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The Conceptual and Procedural Knowledge of Rational Numbers in Primary School Teachers

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Abstract**—**The study investigates the conceptual and procedural knowledge of rational number in primary school teachers, specifically, the primary school teachers level of conceptual knowledge about rational number and the primary school teachers level of procedural knowledge about rational numbers. The study was carried out in Bauchi metropolis in Bauchi state of Nigeria. A Conceptual and Procedural Knowledge Test was used as the instrument for data collection, 54 mathematics teachers in Bauchi primary schools were involved in the study. The collections were analyzed using mean and standard deviation. The findings revealed that the primary school mathematics teachers in Bauchi metropolis posses a low level of conceptual knowledge of rational number and also posses a high level of Procedural knowledge of rational number. It is therefore recommended that to be effective, teachers teaching mathematics most posses a deep understanding of both conceptual and procedural knowledge. That way the most knowledgeable teachers in mathematics deliver highly effective rational number instructions. Teachers should not ignore the mathematical concept aspect of rational number teaching. This is because only the procedural aspect of Rational number is highlighted during instructions; this often leads to rote - learning of procedures without understanding the meanings. It is necessary for teachers to learn rational numbers teaching method that focus on both conceptual knowledge and procedural knowledge teaching.

Keywords**—**Conceptual knowledge, primary school teachers, procedural knowledge, rational numbers.

I. Introduction

MATHEMATICS is the backbone of all scientific and technological activities of human development. It is the science of numbers and space [1]. No nation can develop scientifically and technologically without proper foundation in mathematics, it is due to its indispensability in the nation building that Nigerian made it one of the core and compulsory subjects in primary education and is given the highest number of lesson in primary school time table [2].

Teaching and learning of Rational numbers a topic in primary schools has been problematic [3] and also a major concept which pupils are introduced to primary two. This concept is continuous throughout the mathematical learning in the Primary and secondary schools Rational number is any number that can be expressed as quotient a/b of 2 integers with denominator not equal to zero since it may be equal to one, every integer is a Rational number. A rational number idea plays a major role in the development of proportional reasoning abilities. The pupils' abilities will in turn become the intellectual and mathematical cornerstones of much of what to come in the secondary school years [4].

The effective learning and teaching processes in mathematics comprises not only in the conveying facts from the teachers to pupils, but also provides activities for them so that both their conceptual and procedural knowledge will be enhancing each lesson, they teach and study [5]. Teachers need to encourage pupils to develop their mathematical knowledge and understanding through investigation exploration, problem solving and sharing ideas. They should instill strong Conceptual and Procedural knowledge so that the pupils will be competent and skilled enough to solve all types of problems and tasks.

References [6], [7] identified conceptual and procedural knowledge as the two kinds of knowledge that build up our understanding of a topic especially in mathematic [8] suggests that Procedural knowledge is the mastery of computational skills and familiarity with procedures for identifying mathematical components algorithms and definition, while conceptual knowledge is rich in relationships and refers to the underlying structure of mathematics, the interconnections between ideas and give meaning to mathematical procedures. Teachers' conceptual and procedural knowledge of rational numbers and views influence pupils learning.

In order for teachers to present a conceptually and procedurally based rational number curriculum to their pupils, they themselves must have a conceptually and procedurally based understanding of the topic. Research has shown that the knowledge that elementary school teachers bring to the classroom is procedurally based and largely misunderstood [9], [10]. This is particularly true with teachers' understanding of fraction operations. The report [11] found a compelling relationship between what teachers could do with their students and their own level of competence. The obstacle is that "either teachers do not have enough content knowledge or what they do know is not the "right" content knowledge" [12].

Evidence has shown that teachers have difficulties with the concept of fraction and the meaning of division of fractions [9], cannot understand the operator construct of rational number [13], have difficulty in explaining fractions to children and why algorithms work [14], [15] and could not carry out fractional computation procedures correctly even when they have correct answers [16]. Many teachers think they understand rational numbers because they can perform algorithms for computations with numbers and can teach some procedures. Having rational number sense means more than just manipulating numbers. It involves attaching meaning to the symbols used for rational numbers, moving easily between

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representations, understanding and comparing relative sizes of rational numbers, making mental calculations, and recognizing reasonable solutions [17].

Many researchers indicated that procedural and conceptual knowledge are both important components of mathematical understanding [18], [19]. Therefore, both types of knowledge need to be balanced and emphasized when teachers teach mathematical understanding in rational numbers.

From the forgone submission suffice is to say teachers' possession of conceptual and procedural knowledge of rational number will invariably influence his/her class room instructions in a positive way. Pupils learn mathematics through classroom experience and problems given by their teachers, therefore teachers need to acknowledge and thoroughly understand the mathematical concepts that they teach. Both types of knowledge need to be balanced and emphasized when teachers teach mathematical understanding in rational numbers to the pupils.

II. Statement of the Problem

Mathematics teachers particularly at the primary education level need to posses on appreciable level of conceptual and procedural knowledge to be able to teach mathematics effectively. Teachers at the elementary level face difficulty in interpreting problems in such a way that pupils would be able to relate the mathematics they already know [20]. This makes it difficult for the learners to learn with understanding. Teachers do not know how to interpret the question into ways that enables learners to relate the mathematics to what they already know, they will not learn with understanding and the foundation of the children success lies in the teachers' knowledge of the topic can this problem be attributed to the teachers lack of possession of the required knowledge base on mathematics for effective teaching? The present study is an attempt to explore primary school teachers' conceptual and procedural knowledge of rational numbers

A. Purpose of Study

The main purpose of the study was to explore the primary school teachers conceptual and procedural knowledge of rational numbers, specifically the study intends to

- To determine the primary school teachers level of conceptual knowledge about rational number.
- To determine the primary school teachers level of procedural knowledge about rational number.

B. Research Questions

The following research questions were formulated to guide the study

- What is the level of primary school teachers' conceptual knowledge of rational number?
- What is the level of primary school teachers' procedural knowledge of rational number?

III. Methodology

The design of the study was a descriptive design where the researcher identified teachers' possession of Nigerian Certificate in Education (NCE) which is the minimum teaching qualification at primary school level, and those teaching mathematics. A Conceptual and Procedural Knowledge Test (CPKT) on rational number adapted from [8] was administered to these teachers. The teachers' level of performance in the Conceptual and Procedural Knowledge Test on rational number was used to determine their level of conceptual knowledge in rational number and level of procedural knowledge in rational number.

The study was conducted in Bauchi metropolis, Bauchi state in Nigeria, where the primary school teachers from the primary school were selected.

The population of the study comprised of all the primary school teachers, teaching mathematics that have NCE qualification in Bauchi metropolis.

The sample of the study consists of 54 mathematics teachers in all primary schools in Bauchi metropolis. A purposeful sampling technique was use in selecting the mathematics teachers based on their teaching qualification (NCE) in primary school level.

The instrument used in collecting data for the study was the Conceptual and Procedural Knowledge Test (CPKT) on rational number which was an open – ended questionnaire that was adapted from [8]. The test comprised of 20 items questions which was used to measure the conceptual and procedural knowledge of rational number of the mathematics teachers in primary schools where 10 items questions measured conceptual knowledge in rational number among the sample and the other 10 where designed to measure the of procedural knowledge of rational number. The mean of the conceptual items in the CPKT was calculated and also the mean of the procedural items in the CPKT. Teachers that have scores below the mean is has a low conceptual knowledge or low procedural knowledge and a teacher that scores above the mean has high conceptual knowledge or high procedural knowledge. The test covers the five sub-constructs of rational numbers namely the part-whole, quotients, ratio, operator and measure. The initial draft of the Conceptual and Procedural Knowledge Test consisted of 30 items questions which was modified as a result of face and content validation by 3 experts from the Mathematics Education department in Abubakar Tafawa Balewa University, an expert in mathematics Education, measurement and evaluation from Abubakar Tatari Ali Polytechnic Bauchi, 25 items emerged from the pool after validations. These were used in pilot testing. 20 items emerged as good after the pilot testing and item analysis. The scores obtained in the pilot testing was used to estimate the reliability coefficient of the Conceptual and Procedural Knowledge Test on Rational Number, inter-scorer method of testing reliability was used to determine the reliability, which was found to be 0.76. A coefficient variable that is like this could be considered a good value for research study [21] therefore the test was found to be reliable.

IV. Procedure of Data Collection

The researcher with the help of research assistants administered the instrument to the respondents in the sampled

schools. The research assistants were given clear instructions on how to administer the instruments, the researcher closely supervised the administration of the instrument. The test was administered to only those teachers with NCE teaching mathematics in Primary schools. The teachers were requested by means of instructions, to solve the questions clearly without oversight of any of the diagrams or steps.

The result collected from the test was used to determine the level of performance of the teachers in conceptual knowledge in rational number and the level of performance in procedural knowledge in rational number. The Conceptual and Procedural Knowledge Test was a 20 items open – ended questions which the possible scores obtained in each are 0 – 4 marks accordingly. The maximum score of the conceptual knowledge part and the procedural knowledge part is 40 separately, while the minimum score is 0, the maximum score of the overall test is 80 while the minimum score is 0, the test was scored, recorded and analyzed.

V. Method of Data Analysis

Descriptive statistics of mean and standard deviation were used to describe the level of the primary school teachers' conceptual knowledge and their level of procedural knowledge on rational numbers. The Vassar statistics online software was used to calculate the mean score and standard deviation.

VI. Results

A. Research Question 1

What is the level of primary school teachers' conceptual knowledge of rational number?

From the test presented in Table I, mean and standard deviation where calculated. The mean scores of teachers in the conceptual items in the Conceptual and Procedural Knowledge Test on rational number is 16 and the standard deviation of the score was 10.05, total of 15 teachers (27.78%) scores above the mean and 39 teachers (72.22%) scores below the mean.

TABLE I
Teachers Performance on Conceptual Items in the Conceptual and Procedural Knowledge Test on Rational Number

Mean	Standard Deviation	Number of Teachers Scoring above the mean	Numbers of teachers scoring below the mean	Total
16	10.05	15(27.78%)	39(72.22%)	54(100%)

B. Research Question 2

What is the level of primary school teachers' procedural knowledge of rational numbers?

From the result present in Table II, mean and standard deviation, were calculated. The mean scores of the teachers in the procedural items in the CPKT on rational number is 29.42 and the standard deviation of the scores was 7.28, a total of 42 teachers (77.78%) scores above the mean 12 teachers (22.22%) scores below the mean.

TABLE II
Teachers Performance on Procedural Items in the Conceptual and Procedural Knowledge test on Rational Numbers

Mean	Standard deviation	Number of teachers scoring above the mean	Number of teachers scoring below the mean	Total
29.42	7.28	42(77.78%)	12(22.22%)	54(100%)

VII. Findings and Discussion

Result from Table I shows that a mean of 16 and a standard deviation of 10.05 were obtained, from scores of conceptual items in the Conceptual and Procedural Knowledge Test on rational numbers, it shows that the overall performance of the 15 teachers representing 27.78% the teachers got marks above the mean (16.0). Also 39 teachers representing 72.22% of the teachers got marks below the mean mark. This shows that fewer teachers got marks above the mean than below the mean in the scores of the conceptual items. This indicates that the conceptual knowledge of primary school teacher in rational number is found to be low. This remains a serious problem since all teachers of mathematics in primary school level are expected to have conceptual knowledge of all topics to be taught in their classes. This finding agrees with that of [9], who reported that teachers have difficulties with content of fraction and meaning of division of fraction. Also in another study by [22], it was found that teachers have mastered procedural knowledge more thoroughly than they have mastered conceptual knowledge, teachers perform well on familiar items that required applications of procedural knowledge, they had difficulty in relating different part of their schemata to solve problems that required application or analysis, lack of conceptual knowledge make learning and teaching more difficult and in-effective. This finding is consistent with those of [23] who state the problems in conceptual knowledge given to respondents consisted of mathematical knowledge that should be learned in primary and secondary schools. Teachers also did not utilize or exhibit expertise in conceptual knowledge related to these topics. The teachers were supposed to master conceptual knowledge in order to avoid teaching misconceptions. It was also found that answers given by teachers demonstrated insufficient conceptual knowledge, which was often illogical and even confusing. This result in terms of conceptual understanding of primary school teachers in rational numbers, support the findings that shows that most teachers do not pose a good understanding of content in the subject they teach. References [24], [25], also [26] stated that they are also unable to offer conceptual explanation for procedures and algorithms use to compute mathematics problems. One could assume based on the findings that weakness and incompetency in conceptual knowledge will cause the teacher to provide only logical explanations, which are not based on conceptual knowledge which is wrong and do not display true understanding of rational number. To be effective, teachers must posses' a deep understanding of the mathematics that they teach. Teachers' weakness in conceptual knowledge of rational number seems

to imply that overall conceptual knowledge of rational number might be lacking.

The analyzed result from Table II shows that the mean of 29.42 and a standard deviation of 7.28 were obtained from the score of the procedural items in the conceptual & Procedural knowledge test on rational number 42 teachers representing 77.77% of the teachers scored above the mean mark and 12 teachers representing (27.22% of the teachers) scored below the mean mark it was found out that the primary school mathematics teachers have high level of procedural knowledge of rational number because most of them scored above the mean. This result implies that teachers know how to compute with rational numbers but did not understand the procedural rational behind the procedure used in solving rational number problems. Also that teacher with high level performance can teach the concept of rational number effectively. This finding agrees with that of [27] which states that teachers with high procedural knowledge can solve problem quickly and effectively because it is some extent automated by automation it means execution and activated. This is in line with the findings of [8] which show that teachers exhibited a high level of procedural knowledge of operations using rational number and utilized common representation of rational numbers. Reference [24] found that teachers managed to solve operation involving rational number but failed to explain the procedure that they have and the reason behind "the invert and multiply" algorithm. The procedural knowledge of this teachers' dominated their conceptual knowledge. This implies that teachers believed that teaching an algorithm is sufficient for pupils to be able to understand a concept in mathematics. Additionally, teachers that have procedural knowledge in mathematics believe that a good teacher is one who can show the systematic steps of given computation. For teaching rational number procedurally it is helpful to teachers not to isolate each procedural (steps) independently, but relate the skills to previously taught similar steps (skills), this will be important in teaching rational number and is useful to teach pupils to be aware of similarities among exercise rather than teaching in isolation.

VIII. Recommendations

- Mathematics teachers in primary schools should be encouraged how to enrich their conceptual knowledge. Their low conceptual knowledge will cause their teaching to be ineffective and thus later affect the pupils whom they teach. The teachers themselves need to know their own weakness and overcome them.
- Mathematics teachers in primary school should use teaching methods and techniques that focus in conceptual knowledge in teaching rational number. It is necessary for the teachers to start focusing on conceptual knowledge regarding rational numbers.
- Mathematics teachers should not only focus in teaching of procedural aspect of rational numbers without focusing on the conceptual aspect of teaching rational number and mathematics in general, because most of them teach the pupils to learn by means of computation.
- Evaluation techniques used by mathematic teachers, should be looked into because it focusses is more in evaluating learners, procedural aspect than their conceptual knowledge which result in the lack of opportunities for learners to gain conceptual knowledge as they should. Conceptual knowledge evaluating teachers should be included when evaluating pupils.
- Primary school teachers' high performance in procedural knowledge in rational number should not become an obstruction for understanding. The teachers should balance both conceptual and procedural knowledge of rational numbers so that the understanding of the pupils would in turn enhance better teaching and learning of mathematics in our primary schools.
- Teacher training program should be looked into so that trainee teachers learn how to teach conceptual knowledge and procedural knowledge better. In view of the teachers' educational background many teachers insist on teaching predominantly procedural knowledge. Therefore, they present pupils with generally one type of learning which are the applications of rules are and algorithms is. If teachers learn how to teach conceptual and procedural knowledge, they can change their teaching method towards conceptual and procedural knowledge together.
- Curriculum planners should help in including relevant content in the teacher preparation programs. That is, strong emphasis should be placed in the conceptual and procedural knowledge of rational number particular and mathematics in general.
- The government should organize and sponsor mathematics, workshops, exhibition, quiz, seminars and conference, on regular basis, aimed at upgrading the mathematics knowledge of teachers. The government should also formulate policies that will assist the teachers to deliver their teaching jobs effectively.

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