Low Complexity Peak-to-Average Power Ratio Reduction in Orthogonal Frequency Division Multiplexing System by Simultaneously Applying Partial Transmit Sequence and Clipping Algorithms

V. Sudha, D. Sriram Kumar

Abstract—Orthogonal Frequency Division Multiplexing (OFDM) has been used in many advanced wireless communication systems due to its high spectral efficiency and robustness to frequency selective fading channels. However, the major concern with OFDM system is the high peak-to-average power ratio (PAPR) of the transmitted signal. Some of the popular techniques used for PAPR reduction in OFDM system are conventional partial transmit sequences (CPTS) and clipping. In this paper, a parallel combination/hybrid scheme of PAPR reduction using clipping and CPTS algorithms is proposed. The proposed method intelligently applies both the algorithms in order to reduce both PAPR as well as computational complexity. The proposed scheme slightly degrades bit error rate (BER) performance due to clipping operation and it can be reduced by selecting an appropriate value of the clipping ratio (CR). The simulation results show that the proposed algorithm achieves significant PAPR reduction with much reduced computational complexity.

Keywords—CCDF, OFDM, PAPR, PTS.

I. INTRODUCTION

ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING (OFDM) is the most promising technology for high data rate transmission due to its high spectral efficiency and the immunity to multipath channels. This technique has been widely deployed in many wireless communication standards such as Digital Video Broadcasting (DVB), Digital Audio Broadcasting (DAB), worldwide interoperability for microwave access (WiMAX) [1]. However, OFDM systems still have some challenging unresolved issues on the transmitter side and one of the major issues is high Peak-to-average power ratio (PAPR) of the OFDM signal. Some of the subcarriers have high peak amplitude levels, these high peaks occur when all the subcarriers are added constructively at the output of OFDM modulator. Hence, the system requires a power amplifier with large dynamic range and complex digital-to-analog converter (D/A) at the transmitter. The conventional solution for reducing high PAPR is to back-off the operating point of the nonlinear power amplifier. However, this operation reduces the power efficiency of the amplifier. Therefore, a number of solutions have been proposed in the literature [2] to reduce PAPR in OFDM system, which include clipping [3], [4], clipping and filtering [5]-[7], selected mapping (SLM) [8], partial transmit sequence (PTS) [9], coding [10], tone reservation (TR), tone injection (TI) etc. Each conventional technique has its own merits and demerits in terms of data rate loss, out-of-band interference, and computational complexity. Recently, hybrid schemes have been proposed to reduce PAPR in OFDM system such as SLM and clipping [11], [12], PTS and clipping [13]. However, these schemes suffer by more computational complexity.

In [14], PAPR reduction of OFDM signal has been achieved by simultaneously applying both CSLM and CPTS algorithms. Though, their proposed algorithm reduces PAPR; the computational complexity lies between CSLM and CPTS algorithms. Therefore, to reduce PAPR as well as computational complexity the proposed method tries to apply a simple clipping algorithm for the first half of the input data sequence instead of SLM algorithm in [14] and PTS algorithm for the other half of the data sequence. The clipping noise introduced by clipping operation would be less in the proposed scheme while compared to applying clipping technique alone. Therefore, the impact on bit error rate (BER) performance is not as significant as in the case of clipping technique. The simulation results show that the proposed technique offers low PAPR than conventional clipping and CPTS algorithms with much reduced complexity.

The rest of the paper is organized as follows. In Section II, an overview on the PAPR of OFDM system is provided. Conventional clipping and CPTS algorithms are discussed in Section III and the proposed technique is presented in section IV. The simulation results are shown in Section V. An analysis of computational complexity is given in Section VI. Finally, the conclusion is given in Section VII.

II. PAPR OF OFDM SIGNAL

In an OFDM system, the total number of sub-carriers used are N, the discrete time OFDM signal is given by [15]

$$x(n) = \frac{1}{\sqrt{N}} \sum_{K=0}^{N-1} X(K).\exp\left(\frac{j2\pi k n}{N}\right), \quad n = 0,1,...,N-1 \tag{1}$$

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World Academy of Science, Engineering and Technology
International Journal of Electronics and Communication Engineering
Vol:9, No:4, 2015

where $j = \sqrt{-1}$, $X(K)$, $K = 0, 1, ..., N-1$, are the modulated symbols. The PAPR is defined as the ratio of the maximum power to the average power during an OFDM symbol period and expressed as

$$PAPR = 10 \cdot \log_{10} \frac{\max_{0 \leq n \leq N-1}\{|x(n)|^2\}}{E\left[|x(n)|^2\right]} \, (dB) \qquad (2)$$

where $|x(n)|$ denotes the magnitude of $x(n)$ and E [.] denotes the expectation operation. According to the central limit theorem, for larger value of N the real and imaginary parts of OFDM signal approach a Gaussian distribution with zero mean and variance σ^2. The PAPR value mainly depends on the number of subcarriers (N).

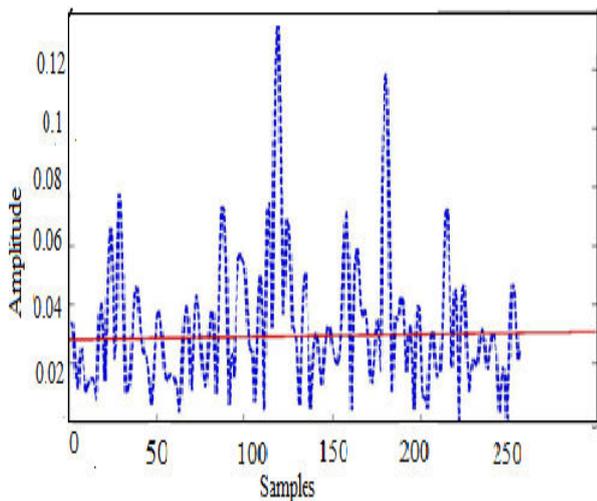Fig. 1 PAPR of OFDM signal

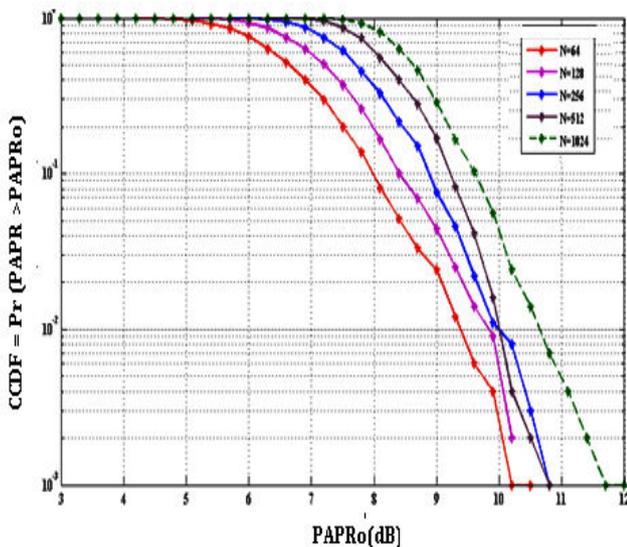Fig. 2 CCDF of PAPR in OFDM with different number of sub-carriers i.e. N=64, 128, 256, 512, and 1024 respectively

Fig. 1 shows the PAPR of OFDM signal with number of subcarrier N=256. From the figure, we observe that high peak

occurs infrequently and randomly. PAPR distribution is characterized by random parameters such as a cumulative distribution function (CDF), complementary cumulative distribution function (CCDF) and crest factor (CF). In general, CCDF is the parameter used to measure PAPR and it is defined as the probability that PAPR of OFDM signals greater than the threshold PAPR value (PAPRo) and it can be defined as

$$CCDF = \Pr(PAPR > PAPRo) = 1 - (1 - e^{-PAPRo})^N \qquad (3)$$

Fig. 2 shows the CCDF of PAPR in OFDM signals with different number of subcarriers N=64, 128, 256, 512, and 1024 respectively. From the graph, it is observed that PAPR increases when the number of subcarrier (N) increases.

III. EXISTING ALGORITHMS

A. Clipping Algorithm

Clipping is a nonlinear signal processing technique that limits the peak amplitude of the OFDM signal to a predetermined value *(A)*. The clipping operation can be defined as

$$\overline{x(n)} = \begin{cases} x(n), & |x(n)| \leq A \\ A \cdot e^{j\phi(n)}, & |x(n)| > A \end{cases} \qquad (4)$$

where $x(n)$ denotes the OFDM signal before clipping and $\overline{x(n)}$ denotes signal after clipping. Clipping ratio (CR) is defined as the ratio of clipping threshold A to the r.m.s value of the OFDM signal σ

$$CR = \frac{A}{\sigma} \qquad (5)$$

where $\sigma = \sqrt{N}$; N is the number of sub carriers in OFDM system.

The abrupt clipping of OFDM signal causes in-band distortion and out-of-band radiation. The out–of-band radiation can be removed by means of filtering process. However, the in-band distortion cannot be eliminated by filtering results in increased BER.

The PAPR of the clipped signal may be bounded as

$$PAPR \leq \frac{A^2}{P_{Out}} = \frac{CR^2}{1 - e^{-CR^2}} \qquad (6)$$

Fig. 3 shows the BER performance comparison of clipping and clipping with the filtered OFDM signal with different clipping ratios (CR). The BER performance of clipped OFDM signal is poorer than clipping with the filtered OFDM signal. However, the filtering of clipped signal increases the PAPR of OFDM signal.

World Academy of Science, Engineering and Technology
International Journal of Electronics and Communication Engineering
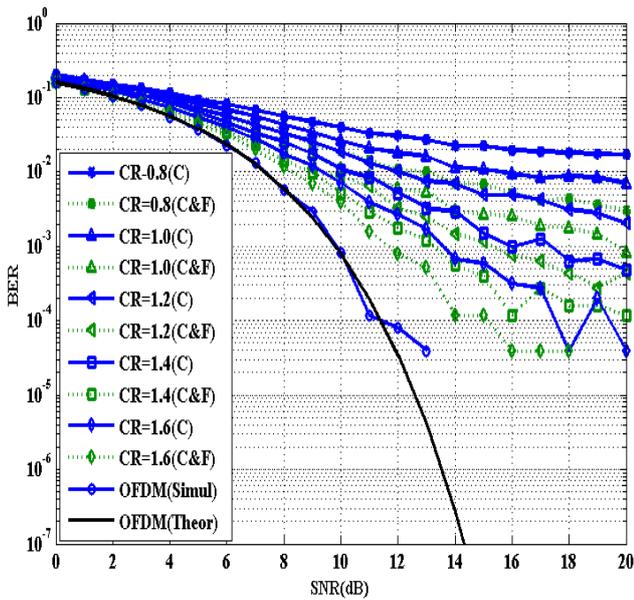Vol:9, No:4, 2015

Fig. 3 BER comparisons of clipping & clipping with filtered OFDM signals
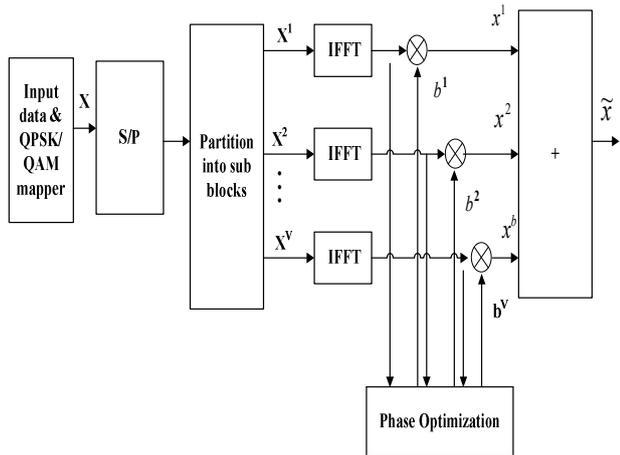
B. C- PTS Algorithm [16]

Fig. 4 Block diagram of partial transmit sequence scheme

In CPTS, the frequency domain input vector X is partitioned into a V number of disjoint sub-blocks as shown in Fig. 4 i.e.

$$X_v = [X_{v,0}, X_{v,1}, ..., X_{v,N-1}]^T, v = 1,2,..,V \qquad (7)$$

Such that

$$\sum_{v=1}^{V} X_v = X \qquad (8)$$

Then, these sub-blocks are transformed into V time domain partial sequences by taking LN point IFFT as

$$x_v = [x_{v,0}, x_{v,1}, .., x_{v,NL-1}]^T \qquad (9)$$

Equation (9) can be referred as partial transmit sequences. To get minimum PAPR, partial transmit sequences are independently rotated by complex phase factors $b_v = e^{j\phi v}, v = 1,2,...V$. The set of phase factor is denoted as a vector $b = [b_1, b_2,...,b_V]^T$. The main objective is to optimally combine V sub-blocks to minimize the PAPR of OFDM signal. The time domain signal after combining is given by

$$x'_v = \sum_{v=1}^{V} b_v x_v \qquad (10)$$

The optimal phase sequences are obtained by an exhaustive search in the peak power optimization process. If the number of available phase vectors is W and the number of sub-block is V, then the number of possible combinations of sub-block becomes $W^{(V-1)}$ and the search complexity exponentially increases with the number of sub-blocks (V) used. The phase sequence corresponding to minimum PAPR is transmitted to the receiver as side information (SI) to reconstruct the original signal. The CPTS algorithms suffer by the following reasons, i.e. more computational complexity in searching of optimal phase factor and the amount of SI required is high. These issues are overcome by the proposed method.
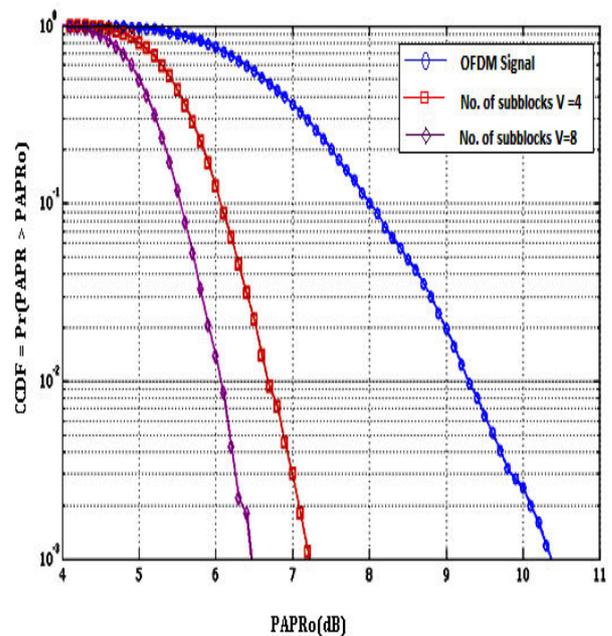

Fig. 5 CCDF of PAPR of CPTS algorithm with N=64, QPSK modulation and L= 4

Fig. 5 shows the PAPR reduction performance of CPTS algorithm with different number of sub-blocks i.e. V= 4, 8 respectively. At $CCDF = 10^{-3}$, The CPTS algorithm provides 3.2 dB, 4dB PAPR reduction for V=4, 8 respectively than the original signal. It can be seen that the PAPR decreases as the number of sub-block increases.

World Academy of Science, Engineering and Technology
International Journal of Electronics and Communication Engineering
Vol:9, No:4, 2015

C. PTS with Clipping (Hybrid Scheme)

Fig. 6 shows a block diagram of a serial combinational /hybrid scheme consisting of CPTS and clipping algorithms. In this technique, CPTS and clipping algorithms are serially combined to further enhance the PAPR reduction of the transmitted OFDM signal. However, this approach does not reduce computational complexity.
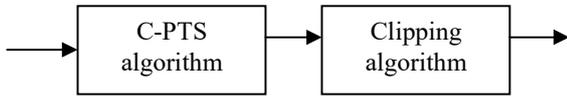
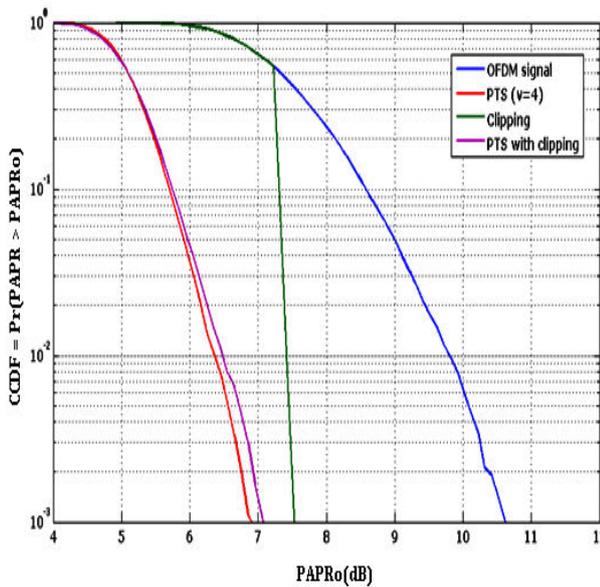Fig. 6 Block diagram of C-PTS with Clipping

Fig. 7 CCDF of PAPR of PTS with clipping hybrid scheme

Fig. 7 shows CCDF of PAPR in PTS with clipping hybrid scheme. For the simulation, the following parameters are considered, i.e. 10^5 OFDM symbols, QPSK modulation, the number of subcarriers N=64, number of sub-blocks V = 4,8 and the maximum allowable phase factor is W=2, $\in$ {1, -1}. The sub-blocks are optimally combined with different phase combination using a sub-combinational algorithm. At CCDF =10^{-3}, PAPR reduction of hybrid scheme is almost same as CPTS algorithm and there is no change in computational complexity.

IV. PROPOSED ALGORITHM

The proposed method intelligently applies both the clipping and CPTS algorithm in order to exploit the advantages of both the algorithms. If these two algorithms are applied serially, then the advantage of first algorithm is mitigated by the second algorithm. In [14], SLM and PTS algorithms are applied simultaneously to enhance the PAPR reduction performance, but the computational complexity is not reduced much and the amount of side information (SI) required is high. Hence, to enhance the PAPR reduction performance and to further reduce the computational complexity, in this paper a method that simultaneously applies clipping and CPTS algorithm is proposed as shown in Fig. 8. The description of three proposed algorithms is given as follows

A. Algorithm I

In this algorithm, the input data sequence $X (n)$ is equally segmented into two half sections of length *(N/2)* and clipping algorithm is applied to the first half section, whereas CPTS algorithm (with less number of sub-blocks) is applied for the remaining half of the input data sequence. Then the time domain output signal from each half section is combined to form an OFDM signal with minimum PAPR.

B. Algorithm II

The input data sequences are grouped as an odd indexed sequence into the first half section and even indexed sequence into the second half section. After the above interleaving process, for the first half section clipping algorithm is applied and CPTS algorithm is applied to the second half section.

C. Algorithm III

The input data sequences are partitioned as discussed in Algorithm I and for the first half section clipping algorithm is applied and for the second half section CPTS algorithm is applied. In addition, the CPTS algorithm is applied to the entire input data sequence and the optimal phase vector is searched for the whole input symbols. Finally, the signal with minimum PAPR is selected for transmission, i.e. either the one processed entirely with CPTS (or) the one with both clipping and CPTS whichever results in minimum PAPR. Therefore, this algorithm has larger computational complexity than the algorithms I, II.

According to our simulation results, the PAPR reduction performance of Algorithm III is similar to Algorithm I, but its computational complexity is very high. Therefore, only two algorithms are considered in this paper.

The algorithm I and II are referred as the algorithm with adjacent segmentation and with interleaving segmentation respectively.

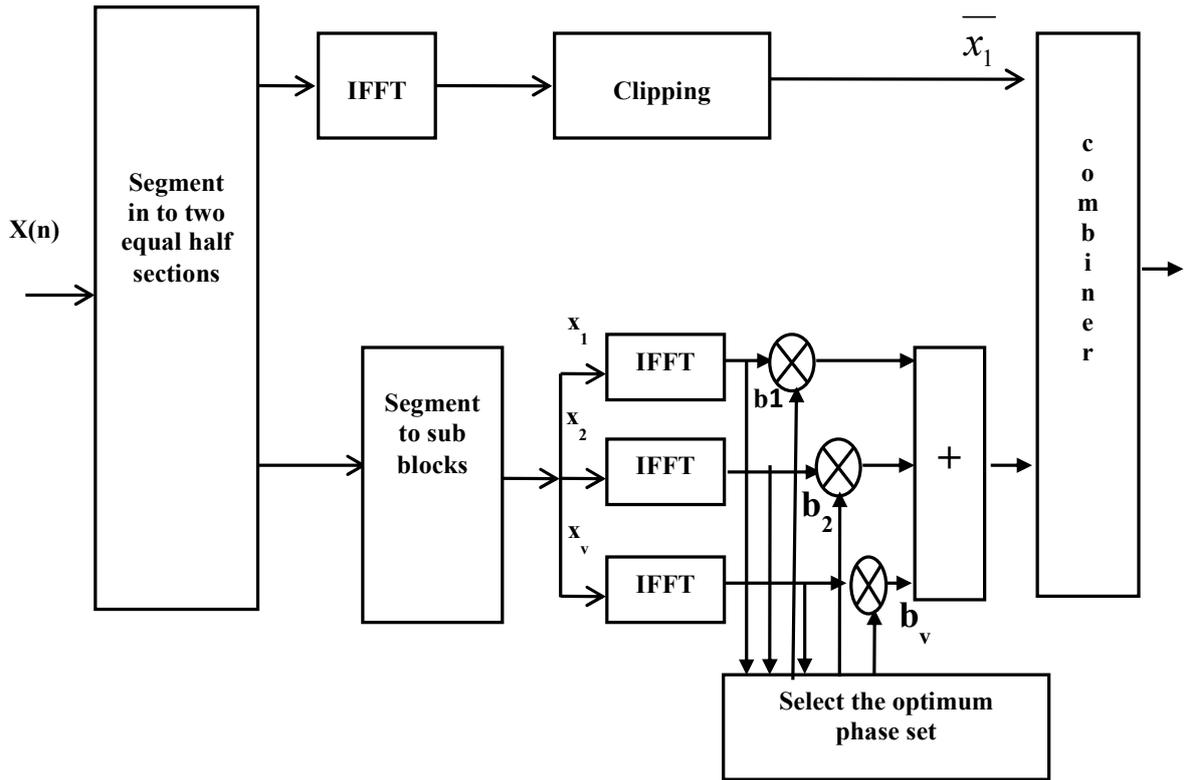World Academy of Science, Engineering and Technology
International Journal of Electronics and Communication Engineering
Vol:9, No:4, 2015

Fig. 8 Block diagram of the proposed algorithm
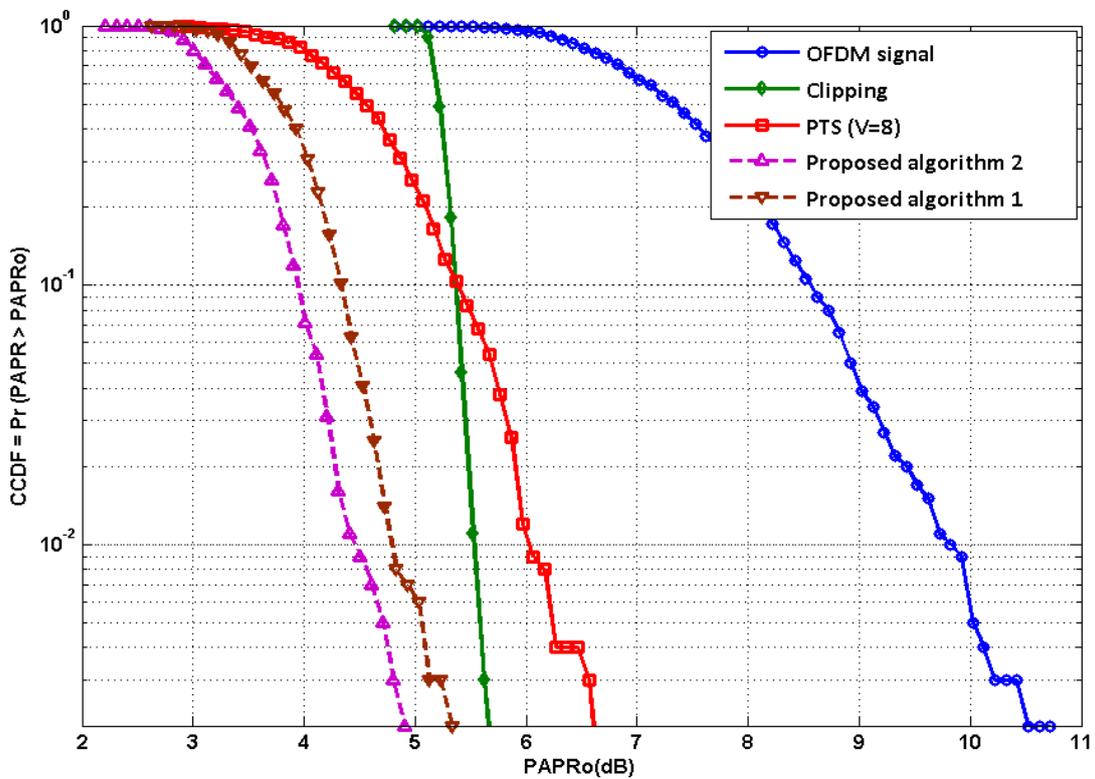
Fig. 9 CCDF of PAPR in proposed algorithm1, 2, when N=64, 16-QAM with L= 4

World Academy of Science, Engineering and Technology
International Journal of Electronics and Communication Engineering
Vol:9, No:4, 2015

V. SIMULATION RESULTS AND DISCUSSION

The PAPR reduction performance of the proposed algorithms has been evaluated using CCDF of PAPR through simulations. The simulations assumed that the data were QPSK/16QAM modulated, $N=64$ subcarriers and over sampling factor $L=4$. The clipping algorithm is applied with the clipping ratio (CR) of 1.2 dB and CPTS algorithm is applied with a number of sub blocks as $V=4$ and the allowable phase vector as $W=2$, i.e., {1, -1}.

Fig. 9 compares CCDF curve of proposed algorithms I, II with the CPTS and clipping algorithms. The simulation result shows that the proposed algorithms I, II achieves better PAPR reduction than conventional algorithms. At CCDF=10^{-3}, the proposed algorithms I, II achieve PAPR reduction of 1.4dB, 1.6dB respectively than CPTS algorithm. Algorithm II (with interleaving segmentation) achieves 0.2dB PAPR reduction gain over the algorithm I.

TABLE I
PAPR COMPARISON OF PROPOSED ALGORITHMS WITH CONVENTIONAL HYBRID ALGORITHM

S. No	PAPR Reduction Methods	PAPR at CCDF=10^{-3}
1	SLM with clipping [11]	6.2 dB
2	PTS with clipping	6.2 dB
3	SLM and PTS [14]	6.5 dB
4	Proposed algorithm	
	I	5.2 dB
	II	5 dB

In Table I, the PAPR reduction performance of the proposed algorithms is compared with other existing hybrid methods. It is observed that the proposed algorithms I, II provide 1dB additional PAPR reduction than the other conventional hybrid algorithms.

VI. ANALYSIS OF COMPUTATIONAL COMPLEXITY

The total complexity of CPTS algorithm with over sampling factor L=1, is given by [16]

$$T = 3VN/2 \, \log_2 N + 2VW^{V-1} N \qquad (11)$$

The computational complexity of the proposed method is expressed as

$$T' = \underbrace{(LN/4*\log_2 LN/2) + (LN/2*\log_2 LN/2) +}_{clipping}$$
$$\underbrace{(3VN/4*\log_2 N/2) + 2VW^{V-1}*N/2}_{C-PTS} \qquad (12)$$

where W is the number of allowed phase factors and V is the total number of sub-blocks. The first three terms in (12) corresponds to the computational complexity of IFFT and the last term is related to search complexity.

The computational complexity reduction ratio (CCRR) of the proposed method over CPTS algorithm is defined as

$$CCRR = \left(1 - \frac{Complexity \ of \ the \ proposed \ a \lg orithm}{Complexity \ of \ C-PTS}\right) X \, 100\% \qquad (13)$$

TABLE II
COMPUTATIONAL COMPLEXITY OF PROPOSED ALGORITHMS

	No. of. Sub-blocks	CPTS	Proposed Algorithm I,II	CCRR (%)
Total complexity	V=4	13,568	6,976	48.50
	V=8	2,72,896	1,36,256	50

Table II presents the computational complexity reduction ratio (CCRR) of proposed algorithms I, II over CPTS for $N=128$ and $W=2$. From the table, the computational complexity of the proposed algorithm I, II is reduced to 50% than CPTS algorithm.

VII. CONCLUSION

In this paper, a low complexity, parallel combinational / hybrid scheme consisting of clipping and CPTS algorithms for reducing PAPR in OFDM system is presented. The simulation result shows that the proposed method is promising to reduce PAPR than other hybrid algorithms with less side information (SI).The computational complexity of the proposed algorithm is reduced to 50% than CPTS algorithm. However, the clipping noise introduced by the proposed scheme is lesser than single clipping technique. Therefore, the proposed algorithm does not affect BER performance much.

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World Academy of Science, Engineering and Technology
International Journal of Electronics and Communication Engineering
Vol:9, No:4, 2015

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