Static Response of Homogeneous Clay Stratum to Imposed Structural Loads

Aaron Aboshio

Abstract—Numerical study of the static response of homogeneous clay stratum considering a wide range of cohesion and subject to foundation loads is presented. The linear elastic–perfectly plastic constitutive relation with the von Mises yield criterion were utilised to develop a numerically cost effective finite element model for the soil while imposing a rigid body constrain to the foundation footing. From the analyses carried out, estimate of the bearing capacity factor, Nc, as well as the ultimate load-carrying capacities of these soils, effect of cohesion on foundation settlements, stress fields and failure propagation were obtained. These are consistent with other findings in the literature and hence can be a useful guide in design of safe foundations in clay soils for buildings and other structure.

Keywords—Bearing capacity factors, finite element method, safe bearing pressure, structure-soil interaction.

I. INTRODUCTION

EXCEPT for floating structures [1], most engineering facilities are largely supported on soils which are quite varied in their inherent properties and response to structural loadings. To avoid failures in foundations and by extension on the super structure, designers normally estimate maximum acceptable bearing pressure between the foundation and the supporting soil using some set of equations, considering its shear strength and settlements that can be allowed for the structure under consideration.

For shallow foundations, the Terzaghi’s [2] equation of computing the ultimate bearing capacity gives reasonably conservative values and does not account for the contribution of the shear strength of the soil above the base of the foundation. It is mostly used in design of foundations bearing pressures for granular and c-ϕ soils. A more accurate equation for estimating bearing capacities of soils is that developed by Meyerhof. The Meyerhof’s equation [3] presented in (1) can be applied to both shallow and deep foundations and may be used for all soil types.

The Meyerhof’s net bearing capacity equation for foundation accounting for the effect of cohesion, surcharge and unit weight of a soil is given by:

\[ q_{\text{ad}} = cN_c + q_s(N_q - 1) + \frac{1}{2}BN_f \]  

where \( N_c \), \( N_q \) and \( N_f \) are the Meyerhof’s bearing capacity factors, \( q_s \) is the soil surcharge and \( B \) is the footing width.

For cohesive soils where \( \phi = 0 \) and \( N_q = N_f = 0 \), the bearing capacity equation reduces to

\[ q_{\text{ad}} = cN_c \]  

Herein, for the numerical analyses of the structure-soil-interactions, the soil constitutive model adopted is such as to account for the effect of geometric non-linearity, hence the use of a non-linear stress-strain relation accounting for both linear elastic and plastic response state of the soil.

The von Mises failure model was also utilised for estimation of the total (un-drained) stress state of the clay which expresses the maximum shearing stress that the clay can support and it is given by the yield function [4];

\[ f(\sigma_y) = J_{\sigma}^{1/2} - c \]

where \( \sigma_y \) is the Cartesian stress tensor, \( J_{\sigma} \) is the second invariant of the deviatoric stress tensor and \( c \) is a material constant (soil cohesion).

II. NUMERICAL MODEL FINITE ELEMENT DISCRETIZATION AND MESH REFINEMENT

For the un-drained soil conditions, a unit width (B = 1) strip footing of semi-infinite length, consistent with conventional practice was employed. However, to ensure meshing efficiency and to optimize computation time, advantage was taken of the structure/loading symmetry, hence only a quarter of the entire structure was modeled.

Preliminary study to establish an optimum depth-to-width ratio for the model (so as to avoid interference of the soil boundaries with the soil deformation and collapse zones) was carried out. A depth-to-width ratio of 5 (H/B=5) was found to provide an optimum solution for stress and deformation fields and hence adopted in this study.

This is apparently more computationally efficient than the width-depth-ratio of 10 as adopted by [6] in a similar study.

The finite element discretization and analyses of the structure-soil-interaction problems were carried out using the PLAXIS code. Unstructured meshes consisting of 15 nodded triangular elements (Fig. 1) were used. This is to take advantage of the traditional characteristics of these elements; i.e. the ease of efficient element arrangement and refinement of the mesh at the vicinity of corners of the footings which is

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crucial for an accurate prediction of the collapse loads [7], as well as stresses at the footing-soil interface. Fig. 1 also shows the model discretization loading and boundary conditions adopted in this study.

III. SOIL MATERIAL PARAMETERS
The initial un-drained clay material model parameters considered in this study are presented in Table I.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>SOIL MATERIAL PARAMETERS</th>
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<tbody>
<tr>
<td>Elastic Modulus</td>
<td>24MN/m²</td>
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<tr>
<td>Cohesion</td>
<td>10 to 120kN/m²</td>
</tr>
<tr>
<td>Poisson Ratio</td>
<td>0.45</td>
</tr>
<tr>
<td>Dry Unit Weight of soil</td>
<td>17 kN/m²</td>
</tr>
<tr>
<td>Saturated Unit Weight of soil</td>
<td>19 kN/m²</td>
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</table>

IV. RESULTS AND DISCUSSION
A. Load-Settlement and Collapse Loads
Results of the soil displacements/settlements under loadings for a wide range of clay soil types i.e. very soft to hard clay is presented in Fig. 2. This gives an insight into the settlement behaviour of the soil when subjected to structural loads. The figure as shows the load-settlement profile of a point directly under the footing centreline for all the soil types considered. The profiles obtained were found to be consistent with submissions in the literature [4], [5] as can be seen from Fig. 3.

The numerical results obtained, indicate the maximum loadings that can be supported by clay soils as they vary only in cohesion values. The collapse/maximum loads thus, increases with increase of the cohesion in the clay material.

In reinforced concrete structures, settlements beyond 20 mm can be adjudged excessive due to serviceability concerns [10]. Hence from the results in Fig. 2, clay soils of cohesion of between 60 and 120 kN/m² can resist settlement of up to 20 mm (though at different collapse loads, i.e. from 150 to 312 kN/m²) before failure, thus can be recommended for use in foundations for reinforced concrete structures. Clay soils however of cohesion values less than the aforementioned may need to be subjected to some kind of soil stabilization before laying structure susceptible to large displacements for the avoidance of failure.

Fig. 1 Typical mesh and boundary condition used for the foundation-soil interaction simulations [8]

Fig. 2 Load-Settlement relation for clay soils of varying cohesion

Fig. 3 Load Settlement Curve [5]
B. Bearing Capacity Factor (Nc)

The bearing capacity factor for clay soils, \( N_c \) as can be seen from the relation in (2) is essential in determining the soil ultimate bearing capacity.

Table II shows values of bearing capacity factor \( N_c \) at internal angle of friction, \( \phi = 0 \), as proposed by Terzaghi and Meyerhof for the analytical/theoretical estimates of the bearing capacities of soils as well as that obtained from the numerical analyses herein conducted.

The \( N_c \) value of 5.2 deduced form the numerical analysis tend to agree more with the Meyerhof’s proposed value of 5.14 than that of Terzaghi. While the difference is not very significant, it thus further buttresses the generally accepted notion that Terzaghi’s approach lead to in overestimation of the bearing capacity of soils.

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>COMPARISON OF THEORETICAL AND NUMERICAL ESTIMATE OF THE BEARING CAPACITY FACTOR (Nc) FOR CLAY SOIL</th>
</tr>
</thead>
<tbody>
<tr>
<td>TERZAGHI</td>
<td>5.7</td>
</tr>
<tr>
<td>MEYERHOF</td>
<td>5.14</td>
</tr>
<tr>
<td>FEM/PLAXIS</td>
<td>5.2</td>
</tr>
</tbody>
</table>

C. Failure Mechanism

The ‘general shear failure’ is known to occur in highly incompressible cohesive soils [5], [7], thus, high contact stresses underneath footings on such soil strata are expected to assume the convex profile as shown in Fig. 3. Apparently, from the figure, the highest stress concentrations will occur at the interface between the triangular wedge and the long spiral shear zone of Terzaghi’s general shear failure mechanism (Fig. 3).

Fig. 3 Terzaghi’s General Shear Failure Mechanism [9]

Fig. 4 shows the general failure mode obtained using the numerical analyses carried out. It is obvious from Figs. 4 (a) and (b) that there exists a close similarity between the failure surface simulated by the Finite Element (FE)-PLAXIS code and that by Terzaghi’s failure analysis for rough footings.

D. Failure Propagation

An assessment of the incremental failure profile was done by comparing the rate of plastic propagation within the model using three loads of 135 KN/m², 280 KN/m², and 312 KN/m². It is interesting to note from Fig. 5 (a) that at a load of 135 KN/m² corresponding to the upper limit of the elastic zone in the load settlement curve (Fig. 3). The entire model was still largely elastic apart from a minute portion at the footing edge where plasticity had just set in represented by the red colouration (Fig. 5 (a)). As the load increased to about 280 KN/m², the plastic zone propagated to the footing centerline along a profile akin to the shear interface between the triangular wedge and the log-spiral shear zone of Terzaghi’s mechanism (Fig. 3). Finally, at the limit load of 312 KN/m², full plasticity developed in the regions down and around the footing centerline and edge resulting in a heave by the sides as the soil undergoes squeezing (Fig. 5 (c)). The dark portions in Fig. 5 represent tension cut-off points.

![Fig. 3 Terzaghi’s General Shear Failure Mechanism](image1)

![Fig. 4 (a) Total incremental displacements contour, (b) Total incremental strains](image2)

![Fig. 5](image3)
V. Conclusion

The numerical model developed and adopted in this study is computationally cost effective and can be used to solve soil-structure interaction problems of varying complexities with reasonable degree of accuracy.

Results of the bearing capacity factor and failure modes of cohesive soils considered agree closely well with analytical/experimental results in the literature.

A useful guide on settlements of un-drained cohesive soils under varying foundation loads presented in this paper can be useful in siting of building and other civil structures.

References