Assessment of Multiscale Information for Short Physiological Time Series

Young-Seok Choi

Abstract—This paper presents a multiscale information measure of Electroencephalogram (EEG) for analysis with a short data length. A multiscale extension of permutation entropy (MPE) is capable of fully reflecting the dynamical characteristics of EEG across different temporal scales. However, MPE yields an imprecise estimation due to coarse-grained procedure at large scales. We present an improved MPE measure to estimate entropy more accurately with a short time series. By computing entropies of all coarse-grained time series and averaging those at each scale, it leads to the modified MPE (MMPE) which provides an enhanced accuracy as compared to MPE. Simulation and experimental studies confirmed that MMPE has proved its capability over MPE in terms of accuracy.

Keywords—Multiscale entropy, permutation entropy, EEG, seizure.

I. INTRODUCTION

OVER a few decades, the most common quantitative Electroencephalogram (EEG) analysis was frequency analysis, which is based on an assumption of stationarity of a signal. To address the nonlinear and nonstationary nature of EEG signals, a number of time-frequency analyses have been recently applied to analysis of EEG [1], [2], [3]. However, the time-frequency analysis methods, which decompose the signal into several stationary monocomponent signals, fail to reflect the dynamic changes of EEG signals effectively. Alternatively, a number of entropy measures have been utilized for dealing with nonlinearity and nonstationarity in EEG signals. Entropy is a measure of the complexity or the regularity of a time series, thus being able to describe nonlinear dynamics [4]. Various studies have found that a reduction in entropy has been generally observed in case when the brain is under an abnormal state [5], [6].

One of the widely used entropy measures for analysis of EEG is permutation entropy (PE). PE has been developed for quantifying the regularity for nonstationary and noisy time-series [7]. Since PE evaluates the probability distribution based on the temporal structure of a time series, the advantage of PE resides in its simplicity and robustness, making it suitable for real-time monitoring. Also, PE is not restricted on the type of a time series, thus implying its appropriateness for analyzing EEG signals [8], [9]. However, since PE is based on single scale computation, it fails to reflect the dynamical characteristics across multiple temporal scales. Motivated by a multiscale entropy approach [10], multiscale based PE has been developed, which consists of a coarse-graining procedure

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and the following estimation of PE, referred to as multiscale PE (MPE) [11]. It is found that MPE is able to measure the values of PE in different scales and is more robust than other multiscale based entropy measures in the presence of artifacts and observational noise [11]. Unfortunately, due to the use of a coarse-grained procedure, MPE suffers from an inaccurate estimation of entropy at large scales, which often occurs in dealing with EEG. Subsequently, MPE results in an imprecise estimation of PE in the case of a short time series. In this paper, to address this unreliable estimation of MPE with a short time series, an improved MPE in terms of accuracy is presented. The proposed MPE takes into account all available coarse-grained time series at each scale, followed by averaging the values of PE of all coarse-grained time series, which is referred to as modified MPE (MMPE). Through the simulations using two synthetic noise signals, i.e., white and $1/f$ noises, the MMPE measure possesses an improved accuracy over the conventional MPE even at large scales. Next, MMPE was applied to the real normal and epileptic EEG recordings to validate its effectiveness in detecting epileptic seizue as compared to MPE.

II. METHODS

A. Permutation Entropy (PE)

PE is a parameter that can quantify the organization degree of a given time series. The key idea of PE proposed in [7] is to associate a symbolic sequence to the time series. The time series is transformed into a series of ordinal patterns which describes the order relations between the present values and a fixed number of equidistant values at a given past times. Based on the counting of ordinal patterns, PE quantifies the relative frequencies of occurrence of the distinct ordinal patterns.

For a given time series $\{x(i), i = 1, \ldots, N\}$, we can embed the time series in a m-dimensional space to obtain a reconstruction vector $\mathbf{X}(i)$,

$$\mathbf{X}(i) = \{x(i), x(i+\tau), \ldots, x(i+(m-1)\tau)\}, \quad (1)$$

where m and τ denote the embedding dimension and the delay time, respectively. Then, reconstruction components of each vector $\mathbf{X}(i)$ can be rearranged in an increasing order. In case when two components in $\mathbf{X}(i)$ are equal, i.e., $x(i + (j_1 - 1)\tau) = x(i + (j_2 - 1)\tau)$, those are sorted according to the values of their corresponding j's, namely, if $j_1 < j_2$, then, $x(i+(j_1-1)\tau) \leq x(i+(j_2-1)\tau)$ is taken. Hence, any vector $\mathbf{X}(i)$ can be uniquely transformed onto a symbol sequence,

$$A(g) = [j_1, j_2, \ldots, j_m], \quad (2)$$

World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:10, No:1, 2016

where $g = 1, 2, \ldots, k$, and $k \leq m!$ and $A(g)$ is one of the $m!$ permutation of m distinct symbol sequences $[j_1, j_2, \ldots, j_m]$. The relative frequency of occurrence of permutation pattern π is denoted as $p(\pi)$ and given by

$$p(\pi) = \frac{\#\{\mathbf{X}(i)|\mathbf{X}(i) \text{ has ordinal pattern } \pi\}}{N - (m-1)\tau}, \quad (3)$$

where $\#$ denotes the number of elements in the set. Thus, a probability distribution $P = \{p(\pi_i), i = 1, \ldots, m!\}$ is defined. Subsequently, for the embedding dimension m, and delay time τ, PE is computed based on the framework of Shannon entropy [4] as:

$$\mathrm{S}_{\mathrm{pe}}(m) = -\sum_{j=1}^{m!} p(\pi_j) \log(p(\pi_j)), \quad (4)$$

where $S_{pe}(m, \tau)$ is denoted as PE. The maximum value of $\mathrm{S}_{\mathrm{pe}}(m)$ is $\log(m!)$ when all permutations of the time series occur with equal probability. The normalized version of $\mathrm{S}_{\mathrm{pe}}(m)$ is obtained as

$$\hat{\mathrm{S}}_{\mathrm{pe}}(m) = \frac{\mathrm{S}_{\mathrm{pe}}(m)}{\log(m!)}. \quad (5)$$

Hereafter, PE refers to the normalized $\mathrm{S}_{\mathrm{pe}}(m)$, namely, $\hat{\mathrm{S}}_{\mathrm{pe}}(m)$. It has been known that the length of time series, N, is constrained as $m! \leq N - (m-1)\tau$ [12].

B. Multiscale PE (MPE)

Based on the framework of multiscale entropy approach [10], MPE has been developed to estimate entropy of a time series over multiple time scales and is conducted as:

1) For a given time series $\{x(i), i = 1, \ldots, N\}$, construct the consecutive coarse-grained time series, $\{y^s(j), j = 1, 2, \ldots, N/s\}$, as

$$y^s(j) = \frac{1}{s} \sum_{i=(j-1)s+1}^{js} x(i), \quad 1 \leq j \leq N/s, \quad (6)$$

where s is the scale factor and $y^s(j)$ denotes a coarse-grained time series at a scale factor of s.

2) Calculate PE of coarse-grained time series $y^s(j)$ on different scales. The resultant entropy measure is referred to as MPE.

C. Modified MPE (MMPE)

Although MPE is capable of measuring the regularity of a time series over multiple scales, the length of the coarse-grained time series is substantially reduced especially for large scales. To address this inaccuracy issue, a modified MPE is presented which consists of two steps: First, all possible coarse-grained time series at each scale are obtained. Next, for each scale, by averaging the values of PE of all coarse-grained time series, it leads to the modified MPE (MMPE). The detailed procedures are as:

1) For a scale factor of s, a lth coarse-grained time series is obtained as

$$y_l^s(j) = \frac{1}{s} \sum_{i=(j-1)s+l}^{js+l-1} x(i), \quad 1 \leq j \leq N/s, \ 1 \leq l \leq s. \quad (7)$$
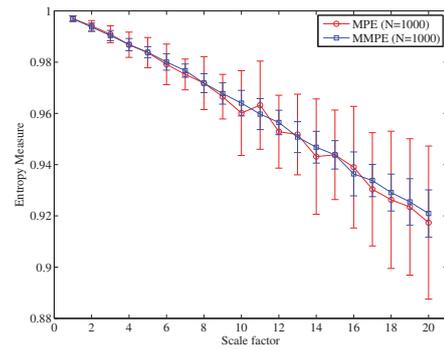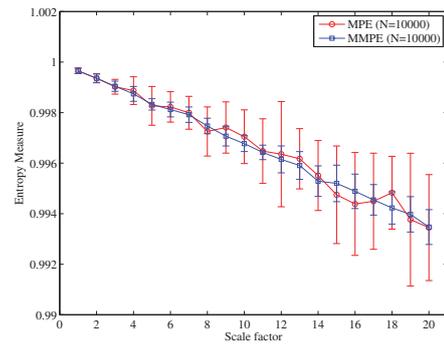
Fig. 1 Results of MPE and MMPE estimation of white noise of different data lengths ($m = 4$): (a) $N = 1000$, (b) $N = 10000$

Note that the all coarse-grained time series correspond to the coarse-grained procedures different starting points. Thus, there exist all s coarse-grained time series at a scale factor of s.

2) Calculate the PEs of s coarse-grained time series at a scale factor s and average all s values of PE. Then, the resultant MMPE at a scale factor of s is given by

$$\mathrm{MMPE}(m, s) = \frac{1}{s} \sum_{l=1}^{s} \hat{\mathrm{S}}_{\mathrm{pe}}(\mathbf{y}_l^s, m), \quad 1 \leq j \leq N/s, \quad (8)$$

where $\mathbf{y}_l^s = \{y_l^s(1), y_l^s(2), \ldots, y_l^s(\lfloor N/\tau \rfloor)\}$ and $\lfloor p \rfloor$ is the largest integer less than p.

It is noted when only the first coarse-grained time series, $\mathbf{y}_1^s$, is used to compute PE, MMPE is equal to MPE.

III. RESULTS

To test the capability of the proposed MMPE, simulation studies using two synthetic noise signals and experimental studies using real EEG signals recorded from healthy and injured subjects were carried out.

A. Simulations

In order to validate the estimation performance of MMPE depending on a data length as compared with the conventional MPE, the white and $1/f$ noise signals with different data lengths were used. To assess the statistical characteristics, the mean and the standard deviation (SD) of the entropy values of
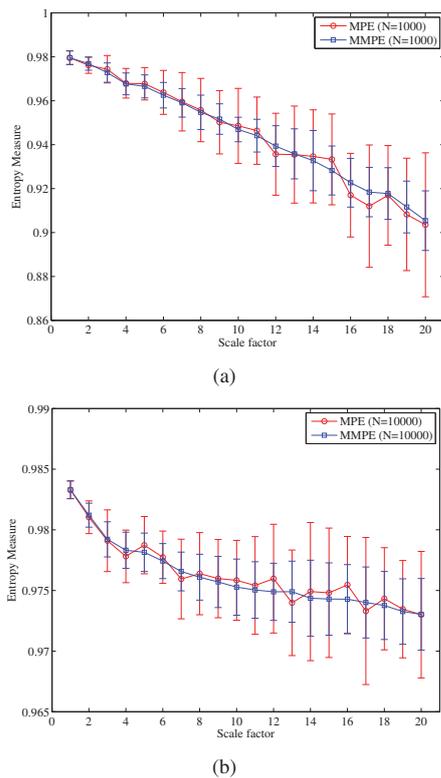
World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:10, No:1, 2016

Fig. 2 Results of MPE and MMPE estimation of $1/f$ noise of different data lengths ($m = 4$): (a) $N = 1000$, (b) $N = 10000$
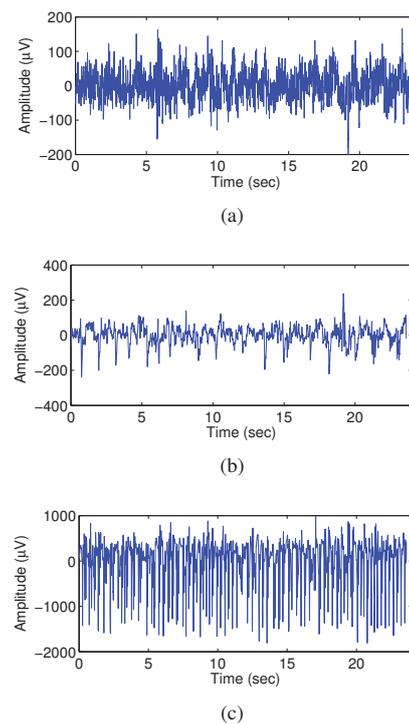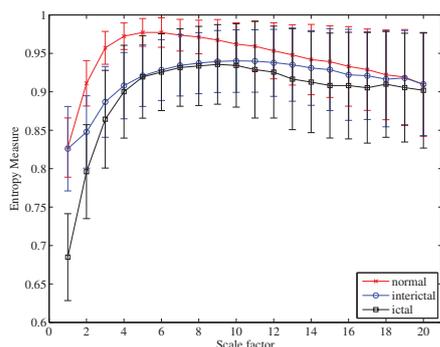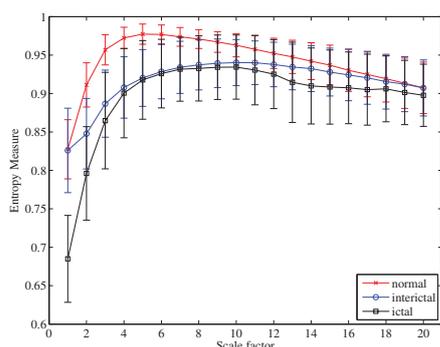

Fig. 3 Representative EEG recordings of three classes: (a) normal EEG recording, (b) interictal (seizure-free) EEG recording, (c) ictal (seizure) EEG recording

MPE and MMPE were taken over 100 independent simulated signals. Here, the embedding dimension $m = 4$ and the delay time $\tau = 1$ were used. Figs. 1(a) and 1(b) show the entropy estimation of white noise with $1/f$ noise with $N = 1000$, and 10000, respectively. The error bar exhibits the SD value of an entropy value at each scale. In the figures, the mean values of MPE and MMPE are not highly discriminative regardless of a data length, while MPE more fluctuates over different scales. On the other hand, it is apparent that the SD values of MMPE are much less than those of MPE for all scales except a scale factor of 1. For MPE, the higher a scale factor is, the larger the SD value of entropy is. Comparing to MPE, the SD values of MMPE are kept smaller even at large scale factors. Figs. 2(a)-2(b) exhibit the entropy estimations of $1/f$ noise with different data lengths. Similar results of Fig. 1 were observed in Fig. 2 in that we observed a noticeable difference of the SD values, especially at large scales. In addition, for both white and $1/f$ noise, it is evident that the larger the scale factor is, the less the variance of MMPE is.

B. Application to Normal and Epileptic EEG Signals

This study used an EEG dataset which is publicly available online for epilepsy research literature [13]. The dataset consists of five subsets (denoted as Z, O, N, F, and S), which each set contains 100 single channel EEG segments of 23.6 s duration. These EEG signals have been selected from continuous multichannel EEG recordings after visual inspection for artifact rejection. The sets Z and O have been obtained from five healthy volunteers with eye open and closed, respectively. The sets N and F have been recorded in seizure-free intervals from five patients in the epileptic zone (set F) and from the hippocampal formation of the opposite hemisphere of the brain (set N), which are referred to as interictal EEG recordings. Finally, the set S have been taken in ictal periods from patients, containing seizure activity. The set Z and O have been measured extracranially with standard electrode locations by international 10-20 system, while the sets N, F and S have been recorded from intracranially. All EEG signals were sampled with 173.61 Hz using 12-bit A/D resolution and ranges in the spectral bandwidth from 0.5 to 85 Hz. In addition, the EEG recordings have been recorded 128 channel amplifier setup with an average common reference. In this study, the sets N (normal), Z (interictal), and S (ictal) are chosen to estimate the multiscale entropies. Figs. 3(a)-3(c) show the representative EEG recordings for normal, interictal and ictal periods, respectively. In this study, the parameters of $m = 3$ and $\tau = 1$ were chosen.

Fig. 4 shows the comparison results of MPE and MMPE for three different groups, i.e., normal, interictal, and ictal EEG recordings. As can be seen, for both MPE and MPE, the mean values of entropies of normal EEG recording are higher than those of other two groups across all scales. Between interictal and ictal groups, the entropy values of ictal EEG recordings indicate lower level than those of interictal ones. In both figures, it is apparent that the variances of MMPE are smaller than those of MPE at most scale factors, implying its improved accuracy for estimating entropy.

World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
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Fig. 4 Results of MPE and MMPE of normal, interictal, and ictal EEG recordings with a window length of 2 sec ($m = 3$): (a) MPE, (b) MMPE
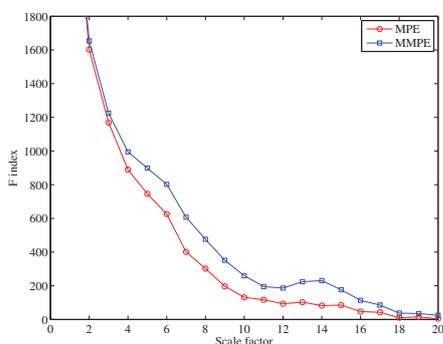

Fig. 5 Results of Fisher's discrimination indexes of MPE and MMPE using ANOVA test

To validate the distinguishability of MMPE with three different EEG recordings, the one-way analysis of variance (ANOVA) test was carried out. The Fishers discrimination index, called F index, which is the ratio of between-group scatter and within-group scatter, was used as a discrimination criterion. The higher the F index is, the better the entropy measure discriminates the different groups,. i.e., normal, interictal, and ictal EEG recordings. Fig. 5 depicts the results of the F indexes of MPE and MMPE with the window lengths of 2 and 3 sec, respectively. In the figures, MMPE exhibits higher values of F index than MPE at all scale factors, implying the improved capability of discrimination for short EEG recordings. This results suggest that MMPE is a potential tool for online and real-time seizure detection with EEG

signal.

IV. CONCLUSION

This paper has presented a multiscale PE which significantly enhances accuracy of estimation of entropy in the case of a short time series. By incorporating PEs of all coarse-grained time series and averaging those, MMPE yields a reliable multiscale based entropy measure for a short time series. Through the simulation and experimental studies, MMPE has shown its effectiveness to estimate entropy more accurately, especially at large scales.

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