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MHD Mixed Convection in a Vertical Porous Channel

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***Abstract*—**This work deals with the problem of MHD mixed convection in a completely porous and differentially heated vertical channel. The model of Darcy-Brinkman-Forchheimer with the Boussinesq approximation is adopted and the governing equations are solved by the finite volume method. The effects of magnetic field and buoyancy force intensities are given by the Hartmann and Richardson numbers respectively, as well as the Joule heating represented by Eckert number on the velocity and temperature fields, are examined. The main results show an augmentation of heat transfer rate with the decrease of Darcy number and the increase of Ri and Ha when Joule heating is neglected.

***Keywords*—**Heat sources, magnetic field, mixed convection, porous channel.

I. Introduction

In recent years, the study of MHD mixed convection heat transfer in porous media has attracted the interest of many researchers due to its different applications in various fields of science and technology such as in geophysics, astrophysics, the design of MHD power generators, the techniques for exploration of geothermal sources, etc. This type of problem also arises in electronic boxes during their operation. MHD mixed convection in oscillating regime was treated by Pan and [1]. for vertical channels without taking into account Joule heating. The analysis showed that the application of a horizontal magnetic field contributes to reduce the intensity of the flow velocity. Tasnim and Mahmud [2], [3] used the first and second law of thermodynamics to characterize the flow and heat transfer under the action of a transverse magnetic field in a vertical porous channel saturated with a fluid generating or absorbing heat. An analysis of entropy generation in a square porous cavity for laminar magnetohydrodynamic natural convection was done by [4]. They concluded that entropy generation rate increases with the augmentation of the magnetic field intensity. Barletta [5] presented a numerical study of MHD mixed convection in a porous annulus which surrounds a cable carrying an electric current that creates a radially varying magnetic field. The conservation equations were written according to Darcy's law with taking into account Boussinesq approximation, Joule heating and viscous dissipation. They found that an electromagnetic force of relatively high intensity tends to inhibit the fluid flow even for a high value of pressure gradient. Raveendra Nath [6] studied the problem of MHD mixed convective heat and mass transfer through a porous channel delimited by plane walls subjected to non-uniform temperatures by taking into account viscous dissipation. A numerical study based on the finite difference method was undertaken by [7] to investigate the mixed convection flow through a flat plate immersed in a porous medium and exposed to a transverse magnetic. It has been found an acceleration of fluid motion by reducing the Hartmann number and increasing the permeability, as well as Grashof and Eckert numbers. The problem of MHD mixed convection stagnation point flow towards a vertical plate embedded in a highly porous medium by taking into account radiation and heat generation was investigated by [8]. The governing equations were transformed into self-similar form before being solved numerically.

The present work is a contribution to the works cited above and its main objective is to analyze the effect of a magnetic field on mixed convection flow in a vertical porous channel.

II. Mathematical Formulation

The physical system considered is a channel constituted by two vertical parallels flat plates of length ℓ and separated by a distant H. A porous medium saturated by an electrically conductive fluid filled the channel. The left plate is thermally insulated, while four discrete heat sources of equal length w and dissipating a uniform heat flux q are mounted at equal spacing s on the right wall which is adiabatic elsewhere. The fluid enters the channel with a uniform velocity U_i and a constant temperature T_i. An external magnetic field of uniform strength B_0 is applied perpendicularly to the plane (x, y).

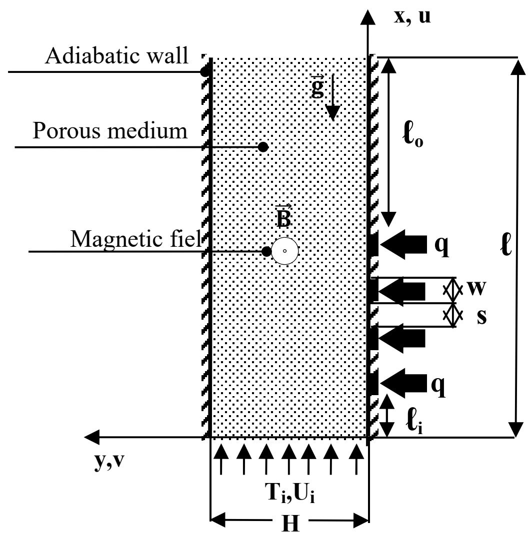

Fig. 1 Physical domain

In order to simplify the problem some assumptions are made: laminar and two-dimensional flow with no internal heat

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generation and neglecting viscous dissipation, incompressible and Newtonian fluid, isotropic and homogeneous porous medium saturated with a single fluid conductor of electricity and in local thermal equilibrium with the solid matrix. The thermo-physical properties of the porous medium and the fluid are taken to be constant except the density variation in the buoyancy force which is determined by using the Boussinesq approximation. The magnetic Reynolds number is taken small enough, so that the induced magnetic field can be neglected in comparison to the applied magnetic field. It is also assumed that there is no applied voltage, which implies the absence of an electrical field, and finally Joule heating is taken into consideration.

The components of the Lorentz force and energy dissipated by Joule effect are deducted from the following equations:

Lorentz Force*:*

$$\vec{F} = \sigma_0(\vec{V} \wedge \vec{B}) \wedge \vec{B} = \vec{J} \wedge \vec{B}$$

Energy dissipated by Joule effect:

$$\frac{J^2}{\sigma_0}$$

σ_0 is the electrical conductivity of the fluid and $\vec{J}$ is the current density vector.

The following variables were used to convert the governing equations and boundary conditions to dimensionless form:

$$X = \frac{x}{H};\ Y = \frac{y}{H};\ U = \frac{u}{\varepsilon U_i};V = \frac{v}{\varepsilon U_i};\ P = \frac{p}{\rho U_i^2};\ \theta = \frac{T - T_i}{\mathrm{qH}/\mathrm{k}}$$

The dimensionless equations are written as follows:

Continuity:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

Momentum:

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{R_\mu}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) - \frac{\varepsilon}{Re\, Da}U - \frac{\varepsilon^2 C}{\sqrt{Da}}|\vec{V}|U + Ri\theta - \varepsilon\frac{Ha^2}{Re}U \tag{2}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{R_\mu}{Re}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) - \frac{\varepsilon}{Re\, Da}V - \frac{\varepsilon^2 C}{\sqrt{Da}}|\vec{V}|V - \varepsilon\frac{Ha^2}{Re}V \tag{3}$$

Energy:

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{R_k}{\varepsilon RePr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) + \varepsilon\frac{Ha^2}{Re}Ec(U^2 + V^2) \tag{4}$$

The dimensionless parameters appearing in the above equations are defined as:

$$R_\mu = \frac{\mu_e}{\mu};\ Re = \frac{\rho U_i H}{\mu};\ Da = \frac{K}{H^2};\ Ha^2 = \frac{\sigma_0 B_0^2 H^2}{\mu}$$

$$R_k = \frac{k_e}{k};\ Pr = \frac{\mu C_P}{k};\ Ec = \frac{kU_i^2}{C_P qH};\ Ri = \frac{g\beta \mathrm{q}H^{4'}}{\nu^2 Re^2}$$

where ρ is the density, μ is the dynamic viscosity, K is the permeability of the porous medium, C_P is the specific heat at constant pressure, k is the thermal conductivity, Re is the Reynolds number, Da is the Darcy number, Pr is the Prandtl number, Ri is the Richardson number, Ec is the Eckert number and Ha is the Hartmann Number. The subscript e indicates porous medium (effective).

The governing equations are associated to the following boundary conditions:

- **At the inlet:** $U = 1$; $V = 0$; $\theta = 0$
- **At the exit:** $\frac{\partial U}{\partial X} = 0$; $V = 0$; $\frac{\partial \theta}{\partial X} = 0$
- **At the right wall**: $U = 0$; $V = 0$

$$\frac{\partial \theta}{\partial Y} = \begin{cases} -\frac{1}{R_k} & sources \\ 0 & elsewhere \end{cases}$$

- **At the left wall:** $U = 0$; $V = 0$; $\frac{\partial \theta}{\partial Y} = 0$

The local Nusselt number is evaluated as:

$$N_u = \begin{cases} \frac{hH}{k} = \frac{1}{\theta_p} & sources \\ 0 & elswhere \end{cases} \tag{5}$$

where h is the local convective heat transfer coefficient.

The average Nusselt number over a heat source "i" is evaluated as:

$$Nu_{mi} = \frac{1}{w}\int_{X_i}^{X_{i+w}} Nu\ dX \tag{6}$$

where Xi is the position of the heat source "i" from the channel inlet.

The global Nussel number is calculated as:

$$Nu_g = \frac{\sum_{i=1}^{4} Nu_{mi}}{4} \tag{7}$$

III. Numerical Procedure

The governing equations with the associated boundary conditions are solved numerically using the finite volume method proposed by [9]. The velocity and pressure are linked by the SIMPLE algorithm, and the power law scheme is employed in the discretizing procedure to treat the diffusion and the convective terms. The obtained algebraic equations are solved using a line by line technique, combining between the tridiagonal matrix algorithm and the Gauss-Seidel method. A non-uniform grid system of 350 × 50 (in X and Y directions, respectively) is employed in both longitudinal and transverse directions with the finer meshes placed near the walls of the channel. For the convergence criteria of the iterative process, the relative variations of velocity components and temperature between two successive iterations are required to be smaller than 10^{-6}.

IV. RESULTS

Due to the great number of control parameters, all computations are performed by keeping fixed the channel length (L = 30), the width (W = 1) and spacing (S = 1) of heat sources with the first one mounted at a distance $L_i = 3$ from the channel inlet, the porosity (ε = 0.9), the Prandtl number (Pr=0.7), the Reynolds number (Re = 250), the inertial coefficient (C = 0.1), the viscosity (R_μ = 1) and the thermal conductivity ratio (R_k = 1). On the other hand, the porous medium permeability translated by the Darcy number Da, the strength of the magnetic field given by the Hartmann number Ha, and the Joule heating and buoyancy represented respectively by the Eckert number Ec and Richardson number Ri are varied.

In order to explain the influence of the magnetic field strength and the buoyancy force on the dynamic regime, we have represented in Figs. 2-4 for Hartmann number varying from 0 to 50 and in Figs. 5-7 for a range of Ri going from 0 to 50, the velocity profiles at the channel exit for different combinations between Da and Ec.

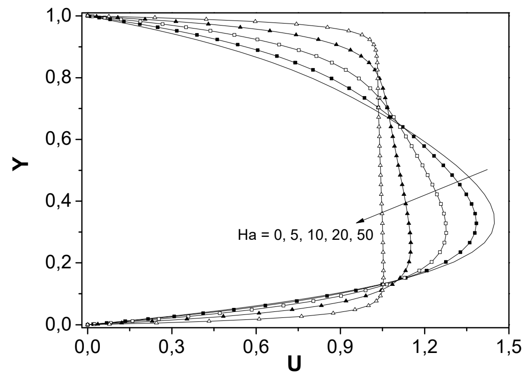

Fig. 2 Velocity profiles at the channel exit for different Hartmann numbers: Ri = 10, Da = 1 and Ec = 0

For the case of a porous medium with a high permeability Da=1, when the Joule heating is neglected, it appears from Fig. 2 that the increase of the Hartmann number has the tendency to slow down the fluid motion at the center of the section and to accelerate it near the walls. This behavior is due to the fact that the application a magnetic field creates an additional resistance to the flow due to the Lorentz force which acts principally in the vertical direction and in the opposite sense of the fluid motion and buoyancy force. We also notice that the velocity profiles from Ha = 0 to Ha = 20 are asymmetrical and the maximum velocity is shifted to the right wall where the heat sources are mounted. This is due to the buoyancy force, which tends to accelerate the fluid flow near this wall further to the increase of its temperature in this region. By increasing Ha beyond the value of 20, the effect of buoyancy force tends to diminish and that of Lorentz force to increase resulting in a symmetrical and flat velocity profile at Ha = 50. The impact of the magnetic field remains similar by introducing a porous medium with moderate permeability in the channel (Da=10^{-3}, Fig. 3). However, the acceleration of the fluid motion at the center is lower than that observed previously and the shapes of velocity profiles are similar to the high permeability case with a magnetic field of great intensity.

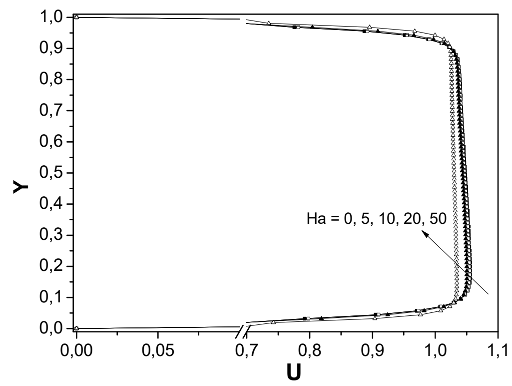

Fig. 3 Velocity profiles at the channel exit for different Hartmann numbers: Ri = 10, Da = 10^{-3} and Ec = 0

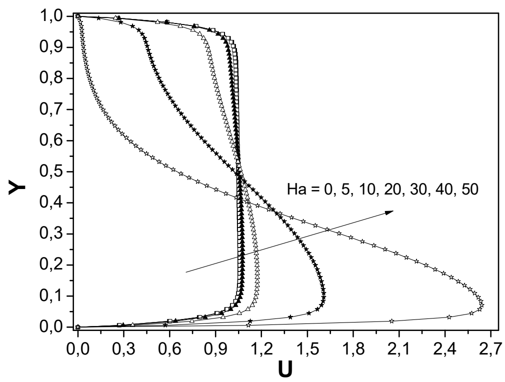

Fig. 4 Velocity profiles at the channel exit for different Hartmann numbers: Ri =10, Da = 10^{-3} and Ec = 0.05

Regarding the influence of the Eckert number, it appears from Fig. 4 that at high values of Ha, the fluid flow is accelerated near the plate where are mounted the heat sources and slowed down elsewhere. This can be explained by the fact that at Ec $\neq$ 0, increasing the magnetic field intensity will cause an accumulation of heat near the right wall coming from the heat sources and Joule heating $(\varepsilon E_C \frac{Ha^2}{Re}(U^2 + V^2))$. The fluid layers will be warmer in this region and their motion will be accelerated under the influence of buoyancy force. At small values of Ha, the velocity profiles tend towards those of case Ec=0.

In order to explain the influence of the buoyancy force, expressed by the variation of Ri, on the dynamic regime, we have illustrated in Figs. 5-7, the velocity profiles at the exit of the channel for different situations of Da and Ec. Globally, the augmentation of the intensity of buoyancy force is manifested by an increase in the fluid velocity near the wall containing the heat sources.

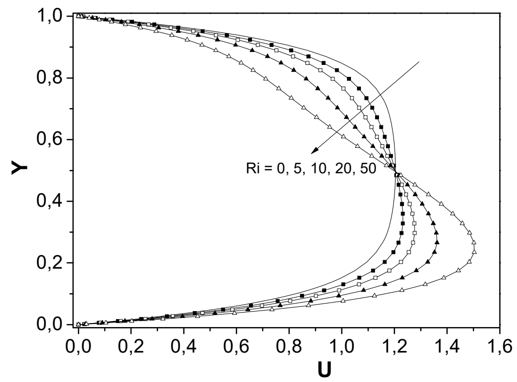

Fig. 5 Velocity profiles at the channel exit for different Richardson numbers: Ha = 10, Da = 1 and Ec = 0

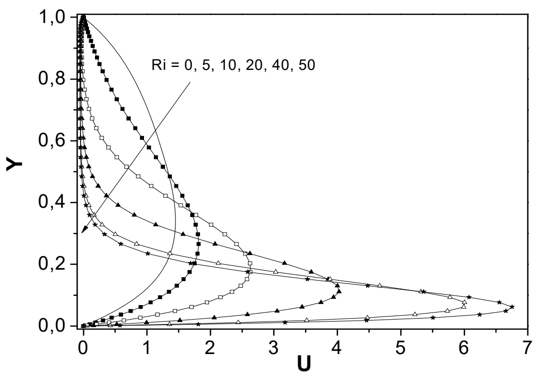

Fig. 6 Velocity profiles at the channel exit for different Richardson numbers: Ha = 10, Da =1 and Ec = 0.05

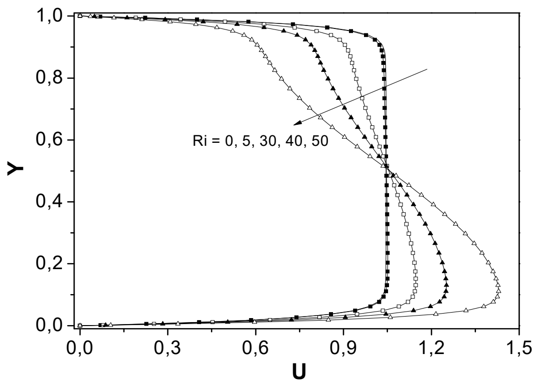

Fig. 7 Velocity profiles at the channel exit for different Richardson numbers: Ha = 10, Da = 10^{-3} and Ec = 0.05

Fig. 8 illustrates the evolution of the global Nusselt number with the Hartmann number for different permeabilities of porous medium. If Joule heating is neglected, it is found an increase of heat transfer with Ha, with the rate of enhancement varying between appreciable for high permeability case (Da=1) and insignificant at low permeability (Da = 10^{-6}). In contrast, when Ec = 0.05 there is a decrease of Nu_g with the strength of magnetic field. This result can be explained by the increase of the amount of heat dissipated by Joule heating to the fluid, which causes a rise in the fluid temperature and consequently leads to a bad cooling of the heat sources.

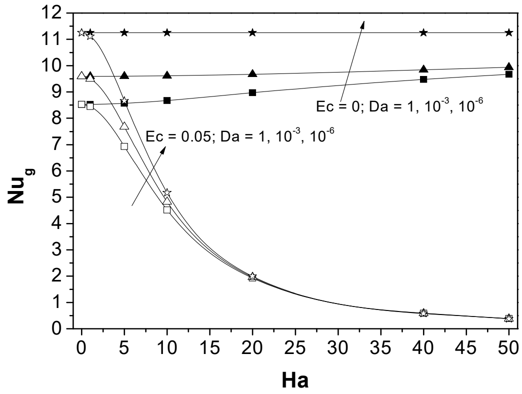

Fig. 8 Variation of Nu_g with Ha for various values of Eckert and Darcy numbers, Ri =10

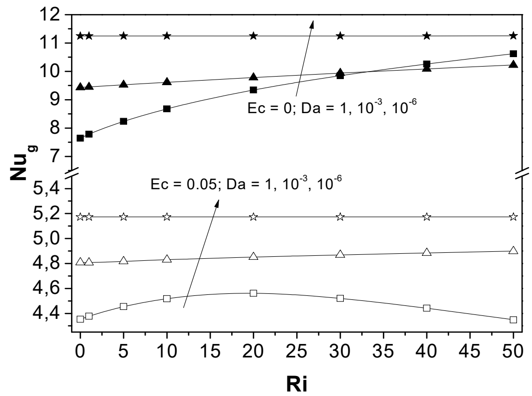

Fig. 9 Variation of Nu_g with Ri for various values of Eckert and Darcy numbers, Ha =10

To highlight the influence of the buoyancy force on heat transfer, we have represented in Fig. 9, the evolution of global Nusselt number with Ri for different situations of Da and Ec. At Ec = 0, the increase of Ri generates a significant heat transfer improvement at Da = 1 where the fluid is easiest to put in motion by the buoyant force; thus providing a better cooling of the heat sources. Taking into account the Joule heating, it is observed for Da = 10^{-3} that increasing Ri is always beneficial to heat transfer despite of heat dissipation by the Joule effect. By against for Da=1, the global Nusselt number initially increases until Ri = 20, then decreases and tends towards a value close to the forced convection case (Ri = 0) at high value of the buoyancy force (Ri = 50).

In order to explain the behavior of Ri at Da = 1 and Ec=0.05, we have depicted in Fig. 10 the evolution of global Nusselt number with Ri for different values of Ha. For Ha less than the value of 8, there is improvement of heat transfer with the increase in the intensity of buoyancy force, and from Ha=9 the change of evolution with Ri begins to appear. This can be explained as follows: the increase of Ri causes an acceleration of the fluid which is normally beneficial to the heat exchange. However, the combination of this situation with a high intensity of magnetic field will generate a strong dissipation of heat by Joule effect which will dominate the heat transfer and lead to a situation of bad cooling of heat sources.

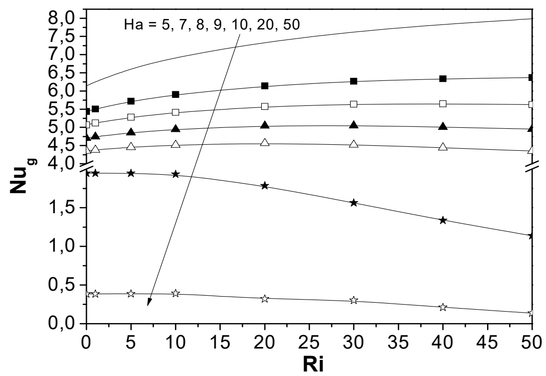

Fig. 10 Variation of Nu_g with Ri for various values of Hartmann, Da=1 and Ec = 0.05

V. Conclusion

The present work is a numerical modeling of mixed convection in a porous channel in the presence of heat sources and an external magnetic field. The highlighting of some control parameters such as Darcy number, Hartmann number, Eckert number and Richardson number, helped us to evaluate the effects caused by these latter on fluid flow and heat transfer in the channel. The main results can be summarized as follows:

- Without Joule heating, increasing the magnetic field intensity has an impact on the dynamic field similar to that of reducing porous medium permeability.
- Increasing Hartmann or Richardson numbers for Ec = 0, enhances the heat transfer.
- Increasing buoyancy force provides a good alternative for heat transfer improvement especially at moderate permeability.
- At large Ha and with the consideration of heat dissipation by Joule effect, it will be interesting to use a porous medium of low permeability in order to avoid situations of bad cooling of heat sources, especially at high Ri.

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