A Mathematical Framework for Expanding a Railway’s Theoretical Capacity

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Abstract—Analytical techniques for measuring and planning railway capacity expansion activities have been considered in this article. A preliminary mathematical framework involving track duplication and section sub divisions is proposed for this task. In railways, these features have a great effect on network performance and for this reason they have been considered. Additional motivations have also arisen from the limitations of prior models that have not included them.

Keywords—Capacity analysis, capacity expansion, railways.

I. INTRODUCTION

THIS article considers the capacity expansion of railways. This is an important topic worldwide and in countries like Australia [26] that have to transport increasing volumes of goods and passengers across large distances by road, rail, air and sea. Railways are considered in this article because of their great importance, and because of limitations in preliminary models and techniques such as those in [3] and [9]-[13],[22],[25]. The approaches developed in those papers are of limited benefit for the problem addressed in this article.

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Several recent articles have considered the related topic of infrastructure expansion and investment. In those articles, optimization models were proposed. Lui et al. [23] considered the capacity expansion of railroads and the spatial configuration of yards in a freight network. In particular, they formulated a model for the yard location problem and demonstrated that railroads can make significant savings by reconfiguring their networks. This outcome provides significant motivation for this article which considers general railway networks and their expansion, and proposes more generic models and techniques for expanding line capacity.

Lai and Barkan [16] and [18] considered strategic capacity planning. Lai and Barkan [17] then extended their previous work and proposed a decision support framework for railway capacity planning. Singh et al. [26] developed a mixed integer linear programming (MILP) model for determining capacity requirements and infrastructure improvements for the Hunter Valley Coal Chain. A trade-off was made between the accuracy of simulation and the use of a more generic mathematical model. Due to its size and complexity however, meta-heuristics were necessary to optimize this bulk material supply chain.

Decision support tools have proliferated greatly in recent years. Kontaxi and Ricci [14] have developed an online tool that compares different railway capacity methods. Abril et al. [1] presented an automated tool to perform several different forms of capacity analysis. They reported the presence of six international companies developing railway capacity software.

Train sequencing and scheduling techniques can be used to verify whether a railway network has sufficient capacity to cater for a specified mix of trains, over a given time period, as determined from an independent capacity determination approach. There are many techniques available for doing this. The latest techniques such as [4]-[6],[20],[21],[8] treat this problem as a hybrid machine scheduling (i.e. job-shop) problem. Solving the train scheduling problem however is difficult as these problems are computationally intractable and become considerably harder to solve as the problem size increases. Hence, they are not good techniques for assessing capacity over longer time periods, nor for performing capacity expansion analysis.
II. METHODOLOGY

The concept of theoretical capacity is utilized in this article. In [3], it is defined as an ideal level that only occurs when critical sections of rail are saturated (i.e. continuously occupied) and train "interaction effects" and "interference delays" that are resolved by proper train scheduling, are ignored. Although theoretical capacity is an overestimation of real "operational" capacity, it is sufficiently accurate for high level planning purposes, for example, that is considered in this article. The purpose of the analytical models in this article is to maximize the theoretical capacity denoted by $\Lambda$.

A. Track Duplications (Static)

Duplicating existing tracks can significantly increase the capacity of a rail network, however as shown in preliminary numerical investigations in [2], choosing the right sections is not entirely transparent, even more so when costs of duplications vary at different locations. Capacity expansion models for doing this are therefore proposed here. The first type of model is "static" and this means that there is no time element to track duplications. The second however considers track duplications over time.

To include track duplication in a static capacity expansion model, the following constraints are necessary.

$$\sum_{ic}(\bar{y}^i_c \bar{t}^i_c + \bar{y}^i_c \bar{t}^i_c) \leq \sum_{i,c} (\bar{t}^i_c) \forall s \in \text{section saturation}$$  \hspace{1cm} (1)

$$\sum_{ic}(\bar{t}^i_c, \bar{t}^i_c) \leq B \text{ [Limitation on spending]}$$  \hspace{1cm} (2)

$$0 \leq p_{ij} \leq p_{ij}^\text{max} \forall s \in S \text{ [Limit on track numbers]}$$  \hspace{1cm} (3)

Here: $\bar{y}^i_c$ and $\bar{y}^i_c$ are the number of trains of type $i$ that traverse section $s$. Their value is determined from the decision variables, $\bar{x}^i_c$ and $\bar{x}^i_c$, which are the number of trains of type $i$ that traverse corridor $c$. In particular, $\bar{y}^i_c = \sum_{ic}(\bar{x}^i_c)$ and $\bar{y}^i_c = \sum_{ic}(\bar{x}^i_c)$ \hspace{1cm} (4)

$$\bar{x}^i_c = \bar{t}^i_c \forall i, \forall c \in S.$$ \hspace{1cm} (5)

Also: $\bar{p}_{ij}$ is an integer decision variable (i.e. $\bar{p}_{ij} \in \mathbb{Z}$) for the number of parallel tracks to add on section $s$. Furthermore $C^s$ is the cost of a single additional track (i.e. cost of duplication), $p_{ij}^\text{max}$ is an upper bound on track numbers, and there is some total budgetary limit (i.e. $B$). Constraint (1) includes the number of newly assigned tracks $\bar{x}^i_c$. The models objective is to maximize the capacity, i.e. $\Lambda = \sum_{ic}(\sum_{i} (\bar{x}^i_c + \bar{x}^i_c))$. It has been assumed that sectional running times are the same on parallel tracks. This assumption must be made otherwise it is necessary to define additional decision variables that describe which parallel track individual trains are assigned to.

From a practical perspective, track duplication cannot be separated from budgetary considerations. This is because the problem would become unbounded, and sections would be duplicated without limit. However, a variant decision making problem is to decide upon what sections should be upgraded in order to achieve a specified level of demand capacity. This variant problem has an alternative objective which is minimization of spending.

B. Track Duplications (Over Time)

Section II. A determines in the preceding section how capacity should be expanded straight away. This is perhaps not entirely useful or realistic. A better approach may be to construct and implement a long term plan of infrastructure expansions that takes into account any intermediate stage capacity requirements, and regular or intermittent budgets and funding. The following questions are, hence, pertinent to infrastructure expansion and modelling activities: i) what is the minimum time required to update the network to a specified level of capacity given a specified budget?, ii) what is the minimum budget required to update the network to a specified level of capacity in a specified time?, and iii) can the network be upgraded to a specified capacity within the given time and with the given budget?. Mathematical models for long term planning are hence investigated to see whether the aforementioned questions can be answered. Those models will be explained in due course. First let $\bar{p} = \{1, 2, ..., \}$ be the set of planning periods (typically in years) available for expansion activities, where $\bar{p} = \bar{p} | \bar{p}|$ is the number of periods. Let $\bar{p} \leq \bar{p}$ be the actual number of periods required or designated. Furthermore, let $\bar{e}_p$ be the expenditure in time period $p$, and let $\bar{t}_{xp}$ be the number of tracks added in time period $p$ in section $s$. The main assumptions for modelling are as:

- A budget $\bar{b}_p$ is provided at various planning periods, not necessarily every period (but could be).
- If the budget is not used then it rolls over into the next period. Hence $\bar{b}_p$ is the cumulative budget up to and including period $p$, i.e. $\bar{b}_p = \sum_{u=1}^{u} \bar{b}_u$.
- There may or may not be intermediate absolute capacity requirements $\Lambda_{p}$, however there is a definite long term goal for capacity. Let $\bar{A}$ be the absolute capacity required at the end of expansion activities. Hence, $\Lambda_{p} = \bar{A}$ for $p = \bar{P}$. If there is no requirement then $\Lambda_{p}$ is defined as the current "initial" system capacity, say $\Lambda_{p0}$. It is necessary for $\Lambda_{p} \geq \Lambda_{p-1} \forall p \in [1, \bar{P}]$.

All track duplications can be constructed within a time period.

Three models can be formulated:

- Model 1. Given $\bar{b}_p$ and capacity requirement $\bar{A}$, create an expansion plan of minimal duration $\bar{P}$.
- Model 2. Given a planning period $\bar{P}$ and capacity requirement $\bar{A}$, create a plan that minimises total spending $\bar{S}$.
- Model 3. Given $\bar{P}$, $\bar{B}$, $\bar{A}$, determine whether a feasible plan can be constructed.

Each of these requires the same set of core constraints:

$$\sum_{ic}(\bar{y}^i_c \bar{t}^i_c + \bar{y}^i_c \bar{t}^i_c) \leq T \times \bar{t}_{xp} \forall s,p \in \bar{P} \text{ [Section saturation]}$$  \hspace{1cm} (4)

$$\bar{y}_{lp} = \sum_{ic}(\bar{x}_{lp}^c) \forall i \in I, c \in C, p \in \bar{P} \text{ [Section usage]}$$  \hspace{1cm} (5)

$$\bar{x}_{lp}^c \geq 0 \forall i \in I, c \in C, p \in \bar{P} \text{ [Positivity]}$$  \hspace{1cm} (6)
The two capacity expansion alternatives investigated in previous sections can be combined into a single decision model. The reason for doing this is that the costs and effects of
duplications and subdivisions are very problem specific. There are many real life technical constraints that may need to be added, that will restrict options at different locations within the railway network.

III. CONCLUSIONS

Railway capacity determination and railway capacity expansion are increasingly important topics as railways will become more developed, sophisticated, and have greater demands placed upon them in the future. To help railway capacity planning activities, a mathematical framework involving optimization models has been introduced to expand the theoretical capacity of a railway. The proposed framework is high level and strategic, and this is why increases to theoretical capacity is concentrated upon. This approach provides a valuable reference point to compare other approaches, for example those for determining operational capacity. The results of simulation activities can also be compared to this reference point.

Two capacity expansion possibilities should be considered in such a framework. The first is track duplications, and the second is section sub divisions. Choosing the right sections to duplicate is not entirely transparent, even more so when costs of duplications, and other restrictions vary at different locations. Capacity is highly related to travel times on critical bottlenecks and other restrictions vary at different locations. Capacity is highly related to travel times on critical bottlenecks sections too. Hence, it is possible to increase capacity by sub-dividing existing sections of track, for example using signals. This alternative is necessary because track duplications are quite costly and permanent and construction times may also be prohibitive. The track sub division approach should utilize a profile of the travelling time, gradient, and/or velocity on all parts of a section if it is available. A model that combines both alternatives is most beneficial.

REFERENCES