Abstract—Optimizing the parameters in the controller plays a vital role in the control theory and its applications. Optimizing the PID parameters is finding out the best value from the feasible solutions. Finding the optimal value is an optimization problem. Inverted Pendulum is a very good platform for control engineers to verify and apply different logics in the field of control theory. It is necessary to find an optimization technique for the controller to tune the values automatically in order to minimize the error within the given bounds. In this paper, the algorithmic concepts of Harmony search (HS) and Genetic Algorithm (GA) have been analyzed for the given range of values. The experimental results show that HS performs well than GA.

Keywords—Genetic Algorithm, Harmony Search Algorithm, Inverted Pendulum, PID Controller.

I. INTRODUCTION

A. Inverted Pendulum

An inverted pendulum is a pendulum which has its centre of mass above its pivot point. It is often implemented with the pivot point mounted on a cart that can move horizontally and may be called a cart and pole. Most applications limit the pendulum to 1 degree of freedom by affixing the pole to an axis of rotation. Whereas a normal pendulum is stable when hanging downwards, an inverted pendulum is inherently unstable, and must be actively balanced in order to remain upright; it can be done either by applying a torque at the pivot point, by moving the pivot point horizontally as part of a feedback system, changing the rate of rotation of a mass mounted on the pendulum on an axis parallel to the pivot axis and thereby generating a net torque on the pendulum, or by oscillating the pivot point vertically. The inverted pendulum is a classic problem in dynamics and control theory and is used as a benchmark for testing control strategies. A second type of inverted pendulum is a tilt meter for tall structures which consists of a wire anchored to the bottom of the foundation and attached to a float in a pool of oil at the top of the structure which has devices for measuring movement of the neutral position of the float away from its original position.

The Inverted pendulum also has certain parameters like mass of the pendulum, length, force to be applied to the cart and so on [1]. Thus the parameter and the value needed for this work are given in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nomenclature</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Mass of the cart</td>
<td>0.5kg</td>
</tr>
<tr>
<td>m</td>
<td>Mass of the pendulum</td>
<td>0.2kg</td>
</tr>
<tr>
<td>B</td>
<td>Friction of the cart</td>
<td>0.1N/m/sec</td>
</tr>
<tr>
<td>L</td>
<td>Length to pendulum center of mass</td>
<td>0.3m</td>
</tr>
<tr>
<td>I</td>
<td>Inertia of the pendulum</td>
<td>0.006 kg*m^2</td>
</tr>
<tr>
<td>Theta</td>
<td>Pendulum angle from vertical</td>
<td>In degree</td>
</tr>
</tbody>
</table>

B. PID Controller

A controller is a device, historically using mechanical, hydraulic, or electronic techniques often in combination, but more recently in the form of a microprocessor or computer, which monitors and physically alters the operating conditions of a given dynamical system. Typical applications of controllers are to hold settings for temperature, pressure, flow or speed. A system can either be described as a Multiple Input and Multiple Output (MIMO) system, having multiple inputs and outputs, therefore requiring more than one controller; or a Single Input and Single Output SISO system, consisting of a single input and single output, hence having only a single controller. Depending on the set-up of the physical system, adjusting the system's input variable (assuming it is SISO) will affect the operating parameter, otherwise known as the controlled output variable. Upon receiving the error signal that marks the disparity between the desired value (setpoint) and the actual output value, the controller will then attempt to regulate controlled output behaviour. The controller achieves this by either attenuating or amplifying the input signal to the plant so that the output is returned to the set point.

A proportional-integral-derivative (PID controller) is widely used in industrial control systems. A PID controller finds an error value as the difference between a measured process variable and a target point. The controller tries to
minimize the error by adjusting the process through use of a variable. It is necessary to tune the controller parameters to achieve good control performance with the proper values of tuning parameters [2].

The PID controller algorithm involves three separate constant parameters such as the proportional, the integral and derivative denoted as P, I, and D. These values can be interpreted in terms of time: P based on the present error, I based on the accumulation of past errors, and D based on the prediction of future errors. By tuning the three parameters in the controller algorithm, it can provide control action designed for specific process requirements. The response of the controller can be described in terms of the responsiveness of the controller to an error. The signal error, e(t), enters the PID controller block and the output signal is the sum of the error signal affected by the proportional, integral and derivative actions [3]. The resulting signal of the PID controller where e(t) = r(t) - y(t) in which r(t) is the input signal and y(t) is the output signal,

\[ u(t) = K_p e(t) + K_i \int e(t) dt + K_d \frac{d}{dt} e(t) \]  

(1)

II. REVIEW ON RELATED WORKS

In the control engineering, the PID controller has been employed for a very long time due to its good characteristics such as the simplicity in architecture, easy implementation, and mature theoretical analysis. Hence, in many real industrial applications, the PID controller is still widely used even though lots of new control techniques have been proposed. The key idea of designing a PID controller is the determination of three parameters of PID controller, proportional gain \( K_p \), integral gain \( K_i \), and derivative gain \( K_d \). The inverted pendulum control problem is usually presented as a pole balancing task. The system to be controlled consists of a cart and a rigid pole hinged to the top of the cart. The cart can move left or right on a one-dimensional bounded track, whereas the pole can swing in the vertical plane determined by the track [4]. Control force is applied to the system to prevent the pole from falling while keeping the cart within the specified limits.

Shen et al. have proposed the general form of PID controller satisfying the robust controller principles and conditions [5]. Since the structure of the PID controller is fixed, the requirement is to find the stable set of values of proportional gain \( K_p \), derivative gain \( K_d \) and the integral gain \( K_i \) by satisfying the various inequalities. The robust stability is ensured by finding the Hurwitz of the closed loop characteristic polynomial.

Gregory et al have proposed that in the analytical method of the tuning, PID parameters are calculated from analytical or algebraic relations between a plant model and an objective [6]. These can lead to an easy-to-use formula and can be suitable for use with online tuning, but the objective needs to be in an analytical form and the model must be accurate. In the Heuristic methods of tuning, these are evolved from practical experience in manual tuning (such as the Z-N tuning rule) and from artificial intelligence (including expert systems, fuzzy logic and neural networks). Again, these can serve in the form of a formula or a rule base for online use, often with tradeoff design objectives.

Ziegler and Nichols proposed one of the classical methods of Ziegler-Nichols which requires very little information about the tuning process [7]. The method is a trial and error tuning method based on oscillations that was first proposed by Ziegler and Nichols. There are several disadvantages in the trial and error tuning process. First, the system needs to be brought its limit of instability and a number of trials may be needed to bring the system to this point. Another disadvantage is that the tunings do not work well on all processes. A third disadvantage of the method is that it can only be used on processes for which the phase lag exceeds beyond -180 degrees at high frequencies.

Gregory et al. have proposed that in the analytical method of the tuning, PID parameters are calculated from analytical or algebraic relations between a plant model and an objective [8]. These can lead to an easy-to-use formula and can be suitable for use with online tuning, but the objective needs to be in an analytical form and the model must be accurate. In the Heuristic methods of tuning, these are evolved from practical experience in manual tuning (such as the Z-N tuning rule) and from artificial intelligence.

PID controllers are tuned either manually or using rule-based methods. Manual tuning methods are iterative and slow, and if used on hardware, they can cause damage. Rule-based methods also have some drawbacks such as they do not support certain types of plant models, or plants with little or no time delay. It can be automatically tune PID controllers to achieve the optimal system design and to achieve design requirements, even for plant models that rule-based methods cannot handle well.

The empirical methods have the advantage of applying easy rules to simple mathematical models, but do not provide as good results as expected [9]. The analytical methods can deal with some higher order models; they are more precise and have the advantage of improving some specific parameters of the system response such as stability, rising time, steady state error, overshoot, etc. Both empirical and analytical methods are time consuming.

III. METHODOLOGY

A. Optimization

An optimization problem with discrete variables is known as a combinatorial optimization problem. At present, there are so many optimization techniques available. The familiar methods linear programming, the quadratic programming, the dynamic programming, the Simplex method and the gradient methods are deterministic methods which make possible to resolve some types of optimization problems in a finished time period. In observation these problems are too complex and require too much time to resolve by deterministic methods. Metaheuristics are stochastic optimization it finds a solution in
a reasonable time. Metaheuristics have usually an iterative behaviour. The same pattern is repeated until a stopping criterion is met at the beginning for optimization. Optimization problems have the following characteristics:

- Different decision alternatives are available.
- Additional constraints limit the number of available decision alternatives.
- Each decision alternative can have a different effect on the evaluation criteria.
- An evaluation function defined on the decision alternatives describes the effect of the different decision alternatives.

For optimization problems, a decision alternative should be chosen that considers all available constraints and maximizes/minimizes the evaluation function. Real-world optimization problems are often very challenging to solve, and many applications have to deal with NP-hard problems. To solve such problems, optimization tools have to be used, though there is no guarantee that the optimal solution can be obtained. In fact, for NP problems, there are no efficient algorithms at all. As a result, many problems have to be solved by trial and errors using various optimization techniques.

### B. Genetic Algorithm

A Genetic Algorithm (GA) is a computational abstraction of biological evolution that can be used to solve optimization problems. The GA, proposed by Holland (1975), is a probabilistic optimal algorithm that is based on the evolutionary theories. GA algorithm is population-oriented. Consecutive populations of feasible solutions are generated in a stochastic manner following laws similar to that of natural selection [10]. Fig. 2 depicts the flow of Simple GA.

There are three operators must be specified to construct the complete structure of the GA procedure; selection, crossover and mutation operators. To successfully apply GA and to solve a problem one needs to determine the following:

- Solution representation or the chromosome encoding.
- Employing the fitness function which accurately represents the value of the solution.
- Identifying the genetic operators to use.
- Discovering the parameter that is suitable.

The procedure for GA can be as follows:

1. It starts with an initial population whose elements are called chromosomes. The chromosome has a fixed number of variables which are called genes.
2. For each chromosome in a population, calculate the fitness function based on the objective function. Each individual is assigned a fitness value according to that it finds good solution to the problem.
3. The selection operator cares with selecting an intermediate population from the current one in order to be used by the other operators; crossover and mutation. In the selection process, chromosomes with higher fitness function values have a greater chance to be chosen than those with lower fitness function values. The least fit members of the population are less likely to get selected for reproduction.
4. Pairs of parents in the intermediate population of the current generation are probabilistically chosen to be mated in order to reproduce the offspring. In order to increase the changeability structure, the mutation operator is applied to alter one or more genes of a probabilistically chosen chromosome.
5. Loop until it meets the termination criteria such that the maximum number of iterations.

### C. Harmony Search Algorithm

Harmony search (HS) is a music-inspired algorithm has been applied to various optimization problems [11]. The harmony in music is similar to find the optimality in an optimization process. Each musician in music performance plays a musical note at a time, and those musical notes together make a harmony. As similar, each variable in optimization has a value at a time, and those values together make a solution vector. As the music group improves their harmonies practice by practice, the algorithm improves its solution vectors iteration by iteration. The new solution is replaced by worst solution in the harmony. The HS metaheuristic algorithm was based on natural musical performance processes that occur when a musician searches for a better state of harmony, such as during jazz improvisation process.

In music improvisation, each player sounds any pitch within the possible range, together creates one harmony. If all the pitches create a good harmony, it is stored in each player’s memory, and the possibility to make a good harmony is increased next time. Parameters of HS are Harmony Harmony Memory (HM), Harmony Memory Size (HMS), Maximum number of Iterations.

First of all, the Algorithm fills the Harmony Memory with random values. The “good” harmonies will be the material (similar with parents in GA) for the creation of new, even better harmonies. In order to use this process effectively, Harmony Memory Considering Rate (HMCR) was defined. This process is defined as Memory Consideration and it is
very important because it ensures that good harmonies (values that give good results) will be considered through the solution. This index will specify the probability that New Harmony will include a value from the historical values that are stored in the Harmony Memory. If this rate is too low, only few elite harmonies will be selected. Every component of the New Harmony chosen from HM, is likely to be pitch-adjusted. The New Harmony will include the value $X_i^{new}$ which is given in (2):

$$X_i^{new} = X_i \pm \text{Random} \times \text{bw}$$

where, $X_i$ is the existing pitch stored in HM, Random is a random number between 0 and 1, and bw is the bandwidth of the adjustment.

Pitch Adjustment is similar to Mutation procedure in GA. Pitch Adjusting Rate (PAR) usually takes small values. Since, PAR as a very important factor, responsible for the convergence. Pitch Adjusting is the local search mechanism which controls the ability for fine-tuning. The next step is to select a totally random value from the possible value range. Although pitch adjustment has a similar role, it is limited in a local area. Randomization can drive the algorithm to explore the whole range and attain the global optimality.

The Pseudo code for HS can be as follows:

Generate an initial solution of Harmony memory size

- Compute the fitness value of each harmony in Harmony Memory (HM)

For $i=1$ to Number of iterations (or stop criteria)

- Choose a value in random and check with HMCR and choose a solution
  - if it satisfies the condition with PAR
    - Update the solution with the bandwidth
  - else
    - choose a new solution within the given bounds
- Evaluate its fitness value
  - if the new fitness value is less than the worst one in HM, replace with the new harmony
- end

D. Solution Representation

The controller values need to be optimized in order to get the optimal solution from all the feasible solution to satisfy the objective function. In the context of PID controller, each solution is constructed as:

$$K_p \quad K_i \quad K_d$$

Fig. 3 Solution representation

E. Fitness Function

One harmony in the harmony memory represents one possible solution for assigning the $K_p$, $K_i$, $K_d$ for controllers. Therefore, a Harmony memory represents a number of candidate solutions for the controller. At the initial stage, each harmony in the harmony memory randomly chooses different $K_p$, $K_i$, $K_d$. The fitness function used is the integral square error and it can be calculated based on (3):

$$\int_0^\infty e^2(t)dt$$

IV. EXPERIMENTAL ANALYSIS

A. Datasets

We have implemented the optimization algorithm on the Simulink model of the inverted pendulum. In order to minimize the error of the inverted pendulum, the genetic algorithm and harmony search algorithm are used. The control parameters of the above mentioned algorithms are given below: GA settings: Mutation Probability=0.1, Crossover Probability=0.8 have been used. HS settings: HMCR=0.9, PAR=0.3 and Bandwidth=0.01 have been used.

An inverted pendulum system is an example commonly found in control system research literature. Its popularity derives in part from the fact that it is unstable without control, that is, the pendulum will simply fall over if the cart isn't moved to balance it. Additionally, the dynamics of the system are nonlinear. The PID controller design is obtained from the MATLAB through Simulink shown in Fig. 4.

An inverted pendulum on a cart consists of having a horizontally moving base. The cart is restricted to linear motion and is subject to forces resulting in or hindering motion. The control system is to balance the inverted pendulum by applying a force to the cart that the pendulum is attached to. The controller needs to obtain the optimal value and to overcome the disturbance in the cart. The model is obtained through the simulink in MATLAB shown in Fig. 8.

Due to its excellent speed control characteristics, the DC motor has been widely used in industry even though its maintenance costs are higher than the induction motor. As a result, Speed control of DC motor has attracted considerable research and several methods have evolved. Proportional-Integral Derivative (PID) controllers have been widely used for speed and position control of DC motor. The control design is obtained from the MATLAB using Simulink shown in Fig. 10.

Figs. 5, 9 and 11 show the comparison of the fitness value obtained for the genetic algorithm and harmony search algorithm applied in the simulink model of the inverted pendulum. The HS algorithm performs well in minimizing the error than GA in the model of the inverted pendulum. The solution is obtained through the model is the optimal value for the controller. The solution obtained through the genetic algorithm causes the premature convergence. It causes the update of the solution within the population. Figs. 6 and 7 shows the response of the system to the optimal values obtained using GA and HS.

Choosing parameters and methods in Harmony Search might result into very different results. The HS algorithm generates a new vector, after considering all of the available vectors and has many advantages over the other meta heuristic algorithms. The initialization of New Harmony, objective function value as fitness, updating of New Harmony and maximum iteration numbers as stopping criterion are used in
the experimental analysis. The algorithm requires many parameters to be chosen such that it will result in achieving the desired output performance. The algorithm uses the given parameter and updates the New Harmony and terminate until it satisfies the termination criteria [12]. The Harmony Search algorithm updates the new solution periodically. The new solutions are generated during the iteration so that there will be a balance between the intensification and diversification in the solution.
V. CONCLUSION

Parameters adjustment at different problems takes more time up by hard mathematical calculation. The manual tuning methods for the parameters in the controller are not efficient. The automatic tuning methods result in the optimal solution and better performance. In this work, the optimization techniques such as genetic algorithm and harmony search algorithm are applied for the simulink model for the inverted pendulum. The experimental results are analyzed based on the error values. The Harmony search algorithm performs well than the genetic algorithm within the given range of values.

REFERENCES