Frequency Transformation with Pascal Matrix Equations

Phuoc Si Nguyen

***Abstract*—**Frequency transformation with Pascal matrix equations is a method for transforming an electronic filter (analogue or digital) into another filter. The technique is based on frequency transformation in the s-domain, bilinear z-transform with pre-warping frequency, inverse bilinear transformation and a very useful application of the Pascal's triangle that simplifies computing and enables calculation by hand when transforming from one filter to another. This paper will introduce two methods to transform a filter into a digital filter: frequency transformation from the s-domain into the z-domain; and frequency transformation in the z-domain. Further, two Pascal matrix equations are derived: an analogue to digital filter Pascal matrix equation and a digital to digital filter Pascal matrix equation. These are used to design a desired digital filter from a given filter.

***Keywords*—**Frequency transformation, Bilinear z-transformation, Pre-warping frequency, Digital filters, Analog filters, Pascal's triangle.

I. INTRODUCTION

CURRENTLY in filter design, a low pass prototype filter (i.e. a low pass, high pass, band pass or band stop filter) is used to convert between types of analogue or digital filters using frequency transformation and bilinear z-transformation with pre-warping frequency techniques [2]. With the support of the Pascal's triangle and two techniques mentioned, this paper will introduce a new method to transform between an analogue low pass prototype or a digital low pass, and a digital filter in a general mathematical way described as Pascal matrix equations. There are two kinds of Pascal matrix equations: the analogue low pass prototype to digital filter matrix equation, which is used for frequency transformation from the s-domain to the z-domain; and the digital low pass to digital filter matrix equation, which is used for frequency transformation in the z-domain. The main purpose of this new method is to provide a new way to design a digital filter with Pascal matrix equations, which facilitates easy hand calculation and programming.

II. TRANSFER FUNCTION

A transfer function is a mathematical way of describing the ratio between output and input of a system, and can be written in matrix form for analogue and digital filters, as shown in (1)-(4). Let A and B represent the coefficient matrices in the numerator and denominator of an analogue filter: a and b are used similarly for a digital filter. The subscript 'g' denotes 'given' and 'd' denotes 'desired'.

The transfer function of a given and desired analogue filter can be written as:

$$H_g(s)=\frac{\sum_{i=0}^{n}(A_g)_i s^i}{\sum_{i=0}^{n}(B_g)_i s^i}=\frac{\begin{bmatrix}(A_g)_0 & (A_g)_1 & \cdots & (A_g)_n\end{bmatrix}\begin{bmatrix}1\\ s^1\\ \vdots\\ s^n\end{bmatrix}}{\begin{bmatrix}(B_g)_0 & (B_g)_1 & \cdots & (B_g)_n\end{bmatrix}\begin{bmatrix}1\\ s^1\\ \vdots\\ s^n\end{bmatrix}}=\frac{A_g S}{B_g S} \quad (1)$$

$$H_d(s)=\frac{\sum_{i=0}^{n}(A_d)_i s^i}{\sum_{i=0}^{n}(B_d)_i s^i}=\frac{\begin{bmatrix}(A_d)_0 & (A_d)_1 & \cdots & (A_d)_n\end{bmatrix}\begin{bmatrix}1\\ s^1\\ \vdots\\ s^n\end{bmatrix}}{\begin{bmatrix}(B_d)_0 & (B_d)_1 & \cdots & (B_d)_n\end{bmatrix}\begin{bmatrix}1\\ s^1\\ \vdots\\ s^n\end{bmatrix}}=\frac{A_d S}{B_d S} \quad (2)$$

The transfer function of a given and desired digital filter can be written as:

$$H_g(z)=\frac{\sum_{i=0}^{n}(a_g)_i z^{-i}}{\sum_{i=0}^{n}(b_g)_i z^{-i}}=\frac{\begin{bmatrix}(a_g)_0 & (a_g)_1 & \cdots & (a_g)_n\end{bmatrix}\begin{bmatrix}1\\ z^{-1}\\ \vdots\\ z^{-n}\end{bmatrix}}{\begin{bmatrix}(b_g)_0 & (b_g)_1 & \cdots & (b_g)_n\end{bmatrix}\begin{bmatrix}1\\ z^{-1}\\ \vdots\\ z^{-n}\end{bmatrix}}=\frac{a_g Z}{b_g Z} \quad (3)$$

$$H_d(z)=\frac{\sum_{i=0}^{n}(a_d)_i z^{-i}}{\sum_{i=0}^{n}(b_d)_i z^{-i}}=\frac{\begin{bmatrix}(a_d)_0 & (a_d)_1 & \cdots & (a_d)_n\end{bmatrix}\begin{bmatrix}1\\ z^{-1}\\ \vdots\\ z^{-n}\end{bmatrix}}{\begin{bmatrix}(b_d)_0 & (b_d)_1 & \cdots & (b_d)_n\end{bmatrix}\begin{bmatrix}1\\ z^{-1}\\ \vdots\\ z^{-n}\end{bmatrix}}=\frac{a_d Z}{b_d Z} \quad (4)$$

In the next sections, the relationship between the coefficients matrix ***A*** and ***B*** of a given analogue low pass prototype and coefficients matrix ***a*** and ***b*** of a desired filter is demonstrated using frequency transformation and bilinear z-transformation with pre-warping frequency.

III. BILINEAR Z-TRANSFORMATION WITH PRE-WARPING FREQUENCY

Bilinear z-transformation is a popular method to transform an analogue filter to a digital filter [1] which preserves the frequency characteristics and it is defined as shown in (5). This technique involves one-to-one mapping from the s-plane onto z-plane, such as the imaginary axis jω (s=jω) is mapped

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into the unit circle |z|=1, the left half-plane s (s = δ+jω) is mapped onto the interior of the unit circle of the z-plane as shown in Fig. 1.

$$s=\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} \tag{5}$$

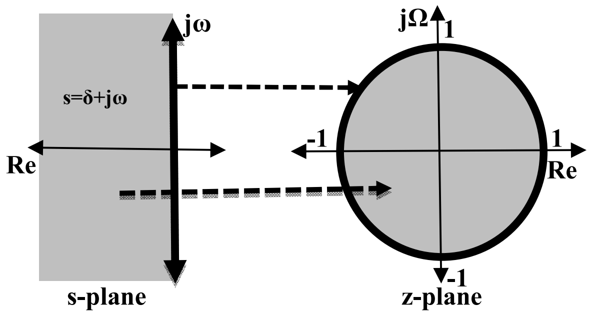

Fig. 1 Mapping from s-plane onto z-plane using bilinear z-transformation

The main advantage of this method is its ability to transform a stable designed analogue filter to a stable digital filter. However, the mapping of the frequency ω in s-plane into frequency Ω in z-plane is a non-linear relationship, called warping frequency as show in (6).

$$\omega=\frac{2}{T}\tan\left(\frac{T}{2}\Omega\right) \tag{6}$$

Let Ω_0 is a cut-off frequency of a desired digital filter, from (6) this frequency is tangentially warped compared with the cut-off frequency ω_0 of a designed analogue filter and this is an undesirable effect when transforming from s-domain into z-domain. To overcome this effect, one method called pre-warping frequency as expressed in (7)

$$s=\frac{\omega_0}{\tan\left(\frac{T}{2}\omega_0\right)}\frac{1-z^{-1}}{1+z^{-1}} \tag{7}$$

In the next section will introduce one new method to transform an analogue filter into a digital filter called bilinear z-transformation with pre-warping frequency.

IV. Bilinear Z-Transform with Pre-Warping Frequency

Bilinear z-transformation with pre-warping frequency is a method to convert an analogue low pass prototype with a corner angular frequency at 1 rad/s to a digital filter [5]. As shown in Table I: f_s is the sampling frequency, ω_c is the corner angular frequency of the desired low pass (Lp) and high pass (Hp), and ω_U and ω_L are the upper and lower angular frequency of the desired band pass (Bp) and band stop filter (Bs).

TABLE I
Bilinear Z-Transformation with Pre-Warping Frequency

Converting types	Analogue low pass prototype H(s)	s = f(z)
Low pass to low pass	H(f(z))	$c\frac{1-z^{-1}}{1+z^{-1}}$
Low pass to high pass	H(f(z))	$t\frac{1+z^{-1}}{1-z^{-1}}$
Low pass to band pass	H(f(z))	$U\frac{1-z^{-1}}{1+z^{-1}}+L\frac{1+z^{-1}}{1-z^{-1}}$
Low pass to band stop	H(f(z))	$\frac{1}{U\frac{1-z^{-1}}{1+z^{-1}}+L\frac{1+z^{-1}}{1-z^{-1}}}$

In the case of a narrow band filter, if f_0 is a centre frequency and Q is a quality factor, the upper frequency (f_U) and the lower frequency (f_L) of the narrow band filter can be found by (5), and the coefficients c, t, U and L can be calculated as:

$$\begin{cases} f_L = f_0\left(\sqrt{1+\frac{1}{4Q^2}}-\frac{1}{2Q}\right) \\ f_U = f_0\left(\sqrt{1+\frac{1}{4Q^2}}+\frac{1}{2Q}\right)\end{cases} \tag{8}$$

$$c=\cot\left(\pi\frac{f_c}{f_s}\right)\ ,\ t=\tan\left(\pi\frac{f_c}{f_s}\right)\ ,\ \begin{cases} c_U=\cot\left(\pi\frac{f_U}{f_s}\right) \\ t_L=\tan\left(\pi\frac{f_L}{f_s}\right)\end{cases}\Rightarrow\begin{cases} U=\frac{c_U}{1-c_U t_L} \\ L=\frac{t_L}{1-c_U t_L}\end{cases}$$

The transformation s=f(z) from an analogue low pass prototype H(s) must satisfy the requirement the digital transfer function H(f(z)) be stable. From Table. I, the inverse bilinear z-transformation can be found as:

$$z=\frac{c+s}{c-s} \tag{9}$$

Sub s = δ + jω into (9),

$$z=\frac{(c+\delta)+j\omega}{(c-\delta)-j\omega}\Rightarrow|z|=\frac{\sqrt{(c+\delta)^2+\omega^2}}{\sqrt{(c-\delta)^2+\omega^2}}$$

If δ = 0, |z| = 1 and δ < 0, |z| < 1, the left haft plane in s-domain maps into the inside of the unit circle. Hence, the bilinear z-transformation with pre-warping frequency is a stable transformation.

From (1)-(4) and Table I, the relationship between the coefficients of a given and a desired filter can be described as a matrix equation, which is a multiplication of some specified matrices introduced in the next section.

V. Definition of Some Specified Matrices

A. Matrix T and D_c

Two kinds of matrix T are matrix T_{LP} and T_{UL}. Matrix T_{LP} and D_c are a diagonal matrix with a size of (n+1;n+1), where n is the given n^{th}-ordered analogue low pass prototype as illustrated below.

$$\boldsymbol{T}_{LP}=\begin{bmatrix}1&0&0&0&0\\0&c&0&0&0\\0&0&c^2&0&0\\0&0&0&\ddots&0\\0&0&0&0&c^n\end{bmatrix}\qquad \boldsymbol{D}_c=\begin{bmatrix}c^n&0&0&0&0\\0&c^{n-1}&0&0&0\\0&0&c^{n-2}&0&0\\0&0&0&\ddots&0\\0&0&0&0&1\end{bmatrix}$$

A neat application of the Pascal's triangle is in the expansion of a binomial expression $(U+L)^n$. Inserting zeros into the Pascal's triangle makes a matrix $\boldsymbol{T}_{UL}$ with a size of (n+1; 2n+1) as shown:

$$\boldsymbol{T}_{UL}=\begin{bmatrix}0&0&0&0&0&1&0&0&0&0&0\\0&0&0&0&U&0&L&0&0&0&0\\0&0&0&U^2&0&2UL&0&L^2&0&0&0\\0&0&U^3&0&3U^2L&0&3UL^2&0&L^3&0&0\\0&\iddots&0&\ldots&0&\ldots&0&\ldots&0&\ddots&0\\U^n&0&\ldots&0&\ldots&0&\ldots&0&\ldots&0&L^n\end{bmatrix}$$

B. Matrix P and P^{tr}

The matrix P contains the positive and negative binomial coefficients of the Pascal's triangle in the first, last row and the first, last column corresponding to the edge size and the nth row of the Pascal's triangle; another element in the matrix P can be calculated from its left, diagonal and above element. There are two different matrices for matrix P: PLP for a low pass filter with size (n+1; n+1), and PHBS for high pass with size (n+1; n+1); the band pass and band stop has a size of (2n+2; 2n+1). The matrices can be defined as:

$$\boldsymbol{P}_{LP}=\left[\begin{cases}(\boldsymbol{P}_{LP})_{i=1;j=1\to n+1}=1\\(\boldsymbol{P}_{LP})_{i=n+1;j=1\to n+1}=(-1)^{j-1}\\(\boldsymbol{P}_{LP})_{i=1\to n+1;j=1}=\binom{n}{i-1}\\(\boldsymbol{P}_{LP})_{i=1\to n+1;j=n+1}=(-1)^{i-1}\binom{n}{i-1}\\(\boldsymbol{P}_{LP})_{i;j}=(\boldsymbol{P}_{LP})_{i;j-1}-(\boldsymbol{P}_{LP})_{i-1;j-1}-(\boldsymbol{P}_{LP})_{i-1;j}\end{cases}\right]\qquad \boldsymbol{P}_{HBS}=\left[\begin{cases}(\boldsymbol{P}_{HBS})_{i=1;j=1\to N+1}=1\\(\boldsymbol{P}_{HBS})_{i=N+1;j=1\to N+1}=(-1)^{j-1}\\(\boldsymbol{P}_{HBS})_{i=1\to N+1;j=1}=(-1)^{i-1}\binom{N}{i-1}\\(\boldsymbol{P}_{HBS})_{i=1\to N+1;j=N+1}=\binom{N}{i-1}\\(\boldsymbol{P}_{HBS})_{i;j}=(\boldsymbol{P}_{HBS})_{i;j-1}+(\boldsymbol{P}_{HBS})_{i-1;j-1}+(\boldsymbol{P}_{HBS})_{i-1;j}\end{cases}\right]$$

Matrix P^{tr} is a transpose matrix of the matrix P and it is used in the case of transforming a digital filter to another digital filter.

VI. Frequency Transformation with Pascal Matrix Equations

The frequency transformation with Pascal matrix equations may be used in the analogue or digital domain by mathematically mapping a designed filter onto another filter with the desired specifications. This section will introduce two algorithms for transformation from a given filter to a digital filter: frequency transformation from the s-domain into the z-domain and frequency transformation in the z-domain.

A. Frequency Transformation from S-Domain into Z-Domain

Frequency transformation from the s-domain into the z-domain is a method to convert an analogue low pass prototype to a digital filter [3], [4]. From Table I and (1), (3), an analogue low pass prototype is converted to another analogue filter by using frequency transformation in the s-domain, then applying bilinear z-transformation with pre-warping frequency to convert it to the desired digital filter. The relationship of the coefficients of a given analogue low pass filter and a desired digital can be described as in (10), which is termed the analogue low pass prototype to digital filter Pascal matrix equation:

$$\begin{cases}\boldsymbol{a}_d=\boldsymbol{P}_d(\boldsymbol{A}_g\boldsymbol{T}_d)\\\boldsymbol{b}_d=\boldsymbol{P}_d(\boldsymbol{B}_g\boldsymbol{T}_d)\end{cases}\tag{10}$$

Equation (10) is a general formula to transform a transfer function H(s) of an analogue low pass prototype in the s-domain into a transfer function H(z) of a digital filter in the z-domain.

B. Frequency Transformation in Digital Domain

Frequency transformation in the z-domain is a method of transforming one digital filter into another. This section will first introduce a new way to transform a low pass digital to a low pass, high pass, band pass and band stop digital filter, from which a general equation will be found for applying this to transform a low pass digital filter to another digital filter as called Low pass digital to digital filter Pascal matrix equation.

Form Table I and the inverse bilinear z-transformation, a digital low pass can be transformed into another digital, as shown in Table II, where $\boldsymbol{c}_N=\mathbf{cot}\left(\pi\frac{\boldsymbol{f}_{CN}}{\boldsymbol{f}_s}\right)$, and f_{CN} is the new corner frequency of a digital low pass filter.

TABLE II
FREQUENCY TRANSFORMATION IN DIGITAL DOMAIN

Converting types	Digital low pass prototype H(z)	$Z(z^{-1})$
Low pass to low pass	$H(Z(z^{-1}))$	$\frac{c-c_N+(c+c_N)z^{-1}}{c+c_N+(c-c_N)z^{-1}}$
Low pass to high pass	$H(Z(z^{-1}))$	$\frac{c-t-(c+t)z^{-1}}{c+t-(c-t)z^{-1}}$
Low pass to band pass	$H(Z(z^{-1}))$	$\frac{c-U-L+2(U-L)z^{-1}-(U+L+c)z^{-2}}{c+U+L-2(U-L)z^{-1}+(U+L-c)z^{-2}}$
Low pass to band stop	$H(Z(z^{-1}))$	$\frac{cU+cL-1-2c(U-L)z^{-1}+(cU+cL+1)z^{-2}}{cU+cL+1-2c(U-L)z^{-1}+cU+cL-1)z^{-2}}$

From Table II, transforming a digital low pass with the transfer function $H(z)$ to a digital low pass, high pass, band pass or band stop requires just the replacement of the variable z^{-1} by a function $Z(z^{-1})$. With the supporting of the Pascal's triangle in binomial expansion, the relationship between coefficients $\boldsymbol{a}_g$, $\boldsymbol{b}_g$ of the given digital low pass and coefficients a_d, b_d of the desired digital filter can expressed as

$$\begin{cases}\boldsymbol{a}_d=\boldsymbol{a}_g\boldsymbol{P}_{LP}^{tr}\boldsymbol{D}_c\boldsymbol{T}\boldsymbol{P}^{tr}\\\boldsymbol{b}_d=\boldsymbol{b}_g\boldsymbol{P}_{LP}^{tr}\boldsymbol{D}_c\boldsymbol{T}\boldsymbol{P}^{tr}\end{cases}\tag{11}$$

Depending on the desired digital filter, P^{tr} and T can be P_{LP} and T_{LP} for low pass, P_{HBS} for high pass, band pass, band stop and T_{UL} for band pass, band stop and narrow band filter.

VII. Design Of A Digital Filer Using Pascal Matrix Equations

As discussed in Section VI, a digital filter can be designed from a given analogue low pass prototype or a digital low pass filter. Fig. 2 illustrates a new algorithm to design a digital filter using Pascal matrix equations, which can be implemented by programming in digital signal processors.

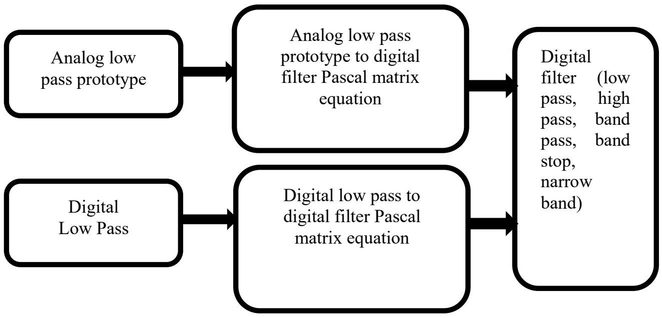

Fig. 2 Block diagram illustrates design a digital filter using Pascal matrix equation

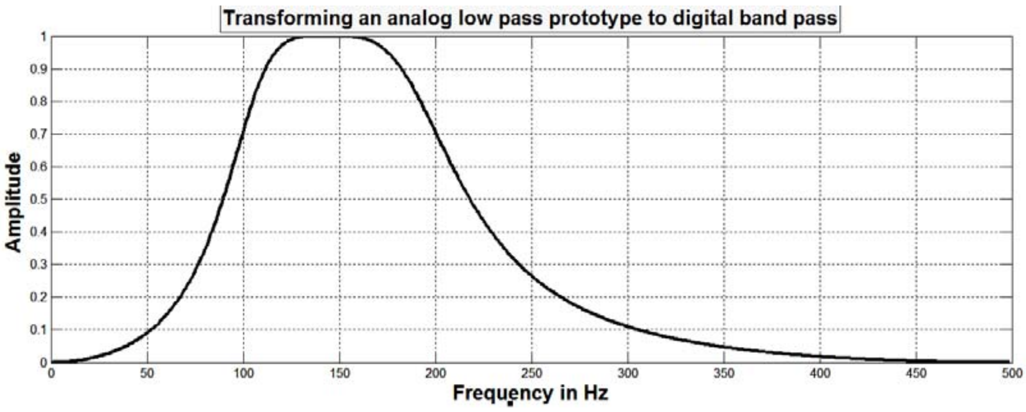

Fig. 3 Transforming an analog low pass prototype to digital band pass with a lower frequency 100Hz, upper frequency 200Hz at sampling frequency 1Khz using Pascal Matrix Equation

- Example 1: Transform a 2nd-order analogue Butterworth low pass prototype to a digital band pass with upper frequency of 200 Hz and lower frequency of 100 Hz, at the sampling frequency at 1 KHz.

$$H(s)=\frac{1}{s^2+1.4141s+1}\Rightarrow\begin{cases}A_g=\begin{bmatrix}1 & 0 & 0\end{bmatrix}\\ B_g=\begin{bmatrix}1 & 1.4141 & 1\end{bmatrix}\end{cases}$$

$$\begin{cases}c=\cot\left(\pi\frac{200}{1000}\right)=1.3764\\ t=\tan\left(\pi\frac{100}{1000}\right)=0.3249\end{cases}\Rightarrow\begin{cases}U=\frac{c}{1-ct}=2.4899\\ L=\frac{t}{1-ct}=0.5878\end{cases}$$

$$Apply(10)\quad\begin{cases}a_d=P_{HBS}\left(A_gT_{UL}\right)\\ b_d=P_{HBS}\left(B_gT_{UL}\right)\end{cases}$$

$$a_d=\begin{bmatrix}1 & 1 & 1 & 1 & 1\\ -4 & -2 & 0 & 2 & 4\\ 6 & 0 & -2 & 0 & 6\\ -4 & 2 & 0 & -2 & 4\\ 1 & -1 & 1 & -1 & 1\end{bmatrix}\times\left(\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}0 & 0 & 1 & 0 & 0\\ 0 & 2.4899 & 0 & 0.5878 & 0\\ 6.1996 & 0 & 2.9271 & 0 & 0.3455\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & 0 & -2 & 0 & 1\end{bmatrix}$$

$$b_d=\begin{bmatrix}1 & 1 & 1 & 1 & 1\\ -4 & -2 & 0 & 2 & 4\\ 6 & 0 & -2 & 0 & 6\\ -4 & 2 & 0 & -2 & 4\\ 1 & -1 & 1 & -1 & 1\end{bmatrix}\times\left(\begin{bmatrix}1 & 1.4141 & 0\end{bmatrix}\begin{bmatrix}0 & 0 & 1 & 0 & 0\\ 0 & 2.4899 & 0 & 0.5878 & 0\\ 6.1996 & 0 & 2.9271 & 0 & 0.3455\end{bmatrix}\right)$$

$$=\begin{bmatrix}14.8246 & -28.7964 & 31.4164 & -18.0364 & 6.1196\end{bmatrix}$$

$$\therefore H(z)=\frac{1-2z^{-2}+z^{-4}}{14.8246-28.7964z^{-1}+31.4164z^{-2}-18.0364z^{-3}+6.1196z^{-4}}$$

- Example 2: Transform the digital low pass H(z) at corner frequency f_c= 50Hz to a digital band pass with upper frequency of 200 Hz and lower frequency of 100 Hz at the sampling frequency 1KHz.

$$H(z)=\frac{1+2z^{-1}+z^{-2}}{49.7925-77.7269z^{-2}+31.9345}\Rightarrow\begin{cases}a_g=\begin{bmatrix}1 & 2 & 1\end{bmatrix}\\ b_g=\begin{bmatrix}49.7925 & -77.7269 & 31.9345\end{bmatrix}\end{cases}$$

$$c=\cot\left(\pi\frac{50}{1000}\right)=6.3138$$

$$Apply\ (11)\quad\begin{cases}a_d=a_gP_{LP}^{tr}D_cT_{UL}P_{HBS}^{tr}\\ b_d=b_gP_{LP}^{tr}D_cT_{UL}P_{HBS}^{tr}\end{cases}$$

$$a_d=\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 1\\ 1 & 0 & -1\\ 1 & -2 & 1\end{bmatrix}\begin{bmatrix}39.8635 & 0 & 0\\ 0 & 6.3138 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 0 & 1 & 0 & 0\\ 0 & 2.4899 & 0 & 0.5878 & 0\\ 6.1996 & 0 & 2.9271 & 0 & 0.3455\end{bmatrix}\times\begin{bmatrix}1 & -4 & 6 & 4 & 1\\ 1 & -2 & 0 & 2 & -1\\ 1 & 0 & -2 & 0 & 1\\ 1 & 2 & 0 & -2 & -1\\ 1 & 4 & 6 & 4 & 1\end{bmatrix}=159.4538\begin{bmatrix}1 & 0 & -2 & 0 & 1\end{bmatrix}$$

$$b_d=\begin{bmatrix}49.7925 & -77.7269 & 31.9345\end{bmatrix}\begin{bmatrix}1 & 2 & 1\\ 1 & 0 & -1\\ 1 & -2 & 1\end{bmatrix}\begin{bmatrix}39.8635 & 0 & 0\\ 0 & 6.3138 & 0\\ 0 & 0 & 1\end{bmatrix}\times\begin{bmatrix}0 & 0 & 1 & 0 & 0\\ 0 & 2.4899 & 0 & 0.5878 & 0\\ 6.1996 & 0 & 2.9271 & 0 & 0.3455\end{bmatrix}\begin{bmatrix}1 & -4 & 6 & 4 & 1\\ 1 & -2 & 0 & 2 & -1\\ 1 & 0 & -2 & 0 & 1\\ 1 & 2 & 0 & -2 & -1\\ 1 & 4 & 6 & 4 & 1\end{bmatrix}$$

$$=159.4538\begin{bmatrix}14.8246 & -28.7964 & 31.4164 & -18.0364 & 6.1196\end{bmatrix}$$

$$\therefore H(z)=\frac{1-2z^{-2}+z^{-4}}{14.8246-28.7964z^{-1}+31.4164z^{-2}-18.0364z^{-3}+6.1196z^{-4}}$$

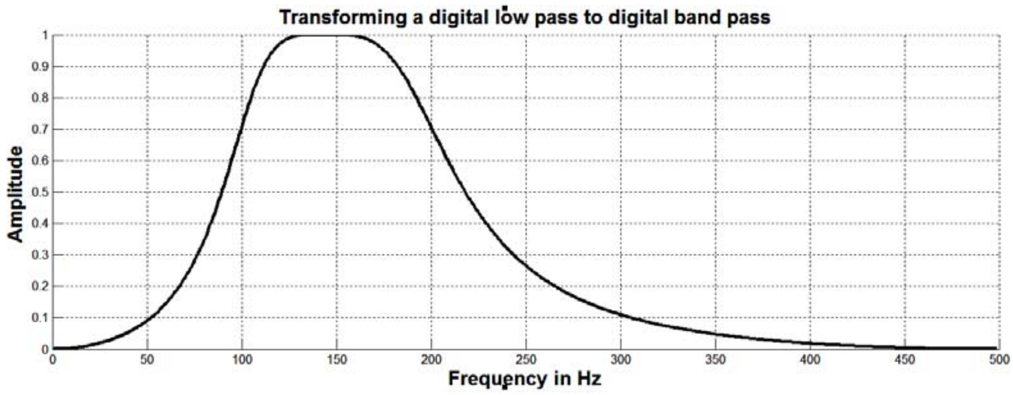

Fig. 4 Transforming a digital low pass prototype at a cut-off frequency 50Hz to digital band pass with a lower frequency 100Hz, upper frequency 200Hz at sampling frequency 1Khz using Pascal Matrix Equation

VIII. Conclusion

Two methods for designing a digital filter based on a given analogue low pass filter or a given digital low pass filter were examined. With the support of Pascal matrix equations, the analogue low pass prototype to digital filter Pascal matrix equation and the digital low pass to digital filter Pascal matrix equation can be implemented by programming using MATLAB, C, C++ or assembly program languages for digital signal processors. Its inherent simplicity could make the algorithm attractive for many applications demanding minimization of computational requirements and workloads.

Acknowledgment

This is a self-research of the author and he would like to share his idea how to apply Pascal's triangle in filters design to whom have the same interesting. He would greatly appreciate all feedbacks from readers to improve this research!

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