World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:9, No:6, 2015

Genetic Algorithm and Padé-Moment Matching for Model Order Reduction

Shilpi Lavania, Deepak Nagaria

Abstract**—**A mixed method for model order reduction is presented in this paper. The denominator polynomial is derived by matching both Markov parameters and time moments, whereas numerator polynomial derivation and error minimization is done using Genetic Algorithm. The efficiency of the proposed method can be investigated in terms of closeness of the response of reduced order model with respect to that of higher order original model and a comparison of the integral square error as well.

Keywords**—**Model Order Reduction (MOR), control theory, Markov parameters, time moments, genetic algorithm, Single Input Single Output (SISO).

I. Introduction

OVER the past few decades the problems on model order reduction have widely studied. The idea behind model order reduction of a system is to change the original system by an approximating system. This approximated system possess smaller state-space dimension, such that, it can significantly reduce design time and allow for belligerent design strategies. Therefore model order reduction technology is a necessity in many fields such as fast simulation of Control Systems, Microelectronics, Analysis of RF circuits, analysis of clock network and delay of signal circuits, etc. Reduced-order modeling is well established for linear systems as well as for dynamic systems [1]-[4].

In system foundation, the Model order reduction is ingenious, sophisticated, powerful and influencing. Lower order approximants, provides ease of simulation and implementation of prototype devices i.e. Controllers in real-time implementation. Since the large scale system requires large simulation time and extensive parallel computing, makes it costly to be implemented in real time. Reduced order modeling provides approximated system which is as much efficient as that of the original system with an additional advantage of small dimensions. This advantage has leaded the generation of model order reduction to an extensively inventive area of research [5].

Many techniques for Model Order Reduction in time domain and frequency domains have been proposed. In time domain Singular method, Modal method, Optimal Solution etc. have been proposed [6]-[11]. Whereas in frequency domain; Padé approximation, Routh approximation, and Continued fraction etc. have been proposed [12]-[15].

Padé approximation is one of the most attractive and powerful tool for model order reduction. It was first proposed by Padé [12], [14]. After this many research has contributed their concepts based on Padé Approximation. In Padé approximation the technique for reduction used is based on the matching the term appearing in the power series expansion about s=0. Padé approximation has advantages that various parameters values of the original system i.e. easy computation, fitting time moments, and steady state are same as that of the reduced order model [12]. Shamsash [16] however has stated a disadvantage of Padé approximation i.e. sometimes it produce unstable model form a stable HOS. With the development of various heuristic approaches in last few decades, have attracted researchers to use these algorithms for model order reduction. Genetic Algorithm is one of the evolutionary approaches [17].

This paper proposes a hybrid method for obtaining the reduced order model of a high-order system (HOS).This method is based on Padé approximation that gives equal emphasis on matching of both Time moments and Markov parameters. Reduced order approximants of denominator polynomial are derived from the above mentioned method whereas, numerator polynomial is evaluated by Evolutionary technique i.e. Genetic Algorithm (GA). Also the error minimization of responses of higher order system and reduced order approximants is further reduced by using Genetic Algorithm (GA).

II. Model Order Reduction Procedure

This section deals with the procedure for model order reduction using proposed method: For an nth order system [18], [19]:

$$G_n(s) = \frac{a_1 s^{n-1} + a_2 s^{n-2} + ... + a_n}{s^n + b_1 s^{n-1} + ... + b_n} \tag{1}$$

$$= t_0 + t_1 s + t_2 s^2 + ... \tag{2}$$

(Expansion about s=0)

$$= M_1 s^{-1} + M_2 s^{-2} + ... \tag{3}$$

(Expansion about s=∞)

The objective is to obtain an approximated rth order model:

Shilpi Lavania is Research Scholar at Faculty of Electronics & Communication Engineering, Uttar Pradesh Technical University, Lucknow, India (e-mail: shilpi.lavania.vlsi@ gmail.com).

Dr. Deepak Nagaria is Associate Professor and Head Department of Electrical Engineering, Bundelkhand Institute of Engineering & Technology, Jhansi, India (e-mail: deepaknagaria@gmail.com).

$$G_n(s) = \frac{e_1's^{n-1} + e_2's^{n-2} + ... + e_r'}{s^n + q_1's^{n-1} + ... + q_r'} \tag{4}$$
$$= t'_0 + t_1's + t_2's^2 + ...$$

(Expansion about s=0)

$$= M_1's^{-1} + M_2's^{-2} + ...$$

(Expansion about s=∞)

A. Steps for Obtaining Reduced Order Denominator

It is easy to validate that the following equations hold true [17]: For i=1;

$$t_r' = \frac{e_r'}{q_r'} \tag{5}$$

For i=2,3,4……..

$$t_i' = e'_{r+1-i} + \sum_{j}^{i-1} (t'_i q'_{r+j-i}) q_r'^{-1} \tag{6}$$

and $e'_i = 0$ for $i \le 0; q'_0 = 1; q'_i = 0$ for $i \le -1$

$$M'_i = e'_i \quad \text{i=1} \tag{7}$$

$$M'_i = e'_i - \sum_{j=1}^{i-1} M'_i \, q'_{i-j} \quad \text{i=2,3,……} \tag{8}$$

The obtained reduced order model must satisfy following equations:

$$t_0' = t_0 \to e'_{r+1} = \sum_{j=1}^{i} t_j q'_{i-j} \,, \qquad i \in \{1,2,..\delta\}$$
$$M_i' = M_i \to e'_i = \sum_{j=1}^{i} M_j q'_{i-j} \,, \qquad i \in \{1,2,..\gamma\} \tag{9}$$

where $\delta + \gamma = 2r$ .By using (5)-(9) approximated reduced order models can be achieved and reduced order approximant of denominator can be derived.

B. Steps to Obtain Reduced Order Numerator Using GA

GA is based on principles inspired from the genetic and evolution mechanisms observed in natural systems. Their basic principle is the maintenance of a population of solutions to the problem that evolves in time. They are based on the triangle of genetic reproduction, evaluation, and selection [20]. Genetic reproduction is performed by means of two basic genetic operators: crossover and mutation. Evaluation is performed by means of the fitness function that depends on the specific problem. Selection is the mechanism that selects parent individuals with probability proportional to their relative fitness [21].

Genetic algorithm (GA) has been used to solve difficult engineering problems that are complex and difficult to solve by conventional optimization methods. GA maintains and manipulates a population of solutions and implements a survival of the fittest strategy in their search for better solutions. The fittest individuals of any population tend to reproduce and survive to the next generation thus improving successive generations. The inferior individuals can also survive and reproduce. Implementation of GA requires the determination of six fundamental issues: chromosome representation, selection function, the genetic operators, initialization, termination and evaluation function. Brief descriptions about these issues are provided in the following sections [22].

1. Chromosome Representation

Chromosome representation scheme determines how the problem is structured in the GA and also determines the genetic operators that are used. Each individual or chromosome is made up of a sequence of genes. Various types of representations of an individual or chromosome are: binary digits, floating point numbers, integers, real values, matrices, etc. Generally natural representations are more efficient and produce better solutions. Real-coded representation is more efficient in terms of CPU time and offers higher precision with more consistent results [22].

2. Selection Function

To produce successive generations, selection of individuals plays a very significant role in a genetic algorithm. The selection function determines which of the individuals will survive and move on to the next generation. A probabilistic selection is performed based upon the individual's fitness such that the superior individuals have more chance of being selected. There are several schemes for the selection process: roulette wheel selection and its extensions, scaling techniques, tournament, normal geometric, elitist models and ranking methods. The selection approach assigns a probability of selection P_j to each individuals based on its fitness value. In the present study, normalized geometric selection function has been used. In normalized geometric ranking, the probability of selecting an individual Pi is defined as [22]:

$$P_i = q^{'}(1-q)^{r-1}$$
$$q' = \frac{q}{1-(1-q)^P} \tag{10}$$

where, q= probability of selecting the best individual, r = rank of the individual (with best equals 1), P = population size [22].

3. Genetic Operators

The genetic operators provide the basic search mechanism of the GA. There are two basic types of operators: crossover and mutation. These operators are used to produce new solutions based on existing solutions in the population. Crossover takes two individuals to be parents and produces two new individuals while mutation alters one individual to produce a single new solution. The following genetic operators are usually employed simple crossover, arithmetic crossover and heuristic crossover as crossover operator and uniform mutation, non-uniform mutation, multi-non-uniform mutation, boundary mutation as mutation operator. Arithmetic crossover and non-uniform mutation are employed in the present study as genetic operators. Crossover generates random number r from a uniform distribution from 1 to m and creates two new individuals by using:

$$x_i' = \begin{Bmatrix} x_i & if & i < r \\ y_i & otherwise & \end{Bmatrix}$$
$$y_i' = \begin{Bmatrix} y_i & if & i < r \\ x_i & otherwise & \end{Bmatrix} \quad (11)$$

Arithmetic crossover produces two complimentary linear combinations of the parents, where r = U (0, 1) [22]:

$$\overline{X}' = r\overline{X} + (1-r)\overline{Y}$$
$$\overline{Y}' = r\overline{Y} + (1-r)\overline{X} \quad (12)$$

Non-uniform mutation randomly selects one variable j and sets it equal to a non-uniform random number [21].

$$x_i' = \begin{Bmatrix} x_i + (b_i - x_i) f(G) & if & r_1 < 0.5 \\ x_i + (x_i + a_i) f(G) & if & r_1 \geq 0.5 \\ x_i & otherwise & \end{Bmatrix} \quad (13)$$

where

$$f(G) = \left(r_2 \left(1 - \frac{G}{G_{\max}} \right) \right)^b \quad (14)$$

$r1$, $r2$ = uniform random numbers (between 0 and 1), G = current generation, Gmax = maximum no. of generations, b = shape parameter [22].

4. Initialization, Termination, and Evaluation Function

An initial population is needed to start the genetic algorithm procedure. The initial population can be randomly generated or can be taken from other methods. The GA moves from generation to generation until a stopping criterion is met. The stopping criterion could be maximum number of generations, population convergence criteria, lack of improvement in the best solution over a specified number of generations or target value for the objective function. Evaluation functions or objective functions of many forms can be used in a GA so that the function can map the population into a partially ordered set [22]. The computational flowchart of the GA optimization process employed in the present study is given in Fig. 1 [21], [22].

III. Application of Proposed Method on Numerical Example

This section defines step by step procedure to achieve reduced order model by using numerical examples.

Example1. Consider the following Higher Order System (HOS) [24].

$$G(s) = \frac{2s^5 + 3s^4 + 16s^3 + 20s^2 + 8s + 1}{2s^6 + 33.6s^5 + 155.94s^4 + 209.46s^3 + 102.42s^2 + 18.3s + 1} \quad (15)$$

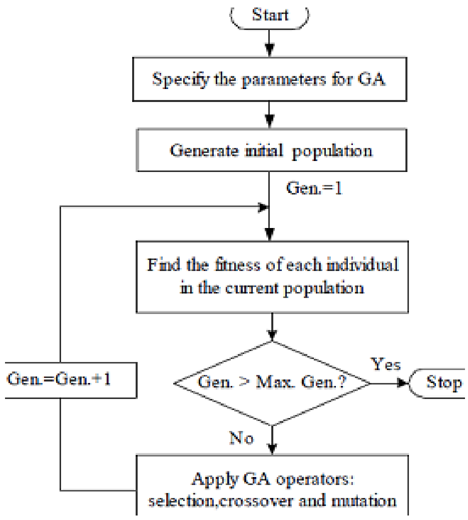

Fig. 1 Flowchart of genetic algorithm [21]

Consider that the reduced order model of the form [19]:

$$G(s) = \frac{e'_1 s + e'_2}{s^2 + q'_1 s + q'_2} \quad (16)$$

A. Reduction Procedure

The denominator of approximated second order model is derived by Padé approximation based Moment Matching technique whereas numerator is derived by Genetic Algorithm (GA). Following the complete procedure for deducing the reduced order model is described below:

1) Steps for Obtaining Reduced Order Denominator

Step1. Expand (10) around $s = 0$ and $s = \infty$:

$$= 1 - 10.3s + 106.07s^2 + ...$$
$$= 1s^{-1} - 15.3s^{-2} + 187.27s^{-3} + ... \quad (17)$$

Step2. On expanding (11) about $s = 0$ and $s = \infty$:

$$= t_1' + t_2' s + t_3' s^2 + ...$$
$$= M_1' s^{-1} + M_2' s^{-2} + M_3' s^{-3} + ... \quad (18)$$

The time moments and the Markov's parameters expansions can be derived byexpending transfer function around $s = 0$ and $s = \infty$.

Step3. In order to get perfect matching 2r terms are required to be matched:

$$t_1 = t_1'$$
$$M_1 = M_1'$$
$$t_2 = t_2'$$
$$M_2 = M_2' \quad (19)$$

Step4. A second order reduced model can be obtained by considering above assumptions and solving the following equations. Using following equations variable for reduced order model (11) can be obtained [19]:

$$\begin{aligned} t_1' &= \frac{e_2'}{q_2'} \\ t_2' &= \frac{e_1' - t_1' q_1'}{q_2'} \\ M_1' &= e_1' \\ M_2' &= e_2' - M_1' q_1' \end{aligned} \tag{20}$$

For better time response approximation it is required to give equal consideration to both time moments and Markov's parameters. As already explained that 2r terms are needed to be matched that must satisfy δ+γ=2r, hence γ=δ=2 is required to be considered. On solving (15):

$$\begin{aligned} e_1' &= 1 \\ e_2' &= 1.53 \\ q_1' &= 16.83 \\ q_2' &= 1.53 \end{aligned} \tag{21}$$

Thus required reduced model can be obtainedas:

$$D^r(s) = s^2 + 16.83s + 1.53 \tag{22}$$

Step5. When the objective function is subjected to Genetic Algorithm the reduced order approximant of numerator is obtained as:

$$N^r = 0.667s + 1.503 \tag{23}$$

Step6. Hence the required reduced order model is obtained as:

$$G_2(s) = \frac{0.667s + 1.503}{s^2 + 16.83s + 1.53} \tag{24}$$

IV. Simulation Results

This section deals with the simulation results for example (1). Fig. 2 provides a comparison of time response of proposed method with time response of other well-known model order reduction methods for the same example (1). This comparison has shown that the proposed method is best approximated to the original model.

A performance comparison of proposed method with other well-known methods is provided in Table I. This performance comparison is made on the basis of an error index which is known as Integral Square Error (ISE) [23], [26] is shown in Table I. This integral square error ISE is an error between the transient part of the original higher order model and the reduced order model. Lower the value of ISE, responses will be more approximated.

$$ISE = \int_0^{t_\infty} \left[y(t) - y_r(t) \right]^2 dt$$

where, $y(t)$ and $y_r(t)$ are the step responses of original and proposed model respectively.

TABLE I
Comparison on the Basis of ISE ofExample1

Method	Model	ISE
Original	$\dfrac{2s^5+3s^4+16s^3+20s^2+8s+1}{2s^6+33.6s^5+155.94s^4+209.46s^3+102.42s^2+18.3s+1}$	-
Proposed	$G_2(s) = \dfrac{0.667s+1.503}{s^2+16.83s+1.53}$	4.166 e-004
Nidhi Singh [24]	$G_2(s) = \dfrac{5.9979s+1}{87.97s^2+15.96s+1}$	0.0650
Mahmoud and M.G.Singh [25]	$G_2(s) = \dfrac{0.0227s+0.0131}{s^2+0.22s+0.0131}$	8.7854

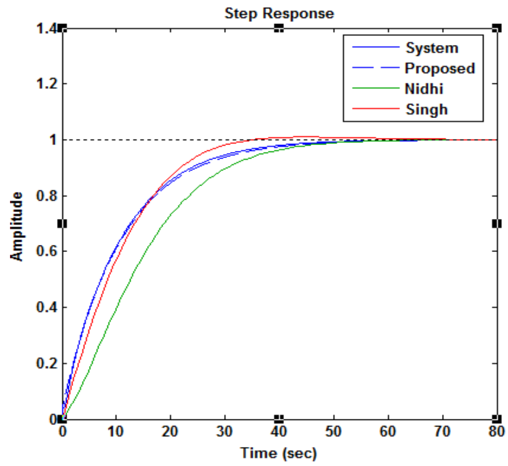

Fig. 2 Comparison of step responses of higher order system, proposed method with previously proposed methods

V. Conclusion

The above proposed method which is based on Padé approximation and evolutionary technique Genetic Algorithm (GA) provides most approximated results than the previously proposed reduced order methods. The presented technique has given equal importance to both time moment matching and Markov's parameters. Also the error minimization is performed using Genetic Algorithm for higher degree of correctness and for getting most approximated results. A comparison on the basis of ISE calculation is presented in this paper. The results and comparison of ISE proves the method to be most approximated. However this technique can be further investigated for MIMO systems.

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