Numerical Study of Mixed Convection Coupled to Radiation in a Square Cavity with a Lid-Driven

Mohamed Amine Belmiloud, Nord Eddine Sad Chemloul

Abstract—In this study, we investigated numerically heat transfer by mixed convection coupled to radiation in a square cavity; the upper horizontal wall is movable. The purpose of this study is to see the influence of the emissivity ε and the varying of the Richardson number Ri on the variation of average Nusselt number Nu. The vertical walls of the cavity are differentially heated, the left wall is maintained at a uniform temperature higher than the right wall, and the two horizontal walls are adiabatic. The finite volume method is used for solving the dimensionless Governing Equations. Emissivity values used in this study are ranged between 0 and 1, the Richardson number in the range 0.1 to 10. The Rayleigh number is fixed to Ra=10^4 and the Prandtl number is maintained constant Pr=0.71. Streamlines, isothermal lines and the average Nusselt number are presented according to the surface emissivity. The results of this study show that the Richardson number Ri and emissivity ε affect the average Nusselt number.

Keywords—Numerical study, mixed convection, square cavity, wall emissivity, lid-driven.

I. INTRODUCTION

The fundamental problem of heat transfer by mixed convection coupled to radiation in a square cavity with a lid-driven has received considerable attention from the researchers. This problem is often encountered in many engineering applications. These types of problems also arise in electronic packages, microelectronic devices during their operations. References [1] and [2] studied the mixed convection in a cavity with shifts of various boundary conditions. Reference [3] studied the flow calculation of a thermally stratified viscous fluid in a square cavity. The fluid flow is from the once the movement of the upper plate and the buoyancy. References [4] and [5] conducted respectively two and three-dimensional numerical simulation of mixed convection in a square cavity heated from the top moving wall. Reference [6] found solutions with the finite element method of the mixed convection in a square cavity heated by the bottom wall. Reference [7] performed a numerical analysis of three-dimensional models of mixed convection in an air-cooled cavity in order to compare the variations in different properties with the results of two-dimensional models. Reference [8] studied the process of laminar mixed convection in shallow two-dimensional rectangular cavities at three different Richardson numbers representing the dominating forced convection, mixed convection, and dominating natural convection using the Fluent commercial code. The effects of the inclination of the cavity on the process result of convection are also studied. The increase in the Nusselt number with the inclination of the cavity is average for the forced convection and significant for the natural convection. Mixed convection of power-law fluids along a vertical wedge with convective boundary condition in a porous medium have been studied by [9]. Reference [10] analyzed the natural convection in a prismatic enclosure; they showed that the temperature decreases as the Rayleigh number increases.

The interaction between laminar and turbulent convection and radiation of non-gray gas was studied by [11] and [12]. Reference [13] has studied numerically natural convection combined with radiation in a square cavity differentially heated. The enclosure is partially divided by a thin vertical baffle. The result shows that the radiation increases the circulation and enhances the heat transfer rate. Reference [14] investigated the turbulent natural convection coupled with radiation in a square enclosure and proposed a new relative correlation between the Nusselt number for isothermal vertical walls levels depending on the Rayleigh number and the optical thickness medium. The numerical analysis of the effects of orthotropic thermal conductivity of substrates in natural....
convection cooling of discrete heat sources was given by [15]. Their results showed that the radiation is the dominant mode of heat transfer and the effect of the thermal conductivity of the substrate is important in the transfer of heat in the enclosure. An experimental and numerical study of combined heat transfer in a three-dimensional enclosure with a protruding heat source was given by [16]. Combined natural-convection and volumetric radiation in an annulus space was studied by [17] using spectral methods and finite volumes and by [24] using a discontinuous finite-element formulation. Y. Reference [18] showed that the radiation of a gray fluid affects the total Nusselt number, tends to reduce the effects of buoyancy, and accelerates the development of the temperature field. Reference [19] studied the effects of surface radiation on conjugate mixed convection in a vertical channel with a discrete heat source in each wall. Reference [20] studied the linear instability and nonlinear stability of a model for convection induced by selective absorption of radiation and demonstrated that theory of linear stability does not make it possible to predict the convection onset. Reference [21] has analyzed one-dimensional linear stability for three-dimension al Rayleigh-Benard convection with a radiatively participating medium using spectral methods. They used a linear stability and weakly nonlinear analysis to determine the critical Rayleigh number for the onset of convection in the combined mode of heat transfer. Reference [22] applied a multidimensional scheme with using the classical discrete ordinates method, found that it is suitable for accurate calculations in radiative transfer, and can minimize the ray effect in complex geometric situations. Reference [23] studied the effects of radiative surface on natural convection in a cavity.

The purpose of this study is to see the influence of the emissivity and the varying of the Richardson number on thermal and hydrodynamic characteristics of the flow.

II. ANALYSIS

For the numerical simulation, the values of emissivity ε in the range from 0 to 1, the Rayleigh number fixed at $Ra = 10^4$ and Richardson number $Ri$ in interval the 0.1 to 10. The geometry considered is a square cavity of which the two horizontal walls are adiabatic, the horizontal top wall is movable in the direction $Ox$, and the two vertical walls are differentially heated $T_h > T_c$. The four walls are assumed gray. The aspect ratio is $A = 1$ and the properties of air are mainly set at the reference temperature $T_0 = 293.5$ K.

A. Governing Equations

The General Equations of Conservation that is, the Continuity Equation, the Equations of Momentum along x and y and the Energy Equation are given in their dimensionless form as:

\[
\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} = 0 
\]  

\[
u^* \frac{\partial u^*}{\partial x} + \frac{\partial P^*}{\partial x} + \frac{1}{Re} \left( \frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2} \right)  + Ri T^* 
\]  

\[
u^* \frac{\partial v^*}{\partial x} + \frac{\partial P^*}{\partial y} + \frac{1}{Re Pr} \left( \frac{\partial^2 v^*}{\partial x^2} + \frac{\partial^2 v^*}{\partial y^2} \right) + \frac{\partial T^*}{\partial y} 
\]

B. Radiation Equation

The General Equations of Radiation are shown below:

- **Radiosity Equation:**

  \[
  J_i^* = \epsilon_i \left( \frac{T_i^*}{T_0^*} + 1 \right)^4 + (1 - \epsilon_i) \sum_{j \neq i} F_{ij} J_j^* 
  \]  

- **Net Flux Equation:**

  \[
  Q_{ad} = \epsilon_i \left( \frac{T_i^*}{T_0^*} + 1 \right)^4 \sum_{j \neq i} F_{ij} J_j^* 
  \]

In (4) and (5), the dimensionless variables are defined by:

\[
x^* = \frac{x}{H}, y^* = \frac{y}{H}, u^* = \frac{u}{u_{ad}}, v^* = \frac{v}{u_{ad}}, P^* = \frac{P}{\rho u_{ad}^2} 
\]

\[
T_i^* = \frac{T_i - T_0}{T_h - T_c}, T_0^* = \frac{T_0}{T_h - T_c}, J_i^* = \frac{J_i}{\sigma T_0^4}, Q_{ad} = \frac{Q_{ad}}{\sigma T_0^4} 
\]

with $T_0 = \frac{T_h + T_c}{2}$ is the reference temperature. The Rayleigh number $Ra$, the Reynolds number $Re$, the Richardson number $Ri$ and the Prandtl number $Pr$ is defined by:

\[
Ra = \frac{g \beta (T_h - T_c) H^3}{\nu \alpha}, Ri = \frac{Ra}{Pr Re^2}, Re = \frac{u_{ad} H}{\nu}, Pr = \frac{\nu}{\alpha} 
\]

C. Boundary Conditions

To solve (2)-(4), the boundary conditions are considered. These conditions are given under their dimensionless form as:

- At the right wall: $T^* = -0.5; u^* = v^* = 0$
- At the left wall: $T^* = 0.5; u^* = v^* = 0$
- In the adiabatic upper horizontal wall:

  \[
u^* = 1; v^* = 0; \frac{\partial T^*}{\partial x} + N_a Q_{ad}^* = 0 
\]

- In the adiabatic lower horizontal wall:

  \[
u^* = v^* = 0; \frac{\partial T^*}{\partial x} + N_a Q_{ad}^* = 0 
\]
III. NUMERICAL ANALYSIS

A. Computational Procedure

The Mass, Momentum, and Energy Equations have been solved by a finite difference algorithm called the Semi-Implicit Method for Pressure Linked Equations (SIMPLE). Details of this method are described by [25]. The Differential Equations are discretized over a control volume. The power law difference scheme (PLDS) has been employed for the calculation of scalar variables and the quadratic upstream-weighted interpolation for convective kinematics (QUICK) scheme for vector variables. The relative tolerance for the error criteria is considered to be $10^{-5}$.

B. Grid Check

The results are given in the form of streamline, isotherm contour, and average Nusselt number in the left wall. The properties of air are linked to the reference temperature $T_0 = 293.5$ K. The Richardson number used in the range 0.1 to 10 for the flow is laminar.

It can be seen from Table I that the differences in $Ri = 0.1$ between the grid sizes of (42×42) and (21×21) are 1.694% which is the higher. However, the differences between $Ri = 1$ and $Ri = 10$ are comparatively the lower. The difference in $Ri = 1$ between the grid sizes of (62×62) and (42×42) is 0.022% is the lowest. However, the differences between $Ri = 0.1$ and $Ri = 10$ are the greatest. The grid used in all subsequent calculations is (42×42).

<table>
<thead>
<tr>
<th>Grid</th>
<th>Average total Nusselt number $Nu_t$</th>
<th>% Change in $Nu_t$ (ABS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ri = 0.1$</td>
<td>$Ri = 1$</td>
<td>$Ri = 10$</td>
</tr>
<tr>
<td>(21×21)</td>
<td>5.783</td>
<td>4.446</td>
</tr>
<tr>
<td></td>
<td>3.888</td>
<td>-</td>
</tr>
<tr>
<td>(42×42)</td>
<td>5.881</td>
<td>4.445</td>
</tr>
<tr>
<td></td>
<td>3.857</td>
<td>1.694</td>
</tr>
<tr>
<td>(62×62)</td>
<td>5.908</td>
<td>4.446</td>
</tr>
<tr>
<td></td>
<td>3.855</td>
<td>0.459</td>
</tr>
</tbody>
</table>

C. Validation of the Code

The first validation of the numerical code was validated against the results of [27] obtained in the case of a square cavity differentially heated. Table II shows the comparison between the values of the average Nusselt number for $Nu_{ev}$, evaluated at the heated wall for $Ra$ varying in the range $10^3 \leq Ra \leq 10^4$. It is seen from the table a fairly good agreement with relative maximum deviations limited to 0.44% for $\varepsilon = 0$ and 1.84% for $\varepsilon = 1$.

<table>
<thead>
<tr>
<th>$Ra$</th>
<th>Present work</th>
<th>Akiyama and Chong [27]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ri = 0.1$</td>
<td>1.123</td>
<td>1.125</td>
</tr>
<tr>
<td>$Ri = 1$</td>
<td>2.26</td>
<td>2.25</td>
</tr>
<tr>
<td>$Ri = 10$</td>
<td>4.616</td>
<td>4.625</td>
</tr>
<tr>
<td>$\varepsilon = 0$</td>
<td>1.248</td>
<td>1.250</td>
</tr>
<tr>
<td>$\varepsilon = 1$</td>
<td>2.232</td>
<td>2.250</td>
</tr>
<tr>
<td>$\varepsilon = 0$</td>
<td>4.173</td>
<td>4.250</td>
</tr>
</tbody>
</table>

The second validation of the numerical code has been validated against the results of [28], which were reported for heat transfer laminar by mixed convection in a cavity heated from below lid-driven. The comparison was conducted while employing the following dimensionless parameters $Re = 500$, $Ri = 0.4$ and $Pr = 1$. Excellent agreement was achieved, as illustrated in Fig. 2, between results of the present work and the numerical results of [28] for both the stream lines and isothermal line inside the cavity.

IV. RESULTS AND DISCUSSION

A. Stream Line and Isothermal Distribution

The effect of emissivity ($\varepsilon$) on the stream lines and isothermal lines for $Ra = 10^6$ and $Ri = 1$ is shown in Fig. 3. The stream lines take the same form for all three cases. However, they are entrained by the mobility of the upper...
horizontal wall. The velocity of circulation increases with increasing the emissivity $\varepsilon$, this shows that the radiative heat transfer affects the velocity contour, we believe that this is due to thermo-physical properties of air.

To see the influence of the emissivity $\varepsilon$ on the isothermal lines, we compared the difference between the two vertical walls; the remark was observed that the heat transfer by convection is good at the hot wall relative to the cold wall. Knowing that the heat transfer from pass the left vertical wall towards the top horizontal wall and then the right vertical wall and finally the bottom horizontal wall, and from the direction of mobility of the upper horizontal wall, this explains why the isothermal lines move away from the cold vertical wall and the convective heat transfer decreases in this wall. For the two horizontal walls, we notice the presence of the force of friction between the fluid and the walls, knowing that the two walls are adiabatic. The frictional force caused a decrease in exchange heat convection.

![Fig. 3 Effect of emissivity $\varepsilon$ on the streamline and isothermal lines for $Ra = 10^4$ and $Ri = 1$](image)

Fig. 4 shows the Richardson number $Ri$ effect on streamlines and isothermal lines for $Ra = 10^4$ and $\varepsilon = 0.5$, the critical values of the convective heat transfer were found by [29]. When the Richardson number is in the interval (0.1 to 16), the convection is mixed, for low values of Richardson ($Ri < 0.1$), forced convection is dominant, the Richardson number higher ($Ri > 16$), the natural convection is dominant.

As mentioned above, the two values of $Ra$ and $Pr$ are constants, and according to the Richardson number ($Ri = \frac{Ra}{PrRe^*}$), the value of Reynolds number $Re$ decreases with increase the number of Richardson $Ri$. For the value of Richardson ($Ri = 0.1$), we noticed a small vortex on the right bottom part of the cavity. This vortex was caused due to the fluid velocity and velocity of shearing the largest upper horizontal wall. For isothermal lines, we find that forced convection dominates the natural convection since we are near the zone of the forced convection. For ($Ri = 1$), we observed that the influence of the moving the upper horizontal wall on the streamlines and the isothermal lines are reduced compared to the first case ($Ri = 0.1$), which indicated that both natural and forced convections exist. We also notice a reduction in the predominance of the forced convection on the natural convection. In the latter case ($Ri = 10$), the streamlines and isothermal lines are almost centered, relative to the middle of the cavity. We believe that this is due to the existence of the gravity which affects the natural convection, and natural convection predominates.
**B. Variation of Heat Transfer**

Figs. 5 (a) and (b) respectively show the variation of convective Nusselt $N_u_c$ and total Nusselt $N_u$ according to different emissivity $\varepsilon$ for $Ra = 10^4$ and for different numbers of Richardson $Ri = 0.1-10$, knowing that all the results that we obtained at the hot wall. We note that Nusselt number $Nu$ decreases with the increase of the Richardson number $Ri$, this shows that the convective heat transfer increases when the velocity of the cavity a lid increases, in the case of the emissivity in the range 0.1 to 1, we note that the convective Nusselt number decreases with increasing $\varepsilon$. The difference between the Nusselt values $N_u_c$ corresponding to $Ri = 0.1$ and $Ri = 1$, takes a maximum value of 1.35. This difference decreases to 0.55 for Nusselt values corresponding to $Ri = 1$ and $Ri = 10$. Fig. 5 (b) shows that passes through a maximum value $\varepsilon = 0.1$ and a minimum value $\varepsilon = 0.4$ to increase then.

![Stream lines](image1.png)

![Isothermal lines](image2.png)

**Fig. 4** Effect of Richardson Number $Ri$ on the stream lines and isothermal lines for $Ra = 10^4$ and $\varepsilon = 0.5$

![Convective Nusselt number](image3.png)

![Total Nusselt number](image4.png)

**Fig. 5** Variation of the Nusselt number as a function of the emissivity $\varepsilon$ for the hot wall to $Ri = 0.1-10$, $Ra = 10^4$: (a) convective Nusselt number; (b) total Nusselt number
A. Variations of the Dimensionless Velocity and Temperature

Fig. 6 shows the variation of the components of $u^+$ velocity and $v^+$ velocity profiles along the vertical and horizontal centerline of the cavity (i.e. $x^+ = 0.5$ and $y^+ = 0.5$), respectively, for $Ra = 10^4$, $Ri = 1$ and for different values of the emissivity $\varepsilon$. The $u^+$ velocity profile is shown in Fig. 6 (a) and it can be seen that are nearly superposed with the values maximum of the $u^+$ velocity obtained to $\varepsilon = 1$. For the $v^+$ velocity component, the same comments as those of the component $u^+$ velocity can be considered (Fig. 6 (b)). The results is showing that the increasing of radiation influence on the velocity of circulation.

![Variation of the velocity components for Ra = 10^4, Ri = 1 and for different values of the emissivity ε: (a) u^+ profile along x^+ = 0.5; (b) v^+ profile along y^+ = 0.5](image1)

Fig. 6 Variation of the velocity components for $Ra = 10^4$, $Ri = 1$ and for different values of the emissivity $\varepsilon$: (a) $u^+$ velocity profile along $x^+ = 0.5$; (b) $v^+$ velocity profile along $y^+ = 0.5$

Fig. 7 shows the variation of the temperature profile $T^+$ along the vertical and horizontal centerline of the cavity, evaluated at $Ra = 10^4$, $Ri = 1$ and for different values of the emissivity $\varepsilon$. The temperature profile $T^+$ as a function of $x^+$ for the position $y^+ = 0.5$, are given by Fig. 7 (a). It can be seen that the profiles are nearly superposed with the maximum value of the temperature for $\varepsilon = 1$, in the range $0 < x^+ \leq 0.19$ and $0.35 < x^+ < 1$. If the value of $x^+$ is in the range $0.19 - 0.35$, we obtained the maximum value of $T^+$ for $\varepsilon = 0$. Fig. 7 (b) is showing the variation of the temperature profile $T^+$ as a function of $y^+$ for the position $x^+ = 0.5$. We remark that the profiles which are distant from each other intersect at $y^+ = 0.35$, remain very close beyond this value. The temperature $T^+$ is decreasing when the emissivity $\varepsilon$ increases.

![Variation of the temperature T' for Ra = 10^4, Ri = 1 and for different values of the emissivity ε: (a) T' profile along y' = 0.5; (b) T' profile along x' = 0.5](image2)

Fig. 7 Variation of the temperature $T^+$ for $Ra = 10^4$, $Ri = 1$ and for different values of the emissivity $\varepsilon$: (a) $T^+$ profile along $y^+ = 0.5$; (b) $T^+$ profile along $x^+ = 0.5$

V. CONCLUSION

The purpose of this study deals with the analysis of the effect of emissivity $\varepsilon$, of the variation in the number of Richardson $Ri$ on mixed convection coupled to a radiation in a square cavity with a lid-driven. From the results obtained we concluded:

- The velocity is increased and the Nusselt number convective $Nu_{cv}$ being reduced when the emissivity $\varepsilon$ is increased, which shows that the convective heat transfer decreases with the increase of the emissivity $\varepsilon$.
- The heat transfer is the dominant mode forced convection, when the values of the Richardson number $Ri$ are very close to those corresponding to the forced convection ($Ri < 0.1$). When the Richardson number $Ri$ increases, the heat transfer is the dominant mode natural convection.
Finally, the velocity lid of the cavity influences the thermal transfer; this is proved by the decrease in the Nusselt number Nu when the Richardson number Ri increases.

REFERENCES


Mohamed Amine Belmiloud was born in 1988, Algeria, received his degree master in mechanical engineering (energy option) at the University Ibn Khaldoun, Tiaret, Algeria and he is currently a Ph.D. candidate in mechanical engineering, University of Ibn Khaldoun, Tiaret, Algeria. His research interests include Heat transfer and the Mass, Computational Fluid Dynamics (CFD).