Subarray Based Multiuser Massive MIMO Design
Adopting Large Transmit and Receive Arrays

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Abstract—This paper describes a subarray based low computational design method of multiuser massive multiple input multiple output (MIMO) system. In our previous works, use of large array is assumed only in transmitter, but this study considers the case both of transmitter and receiver sides are equipped with large array antennas. For this aim, receive arrays are also divided into several subarrays, and the former proposed method is modified for the synthesis of a large array from subarrays in both ends. Through computer simulations, it is verified that the performance of the proposed method is degraded compared with the original approach, but it can achieve the improvement in the aspect of complexity, namely, significant reduction of the computational load to the practical level.

Keywords—Massive multi-input multi-output (MIMO), subarray, zero forcing, singular value decomposition.

I. INTRODUCTION

MULTIPLE input multiple output (MIMO) system is now widely recognized as one promising way to improve the capacity in wireless communications without increasing additional time/frequency resources [1]. Among many topics in MIMO, massive MIMO which adopts large size array one or both of transmitter and receiver sides is a recent active subject [2] as a candidate to transmit the explosively increasing demand of data traffic in cellular system. Researches are on wide variety from device to algorithm; for example, from the aspects of antennas and propagation, development of antenna [3] and measurement of massive MIMO channel [4] are reported. Detection and precoding techniques are also studied; reference [5] and [6] have respectively presented a simplified matrix inversion and a tabu search based detection together with their FPGA implementation. Precoding methods using linear processing [7] and nonlinear vector perturbation [8] have been proposed and their performance has been analyzed.

In those literatures, important subject is the reduction of computational load which is required for design and signal processing, and paying attention to the fact that relatively fewer number of researches have been carried out on design of linear precoder, authors have proposed subarray based low computational version of block diagonalization, which is a widely used technique in conventional moderate-scale multiuser MIMO downlink transmission, for MIMO system using a large array only in transmitter side (the design procedure is described in [9]). In case of communication between access point and user terminals, normally the user end has a small space for the antenna implementation, and the assumption of [9] consists, but when transmission between two large scale arrays are considered, this method cannot be used.

Therefore, in this paper, low computational block diagonalization in [9] is improved to fit to the case large arrays are used both ends of multiuser MIMO. This approach can be applied, for example, to inter-source-relay transmission in the system aided by large scale relay stations. The strategy is that the receiver side also adopts the subarray based processing, but a different types of subarray processing and large array synthesis are required from the case of [9], which is described in this study. The performance degradation caused by using subarrays in both sides of MIMO is also investigated.

The rest of this paper is organized as follows: in section II, the model of downlink multiuser MIMO system considered in this study is briefly described. The proposed design method is presented in section III, and its performance is evaluated by numerical simulations in IV. Finally in section V, conclusions are shown together with some future works.

II. SYSTEM DESCRIPTION

This section briefly describes the model of MIMO downlink communication system considered in this study. In this paper, bold slant font in lowercase letter shows a vector, and a matrix is expressed by uppercase letter with normal slant font.

The model of multiuser MIMO downlink communication system considered in this study is shown in Fig. 1. The system consists of one transmitter and M receivers, where they are denoted by Tx and RX_0, \cdots, RX_{M-1}, respectively. Transmitter Tx has N_t antennas and receiver RX_m is equipped with N_{r,m} antennas. Data streams \{s_{m,\ell}(t); \ell = 0, \cdots, L_m - 1\} are transmitted from the transmitter using N_t-dimensional weight vectors \{w_{t,m,\ell}; \ell = 0, \cdots, L_m - 1\} to receiver RX_m through MIMO channel represented by N_{r,m}-by-N_t matrix H_m whose (n_r,n_t)-th element H_{m,n_r,n_t} denotes the response between the n_t-th antenna of the transmitter and the n_r-th antenna of receiver. Receiver RX_m calculates output signal \hat{s}_{m,\ell}(t), which is the replica of transmitted signal s_{m,\ell}(t), using N_{r,m}-dimensional weight w_{r,m,\ell}.

Those settings are typical in the multiuser MIMO downlink system, but the difference in this paper is that N_t (> 100) and N_{r,m} (> 50) are much larger numbers than the conventional
Here, uniform subarray is considered (indices $2^N$ constructed in the order of the index, for example, for the design of weight vectors $\{w_{r,m,\ell}\}$ and $\{w_{r,m,\ell}\}$ for $m = 0, \cdots, M - 1$); in this study, we consider widely linear processing design [10], but it needs computational load of $O(N_2^s)$ and $O(N_{r,m}^s)$ for the singular value decomposition (SVD) which is repeatedly used $2M$ times in the procedure.

III. SUBARRAY BASED DESIGN METHOD

This section provides a subarray based low computational design method of the system given in section II.

In the first step of the design, subarrays are constructed from the original large size array. In the transmitter $Tx$, the large size array is divided into $S_t$ subarrays $Tx_0, \cdots, Tx_{S_t-1}$, each of which consists of $N_t$ elements with the set of indices $N_t(s)$. In the receiver $Rx$, the large size array is divided into $S_{r,m}$ subarrays $Rx_{m,0}, \cdots, Rx_{m,S_{r,m}-1}$ each of which consists of $N_{r,m}$ elements with the indices of $N_{r,m}(s)$. In the both sides, every antenna elements of the large array should belong only one subarray, which means $Tx = \cup Tx_s$, $Tx_k \cap Tx_{k'} = \emptyset$, $Rx_m = \cup Rx_{m,s}$, and $Rx_m \cap Rx_{m,k} = \emptyset$.

Here, uniform subarray is considered ($N_{t,0} = \cdots N_{t,S_t-1}$ and $N_{r,m,0} = \cdots N_{r,m,S_{r,m}-1}$), and subarrays are simply constructed in the order of the index, for example, $N_t(s) = \sum_{u=0}^{s-1} N_{t,u} + \{0, \cdots, N_{t,s-1}\}$, $s = 0, \cdots, S_t - 1$.

In the second step, receiver weights are first designed, namely, block diagonalization is applied to channel between $Rx_{m,s}$ and $Tx$ (not $Tx_s$). More concretely, $N_{r,m}$-by-$N_t$ channel matrix $H_{m}^{(s)}$ is decomposed by SVD, and its left singular value vectors $\{v^{(s)}_{r,m,\ell}; \ell = 0, \cdots, L_m - 1\}$ corresponding to the largest $L_m$ singular values are derived as the subarray weights $(w^{(s)}_{r,m,\ell} = v^{(s)}_{r,m,\ell})$.

In this process, the transmit array is still large size, but that of $Rx_{m,s}$ is reduced to $N_{r,m,s}$, hence the size of SVD is also reduced from $N_{r,m}$ to $N_{r,m,s}$. After the calculation of receiver weights, virtual channel $H_{v,r,m,\ell} = [H_{m}^{(0)T}w_{r,m,0}^{(0)}, \cdots, H_{m}^{(S_t-1)T}w_{r,m,1}^{(S_t-1)}]^{T}$ is derived by multiplying receive weight to actual channel matrix.

Again by SVD of $H_{v,r,m,\ell}$, its left singular value vector $c_{r,m,\ell}$ corresponding the largest singular value is used for combining subarrays, namely, $w_{r,m,\ell} = [c_{r,m,\ell}^{(0)T}w_{r,m,\ell}^{(0)}, \cdots, c_{r,m,\ell}^{(S_t-1)T}w_{r,m,1}^{(S_t-1)}]^{T}$.

In the third step, transmitter weights are designed. First, virtual channel is calculated by $H_{v,t,m,\ell}^{(s)} = [H_{m}^{(s)T}w_{r,m,0}^{(s)T}, \cdots, H_{m}^{(s)T}w_{r,m,S_{r,m}-1}^{(s)T}]^{T}$. Next, by the SVD of subarray undesired virtual channel matrix $H_{v,t,m,\ell}^{(s)} = [H_{m}^{(s)T}w_{r,m,0}^{(s)T}, \cdots, H_{m}^{(s)T}w_{r,m,L_m-1}^{(s)T}, H_{m}^{(s)T}w_{r,m,L_m+1}^{(s)T}, \cdots, H_{m}^{(s)T}w_{r,m,L_m+S_s}^{(s)T}]$, matrix $V_{t,m,\ell}$ is derived, whose columns span the kernel of virtual matrix $H_{v,t,m,\ell}^{(s)}$. The subarray weight is derived as $w_{t,m,\ell}^{(s)} = V_{t,m,\ell}^{(s)T}H_{m}^{(s)T}w_{r,m,\ell}^{(s)T}$. Then, to reconstruct a large array from subarrays, another virtual channel is calculated as $H_{v,t,m,\ell} = [H_{m}^{(0)T}w_{t,m,0}^{(0)T}, \cdots, H_{m}^{(S_t-1)T}w_{t,m,1}^{(S_t-1)}]$. Subarray combining coefficient $c_{t,m,\ell}$ is derived as the right singular value.
vector of $H_{r,m,t}$ corresponding to the largest singular value, and the large size transmit subarray weight is derived by $w_{t,m,ℓ} = [w_{t,m,ℓ,0}, w_{t,m,ℓ,1}, \ldots, w_{t,m,ℓ,S-1}]$. 

As shown in the above, in this study, as an initial step of transmitter-receiver subarray processing, the subarray construction is carried out based on their indices; the effect of the more sophisticated subarray construction has been verified in our previous works (e.g., [9]), and the performance of the proposed method can be improved using it, but here simply the bare influence of new subarray processing is considered.

Strictly speaking, the block diagonalization in this study is different from the original one which attempts zero forcing to all the antennas of the undesired users, where $N_{\neq m} = \sum_{n \neq m} N_{r,n} (m: desired user index)$ degrees of freedom is consumed regardless of data stream number, which means $N_t \geq N_{\neq m} + L_m$ should be satisfied. But in the method of this study, only one degree of freedom is required for one stream, hence this method is advantageous when total number of data stream is smaller than the maximum possible number of streams $N_{r,m} (N_t > N_{r,m}$ is assumed), ignoring the complexity of the accompanied operation like resource allocation and determination of the modulation scheme, but usually utilization of all the number of streams in large array is impractical. If the number of stream is limited to $L_m \leq \max\{N_{r,m,s}\}$, the proposed method becomes a strong candidate of the design procedure. Consequently, corresponding to the strategy of subarray processing, the conventional method also adopts the above mentioned receiver weight first approach, which has a better performance than the conventional design in [10] under the simulation setting in the next section.

IV. SIMULATION

This section describes the numerical simulations for verifying the effectiveness and features of the proposed design method. It is clear that the performance of the subarray based method is degraded compared with the (receiver weight first design version of) conventional approach [10], so here the main subject is how much degradation is brought by the suboptimality of design procedure.

The simulation conditions are summarized in Table I. The default number of user is $M = 2$ and each of the transmitter and the receivers is equipped with $N_t = 128$ and $N_{r,m} = 64$ antennas assuming that the transmitter has large chassis for many antennas like base station, and the receivers are smaller equipment, but have enough space for large antenna like nonportable relay station (remark that relay system is an example of application, and here we pay attention only to transmitter relay=receiver link). Independent and identically distributed (i.i.d.) Rayleigh fading channels with unit variance are considered between the transmitter and each receiver. (A line-of-sight (LOS) channel with Ricean statistics is sometimes suitable under relay-aided transmission since relay is installed in a location with a direct path, but here we adopt Rayleigh simulations which reflect the effect of subarray processing more clearly considering that there are applications non LOS channels correspond.)

For the evaluation of the performance of the proposed method, the approximated per-user capacity is used, which

![Fig. 2. Procedure of subarray based design](image)

**TABLE I**

<table>
<thead>
<tr>
<th>Simulation Conditions.</th>
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<tbody>
<tr>
<td>Number of Users</td>
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<tr>
<td>Antenna Number</td>
</tr>
<tr>
<td>Receiver $N_{r,m} = 64$</td>
</tr>
<tr>
<td>Number of Data Streams</td>
</tr>
<tr>
<td>Subarray (Transmitter)</td>
</tr>
<tr>
<td>$N_{t,s} = sS_t + {0, \ldots, S_t - 1}$</td>
</tr>
<tr>
<td>Subarray (Receiver)</td>
</tr>
<tr>
<td>$N_{r,m,s} = sS_r + {0, \ldots, S_r - 1}$</td>
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<tr>
<td>Modulation</td>
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<tr>
<td>Energy Constraint</td>
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<tr>
<td>Noise (R.D)</td>
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<tr>
<td>SNR</td>
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<tr>
<td>Fading</td>
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</tbody>
</table>
is defined by

\[ C = \sum_{m=0}^{M-1} \sum_{\ell=0}^{L_m-1} \log_2(1 + \text{SINR}_{m,\ell}). \]  

(1)

In the above equation, notation SINR\(_{m,\ell}\) denotes the signal to interference plus noise ratio (SINR) of the output signal of the \(\ell\)-th stream of the \(m\)-th user, which can be calculated by

\[ \text{SINR}_{m,\ell} = \frac{|\rho_{m,\ell}|^2}{1 - |\rho_{m,\ell}|^2} \]

\[ \rho_{m,\ell} = \frac{\mathbb{E}[|s_{m,\ell}(t)|^2]}{\mathbb{E}[|s_{m,\ell}(t)|^2]\mathbb{E}[|s_{m,\ell}(t)|^2]}. \]

(2)

(3)

On the other hand, signal to noise ratio (SNR) in Table I is given by \(\text{SNR}_m = P_{t,m}/P_{n,m}\) for the \(m\)-th user, where \(P_{t,m}\) is the total transmit powers of Tx for the \(m\)-th receiver, and \(P_{n,m}\) is the noise power generated at Rx\(_m\). The simulations are carried out by changing fading channels 500 times, and the mean value is calculated by averaging over those samples.

Fig. 3 draws distribution functions of the capacity for \(M = 2\)-user system assuming \(N_t = 128\) antennas (curves are plotted for one of \(M\) users, because the curves of all users are similar due to the symmetric channel statistics). Compared with the curve of the conventional method (dotted line), that of the proposed method (solid line) is located on the left, which means the capacity becomes lower. It is a natural result that the difference between two curves is larger than the case of single side subarray processing [9]. Additionally, the slope of the curve is more gradual, so outage characteristics is also worse. Here, it should be remarked that those negative aspects on the performance are at the expense of significantly lower calculation cost which make the use of zero forcing based large array transmission from impossible to practical.

The relation between SNR and capacity is shown in Fig. 4. By this figure, it can be verified that the performance of the proposed methods is not affected by SNR as the case of the previous subarray based design method.

To know the relation between subarray size and capacity, graphs of various settings are drawn in Fig. 5. The number of data streams is changed to \(L_m = 4\) when \(S_r = 32\) is adopted to adjust the condition to case of \(S_r = 16\) as possible. The main result which can be seen from this figure is that the performance degradation becomes larger as the length of subarray size becomes shorter. The proposed method cannot increase the capacity even if the stream number is increased because of the limitation of subarray size which is tight situation about the degrees of freedom (on the other hand, the conventional method is dominant in this point, but its real time design is impractical because of the large size).

Fig. 6 depicts the user number versus capacity curves for \(S_t = S_r = 8\). To deal with larger number of users, the transmit antenna number is changed to \(N_t = 256\) (the receiver antenna number is rest as \(N_r = 64\)). This figure shows that the performance degradation of the proposed method from the conventional one becomes larger as the number of users increases. This is because the proposed method is based on subarray, and increment of the user has a larger impact on shorter array (\(N_{t,s} \ll N_t\) and \(N_{r,m,s} \ll N_r\)) from the viewpoint of degrees of freedom, which brings the result of Fig. 6. Therefore, like previous subarray based design [9], the proposed method is suitable high rate transmission to small number of users.

V. CONCLUSION

This paper has presented a low-computational design method of multiuser massive MIMO system based on subarray processing. Different from the approach in [9] considering large array only in the transmitter, this study deals with the case both of transmit and receive sides are equipped with large arrays assuming, for example, the use in the source-relay transmission in relay-aided systems. In this method, subarray processing is utilized in both of MIMO system, but since the previous method cannot simply applied, the procedure of the original approach is modified to manage the synthesis of subarray in both ends. Through numerical simulations, it has
been shown that the performance of the system designed by the proposed method is degraded from that of conventional one, but it can reduce the computational load in SVD based design to the level practical computation is possible.

A future work is the extension of subarray processing to nonlinear processing represented by vector perturbation. Flexible grouping taking into account the correlation in both sides is an important topic of further investigation, but the problem could become much complex compared with the case of one side subarray [9].

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