Actuator Fault Detection and Fault Tolerant Control of a Nonlinear System Using Sliding Mode Observer

R. Loukil, M. Chtourou, T. Damak

Abstract—In this work, we use the Fault detection and isolation and the Fault tolerant control based on sliding mode observer in order to introduce the well diagnosis of a nonlinear system. The robustness of the proposed observer for the two techniques is tested through a physical example. The results in this paper show the interaction between the Fault tolerant control and the Diagnosis procedure.

Keywords—Fault detection and isolation “FDI”, Fault tolerant control “FTC”, sliding mode observer, nonlinear system, robustness, stability.

I. INTRODUCTION

It is certainly known that an automatic control system requires an efficient fault detection, isolation and accommodation capability. Several conventional linear as well as nonlinear observers have been suggested during the past decades, so different techniques of testing the observability of nonlinear system are presented in literature. For this reason, many observability analyses are developed for these systems to judge the nonlinearity. In this context, we will use a sliding mode observer which is a copy model with a term of correction and can establish the convergence of the estimate state to the real state. Several techniques for monitoring and diagnosis of systems in defaults are developed and have dynamic in recent years. However, most of these techniques are based on monitoring only the process without taking into account its interaction with the control system. The objective of this work is to propose a control policy approach to tolerant default in order to accommodate automatically the effect of certain types of defaults that can appear on actuators or on the system while capable of maintaining the stability of the physical system and avoid catastrophic situations.

II. SYNTHESIS OF SLIDING MODE OBSERVER

It provides robust performance for observing the states of nonlinear systems with uncertainties. It is well known that modelling inaccuracies can have strong adverse effects on nonlinear observer systems. Therefore, any practical design must address them explicitly. Two major and complementary approaches to dealing with model uncertainty are robust and adaptive approaches. In fact, the sliding mode observer was shown successful in dealing with model-free systems [8].

The idea is to define a time-varying surface \( s(x, \dot{x}) \) in the state-space such that the problem of \( s(t) = x(t) - x(t) \) is equivalent to remain on the surface \( s(t) \) for \( t > 0 \) [1], [5]. The major difficulty here is how to find a suitable sliding surface \( s(t) \).

To design a sliding mode observer, let us consider the general state space model:

\[
\dot{x} = f(x, u) + C \dot{x}(t)
\]  

where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R}^m \) is the bounded control input, and \( C \) is known. Then the structure of the Sliding mode observer is given by:

\[
\dot{s}(t) = f(x, u) + \lambda \text{ sign}(s(t))
\]

We choose \( \lambda > 0 \) and properly fixed for the estimation and the diagnosis [3].

III. FAULT DETECTION AND ISOLATION

In this paper, we assume that only actuator faults can occur and no sensor faults are involved. For simplicity, we consider the case that only a single actuator is faulty at one time because extensions to multi-faults situation are straightforward. During this part, we will use a specific algorithm for the FDI scheme [6] which is composed of six steps:

- We form the new faulty model
- We build a bank of m observers for detection and isolation of fault.
- We generate the residuals \( r_i(t) \).
- From the thresholds \( \delta \) and the fault code \( e_i(t) \) we elaborate the structure matrix \( \Phi \).
- We generate the structured residuals for the fault isolation and identification.
- We estimate finally \( f_a \).

To do this, we will determine the appropriate thresholds \( \delta = \) constant and build the vectors of fault codes \( e_i(t) \) composed of \( r_i(t) \) [2].

\[
\begin{bmatrix}
r_1 \\
r_2 \\
r_3 \\
r_4 \\
r_5 \\
r_6
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 & 0 & e_1 \\
0 & e_2 & & & \\
0 & & e_3 & & \\
0 & & & e_4 & \\
0 & & & & e_5 & \\
0 & & & & & e_6
\end{bmatrix}
\begin{bmatrix}
r_a \\
r_a \\
r_a \\
r_a \\
r_a \\
r_a
\end{bmatrix}
\]

\[
a_i(t) = \begin{cases}
1 & \text{if} \quad r_i(t) \geq \delta \\
0 & \text{if} \quad r_i(t) < \delta
\end{cases}
\]
\[
\varepsilon_i(t) = (a_i(t) \alpha_1(t) \ldots \alpha_k(t))\]
\[r_i = R(\hat{y}_i - y) \in m\]

We use \(R\) as an operator of the residual and in our case it is the norm 2 defined as the difference between the estimated output and measured output, so (5) becomes:

\[
r_i(t) = \|\hat{y}_i - y\|, i = 1, \ldots, m
\]

For the isolation and identification of the fault, we must define the threshold, \(i = 1 \ldots m\) as adaptive because of the error measurement [2].

\[
\delta = r_i(t - \tau) + 1
\]

We fix the value of \(\tau\), so we obtain

\[
\Phi = [\varepsilon_i(t) \varepsilon_i(t) \ldots \varepsilon_i(t)] \text{ and } rs \rightarrow \Phi r
\]

**IV. FAULT TOLERANT CONTROL**

The architecture of an active FTC procedure is described in Fig. 1.

![Fig. 1 Process description using the FDI and FTC](image)

In fact, the block "FDI" uses to entry and measured outputs of the system. Its main function is to detect and estimate the default and the state-owned system of variables online.

Once the default appeared, the "FDI" block line provides data for default of the system and state-owned block "FTC". However, the FDI approach allows taking into account the types of different defaults acting on the system and ensuring its friability information to activate the mechanism used reconfiguration in minimal time. Besides, the block "FTC" is based on information delivered by block "FDI" [4], [9].

According to the mechanism and the type of occurred fault, it accommodates or reconfigures online control law to maintain the Stability, the dynamics of the system and its initial performance. Fig. 1 shows that active FTC order contains a supervisor. His principle role is to assure that without fault, the nominal control is determined. We adopt a special command law:

\[
U(t) = - \sum_{i=1}^{N} h_i(z(t))K_i\hat{x}(t)
\]

with \(K_i = (1, \ldots, M)\) are the different gains of the command law that is proposed in order to assure the stability robustness of the model with its sliding mode observer.

**V. APPLICATION**

**A. System Description**

The system of two-level tanks presented in Fig. 2 consists of a set of elementary components: the top tray (BH), the pan bottom (BB), a valve gain \(K_i\) from the top (VKH) and a valve gain \(K_i\) from the bottom (VKB), different sensors that allow us to measure the height of the product in the tank top (H1) and the tank bottom (H2) and the inflow of the product (qH) and (qL) in the tank top and the bottom tank respectively.

The flow admission is controlled directly (open loop) by a digital computer [2], [7].

![Fig. 2 System of two level tanks](image)

The dynamic model of the system can be presented as:

\[
\begin{align*}
S_1 \frac{dh_1}{dt} &= q_e - q_1 \\
S_2 \frac{dh_2}{dt} &= q_1 - q_3 \\
q_e &= K_1 \sqrt{h_1}
\end{align*}
\]
\[ q_1 = K_2 \sqrt{h_1 - h_2} \]

**B. Estimation and FDI Results Using a Sliding Mode Observer**

To compare the performance of the observers, we simulate the observed error \( e_1 \) between \( h_1 \) and its estimated \( \hat{h}_1 \) then the observed error \( e_2 \) between \( h_2 \) and its estimated \( \hat{h}_2 \) so we obtain the Figs. 3 and 4.

In fact, we have chosen \( H \) as a Hurwitz matrix so that the observer has dynamics faster than the system. \( \gamma \) is a constant and \( P \) is a design positive definite matrix solution of Lyapunov equation which is:

\[ H^T P + PH = -Q \] (10)

We should notice that \( Q \) is a definite positive matrix, which can be chosen freely. For the simulation, the values of \( H, P \) and \( \gamma \) used in (10) are given: \( H = -5 I_1, P = I \) and \( \gamma = 2 \) and the injection of a single fault \( f_{ui} \) equal to 2 m/s.

Figs. 5 (a) and (b) show the variation of the residuals especially after the fault injection. However, if we have multiple faults injection on the system we move to the use of structured residuals.

Fig. 5 (a) shows the variation of the residual \( r_1 \) during the time using a sliding mode observer. We find that \( r_1 \) has a negative peak at \( t = 20 \) s the time of actuator fault injection, and it converge rapidly to zero [1].

Fig. 5 (b) shows the variation of the residual \( r_2 \) during the time using a sliding mode observer. We find that \( r_2 \) has a negative peak at \( t = 20 \) s the time of actuator fault injection and it converge rapidly to zero different from the classic one [2].

![Fig. 3 The observed observer \( e_1 \) [1]](image)

![Fig. 4 The observed observer \( e_2 \) [1]](image)
C. Simulation Results According to the Use of FTC Procedure

The injection of a single fault $f_1$ equal to 2 m$^3$/s and at $t_f=4$ s. Fig. 6 shows the variation of the residuals especially after the fault injection using the FTC and without FTC. According to Fig. 3 which presents the results of the work presented in this paper, we notice that the residuals $r_1$ and $r_2$ have been improved in the case of the fault tolerant control. In fact, they are nearer to zero than without FTC approach.

The resolution using the LMI toolbox in Matlab allows the calculation of the gain $\lambda$ of the sliding mode observer and the gain $K_1$ and $K_2$ for the command law.

$$\lambda = \begin{bmatrix} 16.02 & -25.23 \end{bmatrix}$$

$$K_1 = [96.67 \quad 195.03]^T$$

$$K_2 = [-63.62 \quad 170.13]^T$$
VI. CONCLUSION

In this paper, we have used both of the FDI and the FTC using the sliding mode observer due to its great capacity in estimation and diagnosis of nonlinear system although the instability. In fact, from other works realised in literature cited in references we have mentioned the robustness of the specific kind of nonlinear observer for the two techniques. We have tested this fact through a physical example based on a two level tank system. The obtained results show that for a nonlinear system, the fault detection and isolation isn’t sufficient enough for keeping the same performances of the system. For this fact, we add the block "FTC" which is based on information delivered by the block "FDI". According to the mechanism and the type of faults occurred, it accommodates or reconfigures online control law to maintain the Stability, the dynamics of the system and its initial performance. Finally, from the simulation results using the toolbox of Matlab we have determinate the different amirc value of sliding mode ‘s gain and the command law of the FTC procedure, we have founded that the residuals $r_1$ and $r_2$ have been improved in the case of the fault tolerant control. In fact, they are nearer to zero than without FTC approach.

REFERENCES


