Theoretical Model of a Flat Plate Solar Collector Integrated with Phase Change Material

Mouna Hamed, Ammar B. Brahim

Abstract—The objective of this work was to develop a theoretical model to study the dynamic thermal behavior of a flat plate solar collector integrated with a phase change material (PCM). The PCM acted as a heat source for the solar system during low intensity solar radiation and night. The energy balance equations for the various components of the collector as well as for the PCM were formulated and numerically solved using Matlab computational program. The effect of natural convection on heat during the melting process was taken into account by using an effective thermal conductivity. The model was used to investigate the effect of inlet water temperature, water mass flow rate, and PCM thickness on the outlet water temperature and the melt fraction during charging and discharging modes. A comparison with a collector without PCM was made. Results showed that charging and discharging processes of PCM have six stages. The adding of PCM caused a decrease in temperature during charge and an increase during discharge. The rise was most enhanced for higher inlet water temperature, PCM thickness and for lower mass flow rate. Analysis indicated that the complete melting time was shorter than the solidification time due to the high heat transfer coefficient during melting. The increases in PCM height and mass flow rate were not linear with the melting and solidification times.

Keywords—Thermal energy storage, phase change material, melting, solidification.

I. INTRODUCTION

Solar energy plays an important role in replying to the rising demand of energy as well as dealing with pressing climate change and air pollution issues. Its intermittent and dynamic nature makes thermal energy storage systems highly valuable for many solar applications [1], [2]. Latent heat storage using phase change materials (PCM) is one of the most effective methods to store thermal energy due to its high storage density and isothermal behavior during the charging and discharging processes [3]. PCM have been extensively used in practice with an aim to improve the storage capacity of various thermal energy systems. It is possible to use the latent heat of solid-gas, solid-liquid, and liquid-gas transformations; however, only the solid-liquid transformation is used thanks to its lower volume variation [4].

Actually, three different methods to incorporate PCMs into solar thermal system have been suggested: PCM integrated inside the storage tank, the addition of PCM node between the collector and the storage tank and PCM integrated directly in the solar collector [5]. The PCM-integrated solar collector combines collection and storage of thermal energy into a single unit. A system of this type removes the need of conventional storage tanks, thus minimizing space and costs. In recent years, the use of PCMs in solar collector water heater has been developed and analyzed theoretically and experimentally. Tiwari et al. [6] developed a theoretical analysis of PCM storage water heater by incorporating the effect of water flow through a parallel plate placed at the solid-liquid interface. To minimize the night heat loss, the exposed surface was covered by a movable insulation. They proved that during day and night the hot water can be made available at a temperature 15-20°C higher than the ambient temperature, and a rise in the melted region caused a decrease of the water temperature fluctuations. Rabin et al. [7] designed an integrated solar collector storage system with salt hydrate eutectic mixture (48% CaCl₂, 4.5% KCl, 0.4% NaCl and 47.1% H₂O) as PCM. They developed a mathematical model for the charging process based on heat conduction and neglected the effect of natural convection. The results showed that the PCM layer plays an important role in the location of the propagation distance of the solid interface of the solid-liquid PCM interface and minor influence on the liquid interface. Kürklü et al. [8] presented a new type of solar collector consisting of two adjoining sections. One section was filled with water and the other with a paraffin wax. They found that the water temperature exceeded 55°C during a typical day of high solar radiation and remained over 36°C during the whole night after the insulation blanket was applied. The energy efficiency values were between about 22% and 80%. Chaabane et al. [9] conducted a numerical study of an integrated collector storage solar water heater with two kinds of PCM (myristic acid and RT-42 graphite) and three radii of this PCM layer. They concluded that the highest water temperature corresponds to the lowest radius unlike the night thermal losses where the optimum case corresponds to the highest radius. Bouadila et al. [10] studied experimentally the thermal performance of an integrated solar latent storage collector with two PCM-filled cavities incorporating below the absorber. The results showed that the paraffin as a PCM contributes to increase the performance of the solar collector at night. Also, they examined numerically the evolution of the PCM melted volume fraction, liquid-solid interfaces, PCM distribution temperature and melting-solidification flow in the PCM-filled cavity.

The purpose of this work is to study the thermal performance of an integrated collector storage solar heater. A mathematical model based on the energy equations was developed. Effects of inlet water temperature, water mass flow
rate, and PCM thickness were studied and a comparison with a collector without a PCM was made.

II. DESCRIPTION SYSTEM

The proposed integrated collector storage solar water heater is shown in Fig. 1. It consists of a transparent cover, an air gap, tubes, an absorber, a phase change material, insulations and a container. The solar energy transmitted through the transparent cover is absorbed by the absorber; hence, its temperature increases. Part of thermal energy is transferred by convection to the transfer fluid and the other is transferred by both conduction and convection to the phase change material incorporating below the absorber. When the absorber temperature becomes higher than that of the PCM, heat is first stored as a sensible heat until the PCM reaches its melting temperature. As time elapses, the PCM starts to melt and after complete melting, the heat is stored in the molten PCM as a sensible heat. During low intensity solar radiation periods and night the collector solar components starts to cool down and the PCM acts as a heat source for the flat plate solar system. The PCM releases energy to the absorber and from the latter to the transfer fluid until the PCM completely solidifies.

![Fig. 1 Schematic of an integrated PCM collector storage solar heater](image)

III. THEORETICAL MODEL

This section presents equations for mathematical modeling of the heat transfer processes.

A. Solar Radiation

Global solar radiation absorbed by a tilted collector surface depends upon several variables. Perrin Brinchambaut [11] proposed a theoretical model based on the following correlations. The direct irradiance is calculated as:

\[ I = A \cos(\theta) \exp \left( - \frac{1}{B \sin(h + 2)} \right) \]

where \( \theta \) is the incident angle and \( h \) is the sun height.

The diffuse irradiance is given by:

\[ D = \left( 1 + \cos(\beta) \right) D_h + \left( 1 - \cos(\beta) \right) \rho G_s \]

where \( \beta \) is the tilt angle and \( \rho \) is the albedo.

\( D_h \) and \( G_s \) are expressed as:

\[ D_h = A' \left( \sin(h) \right)^{0.4} \]

\[ G_s = A'' \left( \sin(h) \right)^{\beta''} \]

B, A', A'' and B'' are constants which depend on the atmospheric state.

B. Solar Collector

The mathematical model describing the behavior of the integrated collector storage solar water heater is based on the following assumptions:

- The PCM is in perfect contact with the absorber;
- The PCM is homogenous and isotropic;
- The thermo physical properties of the PCM are independent of temperature, but different for solid and liquid phases;
- The PCM behaves ideally, i.e. such phenomena as property degradation and supercooling are not accounted for;
- The PCM is assumed to have a definite melting point.
- No thermal expansion during melting process.
- The heat transfer in the direction of the transfer fluid is ignored.

By writing energy balance equations for all components of the solar collectors: transparent cover, air gap, absorber plate, working fluid and PCM a mathematical model, comprises a set of temperature dependent equations, is obtained.

1. Transparent Cover

\[ M_c C_v \frac{dT}{dt} = \left( h_{c\rightarrow a} + h_{c\rightarrow e} \right) S_c (T_{sh} - T_{c}) + \left( h_{c\rightarrow abs} + h_{c\rightarrow amb} \right) S_c (T_{abs} - T_{c}) \]

\[ + h_{c\rightarrow a} S_c (T_{sh} - T_{abs}) + h_{c\rightarrow a} S_c (T_{abs} - T_{c}) U_{h\rightarrow a} \Delta S_c + \alpha_c \Delta S_c \left( T_{abs} - T_{c} \right) \]

\[ + \alpha \Delta S_C G + \alpha \Delta S_C \left( 1 - \alpha \right) \frac{1}{1 - \phi} \left( 1 - \alpha \right) \]

where \( M, C, T, S, G \) and \( \alpha \) are the mass, the heat capacity, the temperature, the surface, the global solar radiation and the absorption coefficient, respectively. \( h_{c\rightarrow amb} \) and \( h_{c\rightarrow a} \) are the heat transfer coefficients by convection, calculated respectively as [12]:

\[ h_{c\rightarrow amb} = 3.9 \nu_{\text{wind}} + 5.62 \]

\[ h_{c\rightarrow a} = \frac{Nu \lambda_a}{\epsilon_a} \]

\( \nu_{\text{wind}} \) is the wind velocity, \( \lambda_a \) is the air thermal conductivity and \( \epsilon_a \) is the air gap thickness.

The Nusselt number (Nu) is calculated as:

\[ Nu = \left( 0.06 - 0.017 \left( \beta / 90 \right) \right) Gr^{1/3} \]

where \( Gr \) is the Grashoff number expressed as:

\[ Gr = \frac{g (T_{sh} - T_{c}) \epsilon_s}{v^3 T_c} \]
\[ h_{r,C-sky} = e_C \sigma \left( T_{sky}^4 + T_{amb}^4 \right) \left( T_C + T_{sky} \right) \]  
\[ h_{r,C-abs} = \sigma \left( T_{sky}^4 + T_{abs}^4 \right) \left( T_C + T_{abs} \right) \]  
where \( T_{sky} \) is the sky temperature, calculated as [13]:
\[ T_{sky} = 0.0552 \ T_{amb}^{1.5} \] (12)

2. Air Gap
\[ \rho_f C_V \frac{dT_{air}}{dt} = h_{r,C-air} S_{i} (T_i - T_{air}) + h_{r,abs-air} S_{abs} (T_{abs} - T_{air}) + U_{loss} S_{i} (T_{amb} - T_i) \] (13)

3. Absorber Plate
\[ M_{abs} \frac{dT_{abs}}{dt} = h_{r,C-abs} S_{abs} (T_C - T_{abs}) + h_{r,abs-air} S_{abs} (T_{abs} - T_{air}) + \left( T_{amb} - T_{abs} \right) + S_{abs} G \frac{\tau_{air} \sigma_{abs}}{1 - \phi_k (1 - \sigma_{abs})} Q_{abs-MCP} \] (14)

\[ h_{r,abs-air} \] is the heat transfer coefficient by convection between the absorber plate and the air gap, calculated using (7).

3.1. Charging Process
During the charging mode there are three steps: solid phase, solid-liquid phase and liquid phase.

- **Solid Phase \((T_{PCM} < T_{mel})\)**

The temperature distribution in the solid phase is governed by the conduction equation as:
\[ \rho_{PCM} C_{PCM} \frac{\partial T_{PCM,sol}}{\partial t} = \lambda_{PCM} \frac{\partial^2 T_{PCM,sol}}{\partial y^2} \] (20)

The initial and boundary conditions are:
\[ T_{PCMs, sol} \left( y, t = 0 \right) = T_{amb} \] (21)
\[ T_{PCMs, sol} \left( y = 0, t \right) = T_{abs} \] (22)

\[ \lambda_{PCM} \frac{\partial T_{PCM,sol}}{\partial y} \bigg|_{y = \epsilon_{PCM}} = U_{loss,PCM} \left(T_{PCMs, sol} \left( y = \epsilon_{PCM}, t \right) - T_{amb} \right) \] (23)

\( \epsilon_{PCM} \) is the PCM thickness.

- **Solid-Liquid Phase \((T_{PCM} = T_{mel})\)**

Fig. 2 shows the description of the heat transfer process in melting phase.

4. Transfer Fluid
\[ \rho_f C_f \frac{D_{int,Tube}}{4} \left( \frac{\partial T_f}{\partial t} + v_f \frac{\partial T_f}{\partial x} \right) = h_{r,abs-f} \left( T_{abs} - T_f \right) \] (19)

\( v_f \) is the transfer fluid velocity and \( D_{int,Tube} \) is the tube diameter.

5. Phase Change Material

a. Charging Process

Natural convection plays an important role in PCM melting. The importance of convection gradually increases as the molten region grows. Direct simulation of convection in PCM is complicated and needs too long computation time. In order to simplify the simulation, the convection effects are taken into account via the use of an effective conduction coefficient \( \lambda_{eff} \) expressed as [15], [16]:
\[ \lambda_{PCM,eff} = a Re \left( \frac{F_f}{L_{abs}} \right)^{\gamma} \] (24)
modes are analysed under different operating conditions. The parameters that maintained initially fixed are: the ambient temperature $T_{amb}=31^\circ$C, the inlet water temperature $T_{in}=31^\circ$C, the wind speed $v_{wind}=1.5$ m/s and the mass flow rate $mf=0.01$ kg/s. The values adopted for the constants $a$, $b$, $n$ in correlation (24) are 0.1, 0.25 and 0, respectively. The thermo physical properties of PCM used in this investigation are summarized in Table II. The results are presented only for the summer solstice (21st June).

### RESULTS AND DISCUSSION

Climatic data using in this current investigation measured in Gabes ($33^\circ51N$, $10^\circ03E$), Tunisia. Global solar radiation evolution in 21st June is shown in Fig. 3.

#### A. Distribution of PCM and Water Temperatures

Fig. 4 shows the temperature distribution for a 0.018 m of PCM versus time at different axial positions $y=0$ m, $y=0.0053$ m, $y=0.0107$ m, $y=0.0162$ m and $y=0.018$ m. The results present six steps, to complete the charging and discharging processes, namely: solid heating, melting, liquid heating, liquid cooling, solidification, and solid cooling. Firstly, the PCM temperature at each height is increased from the initial value (TPCM=31°C) to the PCM melting point (T mel=44°C) at almost the same speed and the thermal energy is stored as a sensible heat. After that, the PCM starts to melt and energy is stored as a latent heat. During the solid heating step, heat transfer is dominated by conduction while through the phase transition both conduction and natural convection are occurred. The importance of natural convection gradually increases as the melted PCM zone grows. When the PCM is fully melted, free convection within the PCM liquid and the

\[
Ra = \frac{g(T_{amb} - T_{in}) F_{PCM}^2 C_{PCM,liq} F_{PCM}^2 \theta_{PCM}}{\mu_{PCM,liq} \lambda_{PCM,liq}}
\]  

(25)

where $\theta$ is the expansion coefficient, $\mu$ is the dynamic viscosity, $g$ is the gravity acceleration and $T_{mel}$ is the melting temperature.

The analysis of heat transfer problems in melting process, called moving boundary problems in scientific literature, is especially complicated due to the fact that the solid-liquid boundary moves depending on the speed at which the latent heat is absorbed at the boundary, so that the position of the boundary is not known a priori and forms part of the solution. The Stephan condition, describes this process [17]:

\[
\lambda_{PCM,so} \frac{\partial T_{PCM,so}}{\partial y} \bigg|_{y=r} - \lambda_{PCM,liq} \frac{\partial T_{PCM,liq}}{\partial y} \bigg|_{y=r} = \rho_{PCM,so} L_v \frac{\partial F}{\partial t}
\]  

(27)

where $L_v$ is the latent heat absorbed.

The boundary conditions for (26) are:

\[
T_{PCM,so}(y=F, t) = T_{PCM,so}(y=F, t) = T_{amb}
\]  

(28)

\[
T_{PCM,so}(y=0, t) = T_{in}
\]  

(29)

- **Liquid Phase (TPCM>T mel)**

In this stage, the PCM is completely in liquid phase. The temperature distribution and the boundary conditions are expressed respectively by (26), (29) and (30).

\[
\lambda_{PCM,liq} \frac{\partial T_{PCM,liq}}{\partial y} \bigg|_{y=r} = U_{in,so} \left(T_{PCM,so}(y=e_{PCM}, t) - T_{amb}\right)
\]  

(30)

b. Discharging Process

When the absorber attains the melt PCM temperature or when the rate of useful energy gain becomes less than the rate of the thermal losses, the PCM starts to solidify. For this operating mode, the thermal behavior and the boundary conditions of the PCM are governed by (5)-(16) and (18)-(30)

IV. RESULTS AND DISCUSSION

The mathematical model obtained has been transposed into a Matlab computational program. The selected specification and design conditions of the integrated solar collector water heater are shown in Table I. The charging and discharging
absorber plate becomes predominant, the PCM temperature at each height is increased again. The difference in temperature between the top and the bottom of PCM decreases, which is attributed to the enhancement of natural convection in liquid phase.

During the cooling liquid stage, the PCM temperature decreases until its solidification point ($T=44^\circ C$) due to the release of sensible heat stored in the PCM liquid. Fig. 4 also indicates a slight change in temperature curve order. Finally, another temperature drop is observed at the end of the discharging process. Natural convection is insignificant in the solidification phase and the heat transfer is governed only by conduction.

The melt fraction for 0.018 m of PCM during the charge and the discharge modes as a function of time is presented in Fig. 5. The PCM starts to melt at 8:06 am and the time to complete melting is 11:44 am. It is also seen that the melting process of all PCM has ended within 3 h and 38 min. The PCM solidification process starts at 16:39 pm to finish at 21:00 pm. Therefore, the time required to release the latent heat is approximately 4 h and 21 min. It is observed that the complete solidification time is longer than the melting time. This is because of the overall heat transfer coefficient which is low during solidification than the melting mode due to natural convection.

Fig. 6 shows the temperature variations of outlet water temperature without and with PCM as a function of time. For both cases, the temperatures increase with time until a maximum values and began to decrease after that. The highest water temperatures reached for the collector without and with a phase change material are approximately 64.08 and 62.48°C, respectively. This is can be explained by the rise of solar intensity in the morning and its decreasing in the afternoon. It is noticed also that the storage system slows down the increase and the decrease. This can be justified by the fact that for the case without PCM all absorbed solar radiations are used to heat the transfer fluid, contrary to the case where the PCM is used. In this last case, a portion of thermal energy is absorbed by the material in the charging process and this energy is released in the discharging process. As expected, the addition of PCM in solar systems causes a decrease in fluid temperature during charge and a rise during discharge.

**B. Effect of Inlet Water Temperature**

The effect of inlet water temperature on its outlet temperature is presented in Fig. 7. The curves are displayed for inlet temperature of 25, 30, 35, and 40°C. An increase in inlet temperature leads to a rise in outlet water temperature.
The highest temperatures reached are 54, 61.31, 66.76, and 71.50°C, respectively. It is observed that the addition of PCM causes a decrease in temperature during charge and a rise during discharge. The rise is most enhanced for higher inlet water temperature.

The variation of the melt fraction for 0.018 m of PCM during charging and discharging modes versus time and for various inlet water temperatures is shown in Fig. 8. As expected, the higher the inlet water temperature is, the higher the melting speed is and the shorter the melting time is. In fact, for mass flow rates of 0.005, 0.008, 0.01 and 0.015 kg/s respectively whereas the solidification times are 6.06 h, 4.78 h, 4.38 h and 4.16 h. This may be due to the fact that when the mass flow rate is, the higher the water temperature during charging and discharging processes is.

The heat storage system requires a shorter time for the melting process; it spends a longer time to release the latent heat and has a higher fluid temperature at lower mass flow rate. Therefore, the smallest water mass flow rates are suitable for this solar system.

The curves rise to complete solidification position early with the increase in the mass flow rate. Contrary to the case of melting process, the higher the mass flow rate is, the lower the solidification time is. The times required to complete the melting process are 4.32 h, 3.68 h, 3.32 h and 3.07 h whereas the times to release the latent heat are approximately 3.78 h, 4.25 h, 5.64 h and 10.91 h when the inlet temperatures are 25, 30, 35 and 40°C, respectively.

C. Effect of Mass Flow Rate

The effect of the mass flow rate on the storage system performance is investigated in this section. The inlet water temperature and the PCM thickness are assumed to be 31°C and 0.018 m, respectively. Fig. 9 presents the outlet water temperature variation as a function of time. The results are displayed for water mass flow rates of 0.005, 0.008, 0.01 and 0.015 kg/s. The highest outlet fluid temperatures reached are 87.63, 69.70, 62.47 and 49.77°C, respectively. The results show that the peak of outlet temperature increases with the decrease of flow rate. This is simply due to the fact that when the water flow rate rises, the amount of water to be heated with the same quantity of solar energy increases. The lower the mass flow rate is, the higher the water temperature during charging and discharging processes is.

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Fig. 7 Effect of inlet temperature on outlet water temperature

Fig. 8 Effect of inlet temperature on melt fraction during charging and discharging

Fig. 9 Effect of mass flow rate on outlet water temperature

Fig. 10 Effect of mass flow rate on melt fraction during charging and discharging

Fig. 10 predicts the effect of the mass flow rate on the melt fraction. It can be observed that for all mass flow rates the total melt fraction for the initial solid PCM height is 100%. The curves rise to complete solidification position early with the increase in the mass flow rate. Contrary to the case of melting process, the higher the mass flow rate is, the lower the melting speed is and the shorter the solidification time is. In fact, for mass flow rates of 0.005, 0.008, 0.01 and 0.015 kg/s the required melting times are 2.72 h, 3.25 h, 3.63 and 5.07 h, respectively whereas the solidification times are 6.06 h, 4.78 h, 4.38 h and 4.16 h. This may be due to the fact that when the mass flow rate increases the average absorber temperature drops resulting in a rise in the heat transfer between the absorber and the PCM during charging and discharging processes and therefore a rise and a decrease in the melting
and the solidification times, respectively. Numerical results show also that the rise in mass flow rate is not linear with the melting and solidification times. In other words, when the mass flow rate is doubled or tripled the melting and solidification times are smaller than twofold or threefold.

D. Effect of PCM Thickness

Fig. 11 presents the effect of PCM thicknesses on the outlet water temperature. The results are displayed for PCM height from 0 to 0.025 m. The higher temperatures reached are 64.08, 63.97, 63.90, 61.70 and 58.90 °C when the PCM thicknesses are 0, 0.005, 0.01, 0.02, and 0.025 m, respectively. As expected, the smaller the thickness is, the higher the maximum temperature reached is. Furthermore, a rise in PCM thickness leads to a decrease in temperature during charging and an increase during discharging. This is due that an increase in PCM thickness results in a rise in heat absorbed and released by PCM. It is observed also that the PCM thickness slows down the increase and the decrease.

### Fig. 11 Effect of PCM thicknesses on outlet water temperature

![Effect of PCM thicknesses on outlet water temperature](image)

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V. CONCLUSION

A transient one dimensional theoretical model for thermal energy storage solar collector integrated with phase change material is developed to investigate the thermal performance of the system. The model is used to analyze the effect of inlet water temperature, mass flow rate and PCM thickness on the outlet water temperature and the melt fraction during charging and discharging processes. The important results obtained from this study are summarized as:

- **The addition of PCM causes a decrease in temperature during charge and a rise during discharge. The rise is most enhanced for higher inlet water temperature, PCM thickness and for lower mass flow rate.**
- **The complete melting time is shorter than the solidification time due to the high heat transfer coefficient during melting. The natural convection effect in the molten PCM layer is increased with the increase of PCM thickness, thus the difference between the complete melting and solidification times is increased with the PCM height.**
- **The peak of outlet water temperature increases with the decrease of mass flow rate and PCM thickness and the rise of inlet water temperature.**
- **The higher the inlet water temperature is, the higher the melting speed is, the shorter the melting time is and the longer the solidification time is.**
- **The times needed to complete melting and solidification modes are not linear with the mass flow rate and the PCM thickness.**

**REFERENCES**


