Analysis of Slip Flow Heat Transfer between Asymmetrically Heated Parallel Plates

Hari Mohan Kushwaha, Santosh K. Sahu

Abstract—In the present study, analysis of heat transfer is carried out in the slip flow region for the fluid flowing between two parallel plates by employing the asymmetric heat fluxes at surface of the plates. The flow is assumed to be hydrodynamically and thermally fully developed for the analysis. The second order velocity slip and viscous dissipation effects are considered for the analysis. Closed form expressions are obtained for the Nusselt number as a function of Knudsen number and modified Brinkman number. The limiting condition of the present prediction for \( Kn = 0 \), \( Kn^2 = 0 \), and \( Br_{cl} = 0 \) is considered and found to agree well with other analytical results.

Keywords—Knudsen Number, Modified Brinkman Number, Slip Flow, Velocity Slip.

I. INTRODUCTION

Analysis of heat transfer and fluid flow at microscale is of paramount importance due to the rapid development of micro/nano electromechanical systems (MEMS/NEMS) for increasing and widespread application in navigation, spaceflight and industry. At microscale the physics of fluid flow and heat transfer changes substantially. Especially, the need for high heat fluxes in small electronic and other devices has triggered the microscale heat transfer to be the subject of interest in the last decade. As of today, more theoretical studies and devotion are essential to advance the microscale heat transfer field.

In general, the effect of small length scales of micro-devices on the flow condition can be explained by using a parameter termed as Knudsen number \( (Kn) \) and it is aptly defined as the ratio of the mean free path to the characteristic dimension of the system. Beskok and Karniadakis [1] defined four different flow regimes based on the value of the Knudsen number such as: the continuum flow regime for \( Kn < 0.001 \); the slip regime for \( 0.001 < Kn < 0.1 \); transition regime for \( 0.1 < Kn < 10 \); and molecular flow regime for \( Kn > 10 \). The slip flow regime is essentially characterized by velocity slip and temperature jump at the wall and these effects strappingly influence the heat transfer. In order to model the fluid flow in the slip regime, Navier-Stokes and energy equations can be coupled with the appropriate set of hydrodynamic and thermal boundary conditions along with velocity slip and temperature jump effects. In addition, analysis of the heat transfer and fluid flow can be carried out by considering the effect of rarefaction, axial conduction and viscous dissipation. The viscous dissipation features as a source term in the fluid flow due to the conversion of the kinetic motion of the fluid to the thermal energy and causes variation in the temperature distribution. The viscous dissipation can have an antagonist effect on the heat transfer and can distort the temperature profile severely [2].

The increasing trend of miniaturization in devices has led to studies in microscale heat transfer and fluid flow, since thermal behavior in the microdevices may deviate significantly from continuum [3]. In view of this, various experimental and theoretical studies [4]-[16] have been reported to model the fluid flow and heat transfer characteristics in micro-systems. In this paper, an analytical investigation is carried out to investigate the heat transfer characteristics for the fluid flowing between parallel plates by employing the different constant heat fluxes at the walls in the slip flow regime. The application of the constant or different heat flux condition may be observed for different areas such as: electronic cooling, electric resistance heating, and radiant heating. Therefore, interactive efforts have been made to accomplish closed form expressions for the dimensionless temperature distribution and the Nusselt number as a function of Knudsen number and modified Brinkman number, which could be useful for assessing heat transfer characteristics at microscale under asymmetric heating conditions.

II. THEORETICAL ANALYSIS

The flow between parallel plates is assumed to be laminar, steady, fully developed both hydrodynamically and thermally with constant properties.

The axial heat conduction is assumed to be negligible both in the fluid and through the wall. The thermal conductivity and diffusivity of the fluid are considered to be independent of temperature. The plates are distanced at \( W \) or \( 2w \), as shown in Fig. 1.
Utilizing the assumptions made above, the momentum equation can be expressed as:

\[
\frac{d}{dy} \left( \frac{d u}{dy} \right) = \frac{1}{\mu} \frac{dp}{dx}
\]  

Subject to the following boundary conditions:

\[
\frac{du}{dy} \bigg|_{y=0} = 0
\]

\[
\frac{u}{w_{y_{wall}}} = u_s
\]

\[
u = \frac{2-F}{F} \left( \frac{\lambda}{2} \frac{\partial}{\partial y} - \frac{\lambda^2}{2} \frac{\partial^2 u}{\partial y^2} \right)
\]

where, \(u_s\) is the velocity of fluid at the wall and \(F\) is the tangential momentum accommodation coefficient and can be taken as unity for most of the engineering applications [2].

The dimensionless variables are defined as:

\[
y = \frac{y}{W}, \quad Kn = \frac{\lambda}{W}
\]

The fully developed velocity profile for the slip regime can be derived from the momentum equation by employing the second order velocity slip condition. From (1)-(3) the velocity profile can be obtained as:

\[
u = \frac{3}{2} \left[ \frac{1+4Kn-4Kn^2-4Y^2}{1+6Kn-6Kn^2} \right]
\]

Utilizing the assumptions made above, the energy equation can be written as:

\[
\rho u c_p \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + \rho \left( \frac{\partial u}{\partial y} \right)^2
\]

Subject to the boundary conditions, given by:

\[
k \frac{\partial T}{\partial y} = -q_1 \text{ at } y = w
\]

\[
T = T_i \text{ at } y = w
\]

\[
k \frac{\partial T}{\partial y} = q_2 \text{ at } y = -w
\]

In case of internal flow with heat transfer, the fluid temperature at any location of the duct varies in the transverse direction. In such a case, the mean or bulk temperature \(T_m\) of the fluid is usually used to evaluate the heat transfer coefficient and can be expressed as:

\[
T_m = \frac{\int \rho u T dA}{\int \rho u dA}
\]

For the present configuration, with different heat flux walls the fully developed condition can be written as [6]

\[
\frac{\partial T}{\partial x} = \frac{dT_i}{dx} = \frac{dT_m}{dx}
\]

Further, the dimensionless variables can be defined as:

\[
\alpha = \frac{k}{\rho c_p}, \quad Br_{q_1} = \frac{\mu u_s}{q_1 W}, \quad \theta = \frac{T - T_i}{(q_1 W/k)}, \quad \theta_m = \frac{T_m - T_i}{(q_1 W/k)}
\]

Utilizing (4), and (7)-(9), the governing equation in dimensionless form can be written as:

\[
\frac{\partial^2 \theta}{\partial Y^2} = a_1 \left( 1+4Kn-4Kn^2-4Y^2 \right) - 144Br_{q_1} W^2
\]

\[
\left( 1+6Kn-6Kn^2 \right)
\]

where,

\[
a_1 = \frac{3 u_s}{2} \frac{kw}{\alpha q_1} \frac{dT_i}{dx}
\]

Subject to the following dimensionless boundary conditions:

\[
(i) \quad \frac{\partial \theta}{\partial Y} = 1 \text{ at } Y = \frac{1}{2}
\]

\[
(ii) \quad \theta = 0 \text{ at } Y = \frac{1}{2}
\]

\[
(iii) \quad \frac{\partial \theta}{\partial Y} = -q_2 \text{ at } Y = -\frac{1}{2}
\]

For the fully developed condition, the temperature distribution is obtained by solving (10)-(11a-c) and it can be expressed as the function of various modeling parameters such as: Knudsen number \(Kn\), heat flux ratio \(q_2/q_1\), modified Brinkman number \(Br_{q_1}\), and is given below:

\[
\theta(Y) = \frac{-a_{12} Y^4}{2(1+6Kn-6Kn^2)} + \frac{3a_{10} a_{11} Y^2}{4} - \frac{a_2 Y}{2} + \frac{a_3 Y}{q_1} + \frac{a_{14} q_2}{q_1} + \frac{a_{12} Y}{q_1}
\]

\[
\left( 1+6Kn-6Kn^2 \right)
\]

Where, \(a_{10} = \frac{1+q_2}{q_1}, a_{11} = \frac{1+4Kn-4Kn^2}{1+6Kn-6Kn^2}, a_{12} = \left( 1-q_2 \right), a_{13} = \frac{13+72Kn-72Kn^2}{1+6Kn-6Kn^2}, a_{14} = \frac{1+8Kn-8Kn^2}{1+6Kn-6Kn^2}
\]

It may be noted that for \(Kn = 0, Kn^2 = 0\), above expression reduces to (13) and is identical to that obtained by various researchers [5], [9], [10].
\[
\theta(y) = -a_0 y^4 + \frac{3 a_0 y^2}{2} + \frac{a_1 y}{2} + \frac{13}{32} + \frac{3 q_2}{32 q_1} + Br_c q_1 \left(-18y^4 + 9y^2 - \frac{9}{8}\right) \tag{13}
\]

Using (4), (7) and (12), the bulk mean temperature \( \theta_m \) is obtained as:

\[
\theta_m = -\frac{3 a_0 a_m}{1120} + \frac{3 a_0 a_m}{80} \left[ \frac{2 q_1}{q_1 - a_m} \right] - \frac{36 a_m (1 + 14 Kn - 14 Kn')}{1120} + \frac{3 a_m (1 + 10 Kn - 10 Kn')}{20} - \frac{2 (1 + 4 Kn - 4 Kn')}{8} \right] \tag{14}
\]

Where, \( a_m = \frac{1 + 14 Kn - 14 Kn'}{1 + 6 Kn - 6 Kn'} \), \( a_n = \frac{1 + 4 Kn - 4 Kn'}{(1 + 6 Kn - 6 Kn')} \)(1 + 10 Kn - 10 Kn')

For \( Kn = 0 \) and \( Kn^2 = 0 \) bulk mean temperature obtained by the present analysis is identical to the results obtained by other researchers [5], [9], [10].

\[
\theta_m = \left[ \frac{9}{70} q_2 \frac{13}{35} + \frac{27}{35} Br_c q_1 \right] \tag{15}
\]

The Nusselt number is defined as:

\[
Nu = \frac{h W}{k} \text{ or } \frac{q_h W}{k (T_1 - T_m)} \Rightarrow \frac{1}{\theta_m} \tag{16}
\]

Utilizing (14) and (16), the Nusselt can be written as:

\[
Nu = \left[ -\frac{3 a_0 a_n}{1120} + \frac{3 a_0 a_n}{80} \left( \frac{2 q_1}{q_1 - a_n} \right) - \frac{36 a_n (1 + 14 Kn - 14 Kn')}{1120} + \frac{3 a_n (1 + 10 Kn - 10 Kn')}{20} - \frac{8 (1 + 4 Kn - 4 Kn' )}{8} \right]^{-1} \tag{17}
\]

For the case of \( Kn = 0 \) and \( Kn^2 = 0 \), (17) reduces to (19) and is identical to that derived by [5], [9], [10] for the case of \( Br_c q_1 = 0 \).

\[
Nu = \left[ \frac{70}{26 - 9 q_2 q_1 + 54 Br_c q_1} \right] \tag{18}
\]

III. RESULTS AND DISCUSSION

This paper presents the analysis of a gaseous flow between parallel plates by employing different heat flux at the surface of the plates. The influence of velocity slip and viscous dissipation on heat transfer characteristics is investigated and explained by the Knudsen number and modified Brinkman number, respectively. Closed form expressions are obtained for the dimensionless temperature distribution and Nusselt number in the slip range i.e. \( 0.001 < Kn < 0.1 \). The verification of the present results is done for the case that neglect both viscous dissipation and micro scale effects (\( Kn = 0 \)). The results obtained are in good agreement with those reported by earlier researchers [5], [9], [10].
The transverse temperature variation for the given value of the Knudsen number, modified Brinkman number ($Br_q$), and heat flux ratio ($q_2/q_1$) is depicted in Figs. 2 (a)-(d). During fluid flow, the viscosity of the fluid absorbs energy from the motion of the fluid and transforms the same as an internal energy causes the change of the fluid temperature.

The viscous dissipation causes variation in temperature due to internal fluid friction and it is denoted by the modified Brinkman number. Here, positive values of modified Brinkman number correspond to the wall heating case that indicates the heat transfer from wall to the fluid, while the opposite is true for the negative modified Brinkman number.

The effect of viscous dissipation on the heat transfer performance for various values of Knudsen number and heat flux ratio is depicted in Figs. 3 (a)-(e). It may be noted that in the case of viscous dissipation ($Br_q \neq 0$), the profile of the temperature distribution gets altered compared to the case with no viscous dissipation ($Br_q = 0$) for the given heat flux ratio (Fig. 3 (c)). The viscous dissipation increases the bulk temperature of the fluid because of the internal heating of fluid. For the positive modified Brinkman number, increase in the bulk temperature of fluid decreases the temperature difference between the fluid and wall (Figs. 3 (a), (b)). For the negative modified Brinkman number, the heat transfer occurs from fluid to wall leading to a decrease in the bulk temperature of the fluid; while the viscous dissipation increases the temperature of fluid (Figs. 3 (d), (e)).
Fig. 3 (d) Variation of dimensionless temperature with $Y$ at $Kn = 0$ and $Brq_1 = -1$

Fig. 3 (e) Variation of dimensionless temperature with $Y$ at $Kn = 0$ and $Brq_1 = -5$

Fig. 4 (a) illustrates the variation of Nusselt number ($Nu$) with modified Brinkman number for various heat flux ratio ($q_2/q_1 = 0, 1, 26/9, 5$) at $Kn = 0$. It is observed that Nusselt number decreases with the increase in modified Brinkman number. The variation in the Nusselt number is found to be discontinuous and the point of singularity is observed (discontinuity in behavior) at different points for various cases of asymmetric wall heating condition. For $Kn = 0$, the point of singularity with various heat flux ratios can be evaluated from (17). The point of singularity for $q_2/q_1 = 26/9$ and $q_2/q_1 = 5$, is obtained at $Brq_1 = 0.0001865$ and $Brq_1 = 0.35186$, respectively, as shown in Fig. 4 (a). It may be noted that as the asymmetric heating increases, the difference in temperature increases for the given values of the modified Brinkman number at any transverse location. For $Kn = 0.02$ the point of singularity for $q_2/q_1 = 26/9$ and $q_2/q_1 = 5$ is obtained at $Brq_1 = 0.0559$ and $Brq_1 = 0.6566$, respectively and is shown in Fig. 4 (b). Similar observations have been made for $Kn = 0.04$ and are depicted in Fig. 4 (c).

Here, the singularity point signifies that the heat generated due to viscous dissipation balances with the heat supplied by the wall to the fluid. In addition, for a given heat flux ratio $(q_2/q_1)$, there could be a particular modified Brinkman number for which the fluid temperature due to viscous dissipation may
become equal to the imposed wall temperature. In such a case, no heat transfer takes place in either direction and results in an unbounded swing in the Nusselt number as seen in Figs. 4 (a)-(c). However, from the point of singularity, with the increase in $Br_{q1}$ ($Br_{q1}>0$), Nusselt number decreases because of the decrease in the driving potential for the heat transfer and attains an asymptotic value ($Br_{q1} \to \infty$, $Nu \to 0$). With the increase in the heat flux ratio, more heat is conducted to the fluid therefore, more viscous energy is needed to balance the heat supplied by the external source. This results in a shifting of the singularity point from lower to the higher modified Brinkman number ($Br_{q1}$) with the increase in the heat flux ratio from $q_2/q_1 = 26/9$ to $q_2/q_1 = 5$.

IV. CONCLUSIONS

In the present analytical investigation closed form expressions are obtained to study the heat transfer characteristics for the fluid flowing between parallel plates in the slip flow region by employing the asymmetric heat fluxes at upper and lower plates. The expressions are obtained for the temperature distribution and Nusselt number as the function of various modeling parameters including viscous dissipation and second order velocity slip effect. Based on the analysis, following conclusions have been made:

- The viscous dissipation is found to distort the transverse temperature distribution, substantially.
- The heat transfer characteristics are found to depend on various parameters, namely, modified Brinkman number, Knudsen number and heat flux ratio.
- The variation of the Nusselt number is found to be discontinuous having singularities at a specific modified Brinkman number for each Knudsen number.
- With the increase in heat flux ratio and Knudsen number both, the onset of singularity point shifts towards the higher value of the modified Brinkman number.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$a_1$</td>
<td>Parameter defined in (10)</td>
</tr>
<tr>
<td>$Br_{q1}$</td>
<td>Modified Brinkman number,</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Specific heat at constant pressure, J/kg-K</td>
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<tr>
<td>$h$</td>
<td>Convective heat transfer coefficient, W/m²-K</td>
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<td>$k$</td>
<td>Thermal conductivity, W/m-K</td>
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<td>$Kn$</td>
<td>Knudsen Number, $\lambda/W$</td>
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<td>$L$</td>
<td>Width of the plate, m</td>
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<tr>
<td>$Nu$</td>
<td>Nusselt number</td>
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<tr>
<td>$q_1$</td>
<td>Upper wall heat flux, W/m²</td>
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<tr>
<td>$q_2$</td>
<td>Lower wall heat flux, W/m²</td>
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<td>Temperature, K</td>
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<tr>
<td>$T_y$</td>
<td>Lower wall temperature, K</td>
</tr>
<tr>
<td>$u$</td>
<td>Velocity, m/s</td>
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<tr>
<td>$u_s$</td>
<td>Slip velocity, $\left( -\frac{2 - F}{F} \left( \frac{\partial u}{\partial y} \bigg</td>
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<td>$U$</td>
<td>Dimensionless velocity</td>
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<tr>
<td>$w$</td>
<td>Half channel height, m</td>
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<tr>
<td>$W$</td>
<td>Channel height, (= 2w), m</td>
</tr>
<tr>
<td>$x$</td>
<td>Co-ordinate in the axial direction, m</td>
</tr>
<tr>
<td>$y$</td>
<td>Co-ordinate in the vertical direction, m</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Dimensionless vertical co-ordinate, m</td>
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Greek Symbols

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<th>Symbol</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Thermal diffusivity, m²/s</td>
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<tr>
<td>$\theta$</td>
<td>Dimensionless temperature</td>
</tr>
<tr>
<td>$\theta_m$</td>
<td>Mean dimensionless temperature</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Molecular mean free path, m</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity, kg/m-s</td>
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<tr>
<td>$\rho$</td>
<td>Density, kg/m³</td>
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Subscripts

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<tr>
<td>c</td>
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<tr>
<td>e</td>
<td>Fluid entering</td>
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<tr>
<td>$m$</td>
<td>Mean</td>
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ACKNOWLEDGMENT

The authors are thankful to Science and Engineering Research Board (A Statutory body under Department of Science and Technology), India, for providing financial support to attend and present paper in ICFDT 2015: XIII International Conference on Fluid Dynamics and Thermodynamics held at Wembley, London, UK.

REFERENCES


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