which are represented in (9)-(12).

\[
S_{\text{gw},t} - \sum_{i=1}^{m} \text{Supply}_{i,t} = S_{\text{gw},t+1} \quad \forall t \tag{9}
\]

\[
R_{\text{gw},t} \leq \text{Max}_R \quad \forall t \tag{10}
\]

\[
S_{\text{gw},t+1} = S_{\text{gw},t} \quad \forall t \tag{11}
\]

\[
S_{\text{gw},t} \geq S_{\text{gw},t+1} \quad \forall \text{Variables} \geq 0 \tag{12}
\]

where \( S_{\text{gw},t} \) is the storage volume of a groundwater reservoir in month \( t \). Moreover, to include the environmental demand requirements, (13)-(15) are settled as following.

\[
\sum_{t=1}^{T} \text{Supply}_{i,t} + \alpha_d = \text{Total Demand} \quad \forall t \tag{13}
\]

\[
\text{Flow}_{t} + \alpha_e - \beta_e = \text{Env Demand} \quad \forall t \tag{14}
\]

\[
\forall \text{Variables} \geq 0 \tag{15}
\]
where $\alpha$ and $\beta$ are subsequently the shortage and excess of water supply to the corresponding demand node or environmental requirement. The $\text{Total Demand}$ and $\text{Env. Demand}$ parameters are target agricultural and environmental demand values, respectively. The variables of $\alpha$ and $\beta$ are both positive; and therefore, at least one of them should remain zero through the optimization process. To accomplish this statement, (16) limits the multiplication of both variables to zero.

$$\alpha_i \times \beta_i = 0 \quad \forall t$$  \hspace{1cm} (16)

To take account of return flows and insert calibration variables, (17)-(20) have also been considered.

$$RF_i \leq \alpha_i \quad \forall t$$  \hspace{1cm} (17)

$$USF_i + Coef(i) \times RF_i + DSF_i = Outflow_i \quad \forall t$$  \hspace{1cm} (18)

$$\sum_{i} Coef(i) = 1 \quad \forall t$$  \hspace{1cm} (19)

$$\text{AllVariables} \geq 0$$  \hspace{1cm} (20)

where, $RF_i$ stands for return flow at month $t$, $Coef(i)$ is the calibration variable to assign the percent value of $RF_i$ to stream flow $i$, $USF_i$ and $DSF_i$ are, respectively, the upstream and downstream flows prior and after return flows entrance in stream flow $i$ resulting the outflow of it ($Outflow_i$). Equation (17) specifies an upper bound for the return flow, which is the volume of excess water supply. In order to prepare a proper performance of return flow allocation to their available destinations, (18) has been taken into account. Equations (17) and (18) precisely describe the calibration procedure to allocated appropriate water amount as water distribution engines for the system. Equation (19) is a control constraint taking care of a water balance requirement as the sum of calibration coefficients from a specific return flow must equal one.

The formulations advanced within (1)-(20) construct a non-linear program, which is obligated to calibrate the return flow coefficients of the system to achieve the best objective function. Non-linear equations (16), (18), and the objective function are the complicating constraints in the calibration problem.

V. LINEARIZED OPTIMIZATION MODEL

The linear model is developed to describe the GLSWRS’s ability to confront the probable water shortage in future and to adjust the finest operational policies resulting in the best demand and environmental requirement satisfaction. In the linear optimization model, the calibrated variables are fixed and included in the GLSWRS model; thus, leading to the transfer constraint (18) into a linear equation. Additionally, the objective function is modified as a linear function (21) so it eliminates constraint (16) by satisfying it with a change in the problem’s convex hall. The function is formulated as the summation of all both shortages and excesses of water at every demand and environmental site.

$$Z = \min \left\{ \sum_{i=1}^{n} \sum_{j=1}^{m} (\alpha_{i,j} + \beta_{i,j}) + \sum_{i=1}^{m} (\alpha_{i} + \beta_{i}) \right\}$$  \hspace{1cm} (21)

Equation (21) forces the model to satisfy all demands as possible by resulting in a minimum water excess. In this case, one of variables $\alpha$ or $\beta$ in each time-step remains zero due to objective functions minimization, resulting in the most efficient water operation for the water resource system’s management.

VI. CALIBRATION VALIDATION AND SYSTEM OPERATION OUTLINES

Fifty four years of monthly data were used in the calibration of GLSWRS by a non-linear mathematical optimization approach. The objective function generally illustrates the measured error between observed and predicted outflows from seven major rivers in Gilan’s Basin system. The accuracy of the optimized model was significantly outstanding. Fig. 4 shows the out coming results of the calibration model for seven main rivers included in the objective function.
Fig. 4 (a)-(g) Fifty four years of predicted outflows compared to observed outflows for seven main rivers in GLSWRS

(d) Pasikhani River outflow comparision

(e) Shanderman River outflow comparision

(f) Masal River outflow comparision

(g) Masoule Roudlkhan River outflow comparision

One can see from Fig. 4 that the calibration results seem to be reasonable enough. The ability of the model to match itself delicately with observed parameters is principally because of the large size of the model which leads to a high degree of freedom for the search algorithm to find an optimum solution. Although the high flexibility of the model has the potential to result in better solutions, computational load may exceed high enough to become as an obstacle for modeling procedures. Among large number of solvers available in GAMS, the BARON solver was able to accomplish the calibration model and to reach the global optimum, while other solvers were not able to perform well enough and usually failed searching the feasible space for the global optimum. The run time of Gilan’s large-scale water resources calibration model took 2.2 hours for a 64-bit windows based general i5-3210M 2.5GHz computer with 4Gb installed memory to execute without crashing. This, apparently, was the maximum capability taken into effort from BARON’s search algorithm to optimize GLSWRS since the model was not capable to furtherance its search when it faces additional non-linear constraints.

The modeling approach for evaluating future water shortage was also performed by linear programming. In this model, mainly, 30 percent of Sefidroud’s River inflow to Gilan’s water resource system was cut off due to upstream demand consummations. The linear model was run by forcing water demands to remain satisfied as much as possible with the minimum excess water supply. Results show the systems optimal operation has marked 95.2 percent of demands satisfied for urban, agricultural, industrial and aquaculture demands while environmental minimum water flows were met without discounts. Fig. 5 shows the demand fulfillment for major demand sites in percent.

Sefidroud reservoir is the most prominent structural element in the system governing the water allocation and distribution within the system. The operational rule curve for managing Sefidroud reservoir’s operation is extremely essential to the system’s demand satisfaction. The operational monthly average rule curve for Sefidroud’s reservoir is shown in Fig. 6.

VII. CONCLUSION AND SUGGESTIONS

We formulated and solved a non-linear mathematical program to calibrate the Gilan Large-Scale Water Resource System (GLSWRS). The resulting optimized parameters of return flows were utilized then in a linear optimization model
so as to evaluate the consequences of reduced inflow to the system in future due to upstream withdrawals. The results showed that although the calibration problem's size was extremely large, the solution obtained was well desiredly validated by its objective function. Note that the exceedingly large connectivity among the rivers, channels, and water resources in the GLSERS practically points at the vastly extended feasible space which may result in the existence of multiple solutions in the linear optimization model.

The linear model results marked out appropriate implications about the system's future performance. Further studies could be carried out upon defining and assessing future circumstances and scenarios on possible structural and nonstructural measures to face future water challenges.

As a finalization approach, a Decision Support System (DSS) can be developed according to the calibration-simulation procedure introduced in this research in order to prepare a more flexible decision making platform for users to take under consideration.

ACKNOWLEDGMENT

It is desired to thank Mahabghods consulting engineering company, Iran, which has been an intense assistance to this research by gathering the initial data sets and developing initial simulation models.

REFERENCES


