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Regionalization of IDF Curves with L-Moments for Storm Events

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***Abstract*—**The construction of Intensity-Duration-Frequency (IDF) curves is one of the most common and useful tools in order to design hydraulic structures and to provide a mathematical relationship between rainfall characteristics. IDF curves, especially those in Peninsular Malaysia, are often built using moving windows of rainfalls. However, these windows do not represent the actual rainfall events since the duration of rainfalls is usually prefixed. Hence, instead of using moving windows, this study aims to find regionalized distributions for IDF curves of extreme rainfalls based on storm events. Homogeneity test is performed on annual maximum of storm intensities to identify homogeneous regions of storms in Peninsular Malaysia. The L-moment method is then used to regionalized Generalized Extreme Value (GEV) distribution of these annual maximums and subsequently. IDF curves are constructed using the regional distributions. The differences between the IDF curves obtained and IDF curves found using at-site GEV distributions are observed through the computation of the coefficient of variation of root mean square error, mean percentage difference and the coefficient of determination. The small differences implied that the construction of IDF curves could be simplified by finding a general probability distribution of each region. This will also help in constructing IDF curves for sites with no rainfall station.

***Keywords*—**IDF curves, L-moments, regionalization, storm events.

I. Introduction

EXTREME rainfall events such as flood are usually caused by excessive rainfalls that results in a large magnitude of water. The large amount of water exceeds the maximum capacity of irrigation and drainage system at the site which then causes extreme events. These extreme events usually come with a lot of disaster such as the loss of life and the damage to infrastructure, crops as well as properties. Hence, extreme rainfall analysis is important in order for researchers and experts from various field could model and understand rainfall characteristics and consequently prepare and produce counter measures for these events in the future.

There are a lot of research regarding extreme rainfall analysis using either annual maximum series (AMS) [1]-[3] or partial duration series (PDS) [4]-[6]. Extreme rainfall analysis is a frequent study including analyses performed using storm event analysis (SEA). SEA is a method of extracting information from rainfall data whereby rainfalls are analyzed as storm events. Storm events are defined based on the inter-event storm definition (IETD). IETD is the minimum duration between two storm events such that the serial correlation between two events are minimized [7]. In this study, the value chosen for IETD is six hours since the difference between the mean annual total storms with IETD value of six hours and the mean annual total storms with IETD value of seven hours are not significant. A storm event is a rainfall event that does not contain any dry period of more than or equal to the value of IETD. Hence, storm duration is known as the length of time for the rainfall event while storm amount is the accumulated rainfalls within a storm event. Meanwhile, storm intensity is the ratio of storm amount against storm duration [8].

IDF curves model the relationship between intensity, duration and return period of storm intensity (time intervals between two storm events with the same magnitude of storm intensity). IDF curves are also a graphical representation that summarizes the important statistical properties of storm events [9]. The IDF curves have been used and updated regularly by many countries such as Taiwan [10], Denmark [11] and Canada [12]. Regionalization of IDF curves is important in order to minimize computational time and effort as well as to obtain the IDF curves for areas with no rainfall stations. Yu and Chen have tried to get the regional IDF through the use of regression analysis [13] while Willems searched for the scaling formula for the regional IDF [14]. In this study, the regionalization of IDF curves is obtained by searching for regional probability distributions for homogeneous regions through the method of L-moments.

II. Homogeneous Regions

The first step in obtaining regional probability distributions for annual maximum storm intensities and regionalized IDF curves is to perform the regional homogeneity test. The regional homogeneity test is performed to determine whether a region or an area consisting of a few rainfall stations may be considered as a homogeneous region. The test is done using the method of L-moments. The L-moment method is an alternative method for describing the characteristics of probability distribution a series of data set. L-moments are functions which are constructed using the expected values of linear combinations of order statistics. For an ordered sample $X_{1:n} \leq X_{2:n} \leq \ldots \leq X_{n:n}$ with size n, $X_{j:n}$ denotes the j^{th} smallest observation for the sample. Hence, the h^{th} order L-moment, λ_h,

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can be written as [15]

$$\lambda_h = h^{-1} \sum_{k=0}^{h-1} (-1)^k \binom{h-1}{k} \mathrm{E}\left[X_{h-k:h}\right],\ h=1, 2, \ldots \tag{1}$$

with the expectation of an order statistic is [16]

$$\mathrm{E}\left[X_{j:h}\right] = \frac{h!}{(j-1)!\,(h-j)!} \int x\,(F_X)^{j-1} \left(1-F_X\right)^{h-j} dF_X. \tag{2}$$

In general practice, L-moments are estimated using a finite sample whose distribution is unknown. Thus, for an ordered sample $x_{1:n} \leq x_{2:n} \leq \ldots \leq x_{n:n}$ of size n, the h^{th} order sample L-moment, l_h, is written as

$$l_h = h^{-1} \binom{n}{h}^{-1} \sum_{i=1}^{n} \left[\sum_{k=0}^{h-1} (-1)^k \binom{h-1}{k} \binom{i-1}{h-k-1} \binom{n-i}{k}\right] x_{i:n},$$

$$h = 1, 2, \ldots, n \tag{3}$$

The first order L-moment is known as the L-location while second order L-moment is the L-scale. L-CV, denoted by τ, is the ratio of L-scale against L-location. Similarly, the sample L-CV, t, is the ratio of sample L-scale against sample L-location; i.e.

$$\tau = {}^{\lambda_2}/_{\lambda_1} \text{ and } t = {}^{l_2}/_{l_1}. \tag{4}$$

L-moment ratios are dimensionless version of L-moments where higher-order L-moments are divided by L-scale. Two most common L-moment ratios are L-skewness, τ_3, and L-kurtosis, τ_4, with the sample L-skewness and L-kurtosis both denoted by t_3 and t_4 respectively. L-moment ratios and sample L-moment ratios are obtained as:

$$\tau_h = {}^{\lambda_h}/_{\lambda_2} \text{ and } t_h = {}^{l_h}/_{l_2},\ \ h = 3, 4 \tag{5}$$

The regional homogeneity test is performed by comparing the variation in the values of sample L-moment ratios between rainfall stations and the value expected for a homogeneous region. This is because the L-moment ratio for all rainfall station in a homogeneous region should be the same. If all the stations in a region form a homogeneous region, then the value of the L-moment ratio for that particular region is close to the average value of the sample L-moment ratios for all rainfall stations that make up the region [15].

A simple way to measure the variation of sample L-moment ratios is by calculating the standard deviation of L-CV (4) for each rainfall station in the region. These standard deviations are weighted so that they are proportional to the sample size of rainfall data at each rainfall station. In order to obtain the value for the expected dispersion for a homogeneous region, repetitive simulations are done with the same sample size as the length of rainfall data at each station in the region. Then, the average and standard deviation for the dispersion measure are calculated based on all the simulations [17].

A probability distribution is chosen to generate these simulations and in order to avoid being committed to a certain two-parameter or three-parameter probability distribution, the four-parameter Kappa distribution is used [18]. The cumulative distribution function, $F(x)$, and the quantile distribution function, $Q(u)$, for the Kappa distribution can be written as:

$$F(x) = \left\{1-\eta\left[1-\tfrac{\kappa}{\alpha}\left(x-\xi\right)\right]^{1/\kappa}\right\}^{1/\eta} \tag{6}$$

and

$$Q(u) = \xi + \frac{\alpha}{\kappa}\left[1-\left(\frac{1-u^{\eta}}{\eta}\right)^{\kappa}\right]$$

with ξ and α are the location and scale parameters respectively while κ and η are shape parameters for the Kappa distribution. The Kappa distribution is a general distribution to many other distributions. Special cases of Kappa distribution include the generalized logistic distribution (GLO) when η = -1, the generalized extreme distribution (GEV) when η = 0, and the generalized Pareto distribution (GPA) when η = 1. Hence, Kappa distribution is able to represent a lot of probability distributions which are commonly used in extreme event analysis. In general, Kappa distribution is a suitable distribution to be used as a general distribution in simulations.

Let a region contains N_S rainfall stations with each station j have a sample size of n_j. Sample L-CV, L-skewness and L-kurtosis for each station j are denoted as $t^{(j)}$, $t_3^{(j)}$ and $t_4^{(j)}$ respectively. Meanwhile t^R, t_3^R and t_4^R refer respectively to the average values of L-CV, L-skewness and L-kurtosis for the region. These average values are weighted proportionally with the sample size of each rainfall station which can be written as [1]:

$$t^R = \sum_{j=1}^{N_S} n_j t^{(j)} \Big/ \sum_{j=1}^{N_S} n_j \text{ and } t_h^R = \sum_{j=1}^{N_S} n_j t_h^{(j)} \Big/ \sum_{j=1}^{N_S} n_j,\ h = 3, 4 \tag{7}$$

Based on the value of t^R, the weighted standard deviation of L-CV, denoted by V, can be obtained as [1]:

$$V = \left(\sum_{j=1}^{N_S} n_j \left(t^{(j)}-t^R\right)^2 \Big/ \sum_{j=1}^{N_S} n_j\right)^{1/2} \tag{8}$$

The average L-location for the region, l_1^R, is fixed to one. Then, the values of l_1^R, t^R, t_3^R and t_4^R for the region are fitted to the L-location, L-CV and L-moment ratios of Kappa distribution in order to estimate the suitable parameters of Kappa distribution that is needed to simulate the region. Functions for L-location, L-CV and L-moment ratios of Kappa distribution are written as [15]:

$$\begin{gathered}\tau = [\alpha(m_1\text{-}m_2)]/[\xi+\alpha(1\text{-}m_1)] \\ \tau_3 = (\text{-}m_1+3m_2\text{-}2m_3)/(m_1\text{-}m_2) \\ \tau_4 = (\text{-}m_1+6m_2\text{-}10m_3+5m_4)/(m_1\text{-}m_2)\end{gathered} \tag{9}$$

with

$$m_h = \begin{cases} \dfrac{m\Gamma(1+\kappa)\Gamma(m/\eta)}{\eta^{1+\kappa}\Gamma(1+\kappa+m/\eta)}, & \eta > 0 \text{ and } \kappa > -1 \\ \dfrac{m\Gamma(1+\kappa)\Gamma(-\kappa-m/\eta)}{(-\eta)^{1+\kappa}\Gamma(1-m/\eta)}, & \eta < 0 \text{ and } -1 < \kappa < -1/\eta \end{cases},$$

and $\Gamma(.)$ is the gamma function.

The estimated parameters obtained for the Kappa distribution are then used to generate a large number of simulation for the region, N_{sim}. Each simulated region is homogeneous with no cross correlation or serial correlation and contains the same length of recorded data as the original region [15]. Then, the value of V is calculated for each of the simulated regions. Hence, there will be N_{sim} values of V and thus the average of V, μ_V, and the standard deviation of V, σ_V, can be computed. Subsequently, the regional homogeneous test statistic is obtained as [19]:

$$H^R = \frac{V-\mu_V}{\sigma_V} \quad (10)$$

If $H^R < 1$, then the region is considered homogeneous; if $1 \leq H^R \leq 2$, then the region has a probability of not being homogeneous while if $H^R \geq 2$, then the region is deemed heterogeneous.

Based on the geographical locations and storm events characteristics, four regions are identified for Peninsular Malaysia as shown in Fig. 1 [20]. Fig. 1 also shows the locations of all 45 rainfall stations under consideration. The regional homogeneous test is performed on the annual maximum storm intensities of certain storm durations for the four said regions; north, west, easy and south of Peninsular Malaysia. The annual maximum storm intensities series used in this study are the annual maximums of storm intensities with storm durations of 1, 2, 3, 4, 6, 8, 9, 12, 16 and 24 hours. The regional homogeneous test is done on all ten series to make sure that each series has a homogeneous relationship between rainfall stations in each of their respective region. The regional homogeneous test statistics for all ten series of annual maximum storm intensities of all four regions are given in Table I.

TABLE I
TEST STATISTIC H^R OF TEN SERIES OF ANNUAL MAXIMUM STORM INTENSITIES FOR FOUR REGIONS IN PENINSULAR MALAYSIA

Storm duration, d (hours)	Test statistic H^R			
	North	West	South	East
1	0.43	-0.99	0.95	-0.30
2	0.74	0.51	0.64	-1.28
3	0.76	0.47	1.63	-1.43
4	-1.12	0.37	0.67	-1.23
6	-0.96	-0.35	0.96	-0.01
8	-0.01	0.55	0.01	0.37
9	0.61	0.13	0.17	-1.23
12	0.76	0.83	-0.73	0.61
16	-1.27	0.33	0.69	-1.43
24	-0.49	0.06	-0.44	-0.24

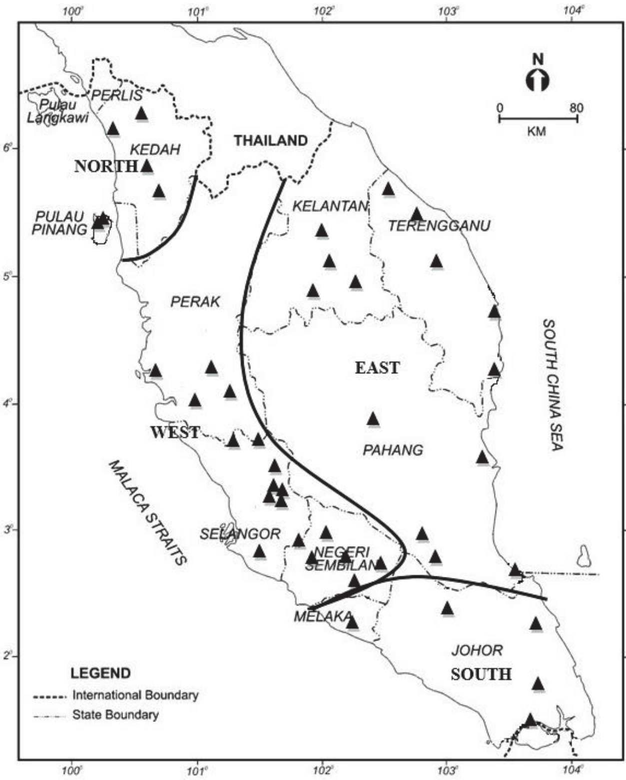

Fig. 1 Rainfall stations in Peninsular Malaysia

Table I shows that almost all of the annual maximum storm intensities series for all four regions have values less than one for the test statistic H^R. This means that the annual maximums for the four regions are homogeneous for various storm durations. Only the series of annual maximum storm intensities with a 3-hour storm duration in the southern region has a value of H^R equals to 1.63. However, it is still less than two and thus not significantly heterogeneous.

III. Regional Probability Distribution

Regional L-moment algorithm is used to determine a regional probability distribution for a homogeneous region. This regional probability distribution is able to represent the observed rainfall data at each rainfall station in the region when scaled with the scaling factor of the station. The quantile function for a station j which has a sample size of n_j from a region with N_S rainfall stations is denoted as Q_j. An observed rainfall data, x_j, for the station at a certain quantile u can be approximated using the quantile function as $x_j = Q_j(u)$ for $0 \leq u \leq 1$. Hence the observed rainfall series for station j can be written as $x_{j,k} = Q_j(u_k)$ for $k = 1, \ldots, n_j$ and $j = 1, \ldots, N_S$. If the region is homogeneous, then the quantile function for rainfall station j in the region can be written as [21]:

$$Q_j(u) = \mu_j q(u), \quad j = 1, \ldots, N_S \quad (11)$$

with μ_j is the scaling factor for station j and $q(u)$ is the regional quantile function. Subsequently, the regional data which have been scaled, x^R, at a certain quantile u is computed as $x^R = q(u)$ with $x_k^R = x_{j,k}/\mu_j$ for $k = 1, \ldots, n_j$ and $j = 1, \ldots, N_S$. The values for the sample L-CV and sample L-moment ratios for station j using x_j is the same as the values found using the regional data x^R. For simplicity purposes, the scaling factor μ_j, which is also known as the flood index, is taken as the mean of the rainfall series at rainfall station j (sample L-location), i.e. $\mu_j = l_1^{(j)}$.

The estimated regional quantile function, $\hat{q}$, is obtained by fitting l_1^R, t^R, t_3^R and t_4^R for the region with the mean, L-CV and L-moment ratios of the probability distribution considered as the regional probability distribution of the region under study. In this study, GEV distribution is chosen to represent the series of annual maximum storm intensities for certain storm durations. This is because the GEV distribution is a suitable distribution for extreme storm events in Peninsular Malaysia [22]. The definition for L-moments and L-moment ratios of GEV distribution is given as [15]:

$$\lambda_1 = \xi + \frac{\alpha}{\kappa}[1 - \Gamma(1+\kappa)]$$

$$\lambda_2 = \frac{\alpha(1-2^{-\kappa})\Gamma(1+\kappa)}{\kappa}$$

$$\tau_3 = 2\left(\frac{1-3^{-\kappa}}{1-2^{-\kappa}}\right) - 3$$

$$\tau_4 = \frac{5(1-4^{-\kappa}) - 10(1-3^{-\kappa}) + 6(1-2^{-\kappa})}{1-2^{-\kappa}}$$

where ξ and α are the location and scale parameters respectively while κ is the shape parameter with $\kappa > -1$. The estimated parameters of GEV distributions for annual maximum storm intensities with storm duration 1, 2, 3, 4, 6, 8, 9, 12, 16 and 24 hours for four regions in Peninsular Malaysia are given in Table II.

TABLE II
ESTIMATED PARAMETERS OF GEV DISTRIBUTIONS AS THE REGIONAL PROBABILITY DISTRIBUTIONS OF ANNUAL MAXIMUM STORM INTENSITIES FOR FOUR REGIONS IN PENINSULAR MALAYSIA

d (hours)	North			West		
	ξ	α	κ	ξ	α	κ
1	0.75	0.43	0	0.76	0.42	0
2	0.81	0.34	0	0.81	0.32	-0.01
3	0.82	0.31	0	0.82	0.29	-0.04
4	0.82	0.30	-0.01	0.83	0.30	0
6	0.83	0.30	0	0.83	0.29	0
8	0.82	0.31	0	0.82	0.31	0
9	0.81	0.33	0	0.81	0.32	0
12	0.78	0.39	0	0.77	0.39	0
16	0.71	0.49	0	0.68	0.50	-0.06
24	0.67	0.49	-0.09	0.62	0.51	-0.14
d (hours)	South			East		
	ξ	α	κ	ξ	α	κ
1	0.70	0.48	-0.06	0.71	0.44	-0.08
2	0.74	0.38	-0.11	0.77	0.33	-0.12
3	0.74	0.35	-0.14	0.79	0.31	-0.10
4	0.79	0.37	0	0.82	0.31	0
6	0.78	0.37	0	0.83	0.30	0
8	0.78	0.37	-0.02	0.81	0.33	0
9	0.77	0.39	0	0.80	0.34	0
12	0.74	0.46	0	0.76	0.41	0
16	0.66	0.51	-0.08	0.68	0.46	-0.12
24	0.61	0.47	-0.20	0.63	0.48	-0.17

IV. REGIONALIZATION OF IDF CURVES

In order to find the IDF function for annual maximum storm intensities to build IDF curves for rainfall stations, sets of values (i, d, T) which correspond to values of storm intensities, i, at various values of storm duration d and return period T. The value of i for a certain d and T is determined using:

$$i = Q_d\left(1 - \frac{1}{T}\right) \tag{12}$$

where Q_d is the quantile function for the series of annual maximum storm intensities with storm duration d. By obtaining the quantile functions, q, for the series of annual maximum storm intensities with various values of storm durations and return periods for a homogeneous region; as well as determining the scaling factor, μ_j, for a rainfall station j in that region, various sets of (i, d, T) can be found for the station by using (11) and (12). Hence, it is not necessary to search for the at-site probability distributions for the series at each rainfall station in the region.

The sets of (i, d, T) for each station are then fitted to the IDF equation. In this study, a simple IDF equation, known as the Sherman equation, is used. The Sherman equation can be written as:

$$i = \frac{aT^e}{(d+b)^c} \tag{13}$$

with a, b, c and e are constants. The values for these constants are estimated using the least squares method which is performed on (13) with values of (i, d, T) found for each station. With the estimated values, the IDF equation can be used to build the IDF curves for each rainfall station under study.

IDF curves found based on these regional probability distributions are compared to IDF curves obtained by searching for at-site probability distributions at each rainfall station. The differences between the two sets of IDF curves found for each station are examined by using three goodness-of-fit indices which are coefficient of variation of root mean square error, CV_{RMSE}; mean percentage difference, Δ; and the coefficient of determination, R^2. If we denote the IDF curves found from the at-site and regional probability distributions as the series of X and Z respectively, these three indices are calculated as:

$$CV_{RMSE} = \frac{\sqrt{\frac{1}{N_d N_T}\sum_{d=1}^{N_d}\sum_{T=1}^{N_T}\left(x_{d,T} - z_{d,T}\right)^2}}{\frac{1}{N_d N_T}\sum_{d=1}^{N_d}\sum_{T=1}^{N_T} x_{d,T}} \times 100\%, \tag{14}$$

$$\Delta = \frac{1}{N_d N_T}\sum_{d=1}^{N_d}\sum_{T=1}^{N_T}\frac{|x_{d,T} - z_{d,T}|}{x_{d,T}} \times 100\% \tag{15}$$

and

$$R^2 = \frac{\sum_{d=1}^{N_d}\sum_{T=1}^{N_T}(z_{d,T}-\bar{z})^2 - \sum_{d=1}^{N_d}\sum_{T=1}^{N_T}(z_{d,T}-\hat{z}_{d,T})^2}{\sum_{d=1}^{N_d}\sum_{T=1}^{N_T}(z_{d,T}-\bar{z})^2} \times 100\%,$$
$$\bar{z} = \frac{1}{N_d N_T}\sum_{d=1}^{N_d}\sum_{T=1}^{N_T} z_{d,T} \quad (16)$$

with N_d and N_T are the number of storm durations and the number of return periods considered in the analysis. The values of $x_{d,T}$ and $z_{d,T}$ are storm intensities with storm duration d and return period T from the series of X and Z respectively. Meanwhile, $\hat{z} = \beta_1 + \beta_2 x$ where β_1 and β_2 are approximated through the regression analysis for Z against X.

For CV_{RMSE} and Δ, the smaller the values obtained show smaller differences between the two sets of IDF curves at each rainfall stations. Meanwhile, the larger the value for R^2 show that the two sets of curves are more similar. The values of the three indices for the differences between the IDF curves based on the at-site and regional probability distributions at 45 rainfall stations under study are given in Table III.

TABLE III
VALUES FOR GOODNESS-OF-FIT INDICES OF THE DIFFERENCE BETWEEN IDF CURVES BASED ON REGIONAL AND AT-SITE PROBABILITY DISTRIBUTIONS AT 45 RAINFALL STATIONS

Stn.	CV_{RMSE}	Δ	R^2	Stn.	CV_{RMSE}	Δ	R^2
Kh1	8.02	2.22	99.97	N3	6.94	7.45	99.73
Kh2	4.90	1.69	99.98	N4	12.60	6.93	99.20
Kh3	4.31	2.14	99.93	N5	5.87	3.15	99.94
Kh4	11.41	7.57	99.76	J1	11.30	4.24	99.86
PP2	5.21	5.03	99.64	J2	14.86	8.83	99.39
PP3	2.67	0.89	99.98	J4	16.30	7.15	99.52
Pk1	13.76	5.11	99.72	J5	27.95	18.63	98.70
Pk2	4.14	2.78	99.97	J6	11.30	4.87	99.80
Pk3	4.35	5.29	99.92	M4	25.04	12.54	99.10
Pk4	9.39	8.08	99.66	Pg1	18.83	7.55	99.49
Pk5	7.40	8.82	99.87	Pg2	8.43	4.58	99.84
S1	3.45	1.16	99.99	Pg3	6.69	4.69	99.90
S2	3.75	2.48	99.98	Pg4	4.56	4.37	99.94
S3	10.78	4.78	99.68	T1	11.73	6.32	99.79
S4	15.54	12.03	99.28	T2	7.28	6.15	99.85
S5	1.00	0.69	100.00	T3	7.94	3.61	99.86
W1	2.31	1.54	99.99	T5	6.74	3.55	99.89
W2	8.72	8.12	99.39	T6	5.94	3.28	99.80
W3	2.29	2.55	99.98	Kn1	8.87	11.27	99.83
W4	6.78	7.45	99.88	Kn2	3.67	2.51	99.84
W5	12.02	9.39	99.64	Kn3	7.29	8.56	99.83
N1	77.91	25.61	94.09	Kn4	3.93	3.52	99.93
N2	25.06	12.15	99.41				

Both the values of CV_{RMSE} and Δ are small for most rainfall stations under consideration with an average of 10.6 and 6.3 percent respectively for all 45 stations. This shows that storm intensities obtained from IDF curves found from the regional probability distributions are equivalent to the storm intensities obtained from IDF curves built by using at-site probability distributions. The relationship between the two sets of IDF curves are also seen by the value of R^2 which has an average of 99.6 percent for all the rainfall stations under study. This implies that almost all the variations found from IDF curves through regionalizing probability distributions can be explained and determined from the variations observed from IDF curves using at-site probability distributions.

The values of Δ for the differences between storm intensities of the two sets of IDF curves at each station are looked at more closely by looking at the average values of Δ for storm intensities with storm duration 1, 3, 6, 9, 12 and 24 hours as well as for return periods of 2, 5, 10, 25, 50 and 100 years. These values are shown in Table IV.

TABLE IV
AVERAGE VALUES OF Δ FOR DIFFERENCES OF STORM INTENSITIES FOUND FROM THE TWO SETS OF IDF CURVES AT 45 RAINFALL STATIONS

d (hours)	*T* (years)					
	2	5	10	25	50	100
1	4.46	4.65	5.95	8.50	10.39	12.25
3	3.66	2.65	3.44	6.08	8.14	10.17
6	5.95	3.08	2.93	4.72	6.75	8.70
9	7.79	4.50	3.46	4.61	6.16	8.10
12	9.15	5.67	4.20	4.95	6.05	7.73
24	12.65	8.84	6.94	6.16	6.89	7.86

Based on Table IV, the average values of Δ for storm intensities obtained from both sets of IDF curves have very small difference since most of Δ are less than ten percent. Hence, this shows that the regionalization of IDF curves by using regional probability distributions provide similar IDF curves to those obtained by finding individual probability distributions at each rainfall station. The comparison between the two sets of curves from selected rainfall stations are shown in Fig. 2.

V. CONCLUSION

The method of L-moments is used in this study to determine homogeneous regions and to find the regional probability distributions for all the homogeneous regions obtained. Regionalization of IDF curves are then performed by building IDF curves using these regional probability distributions instead of using at-site probability distributions for rainfall stations in the regions. The IDF curves obtained from both the regional and at-site probability distributions are compared through three goodness-of-fit indices. These indices show that the regionalization of IDF curves for storm events in Peninsular Malaysia results in similar IDF curves to those obtained by fitting probability distributions at each rainfall station. This helps to simplify the process of building IDF curves for rainfall stations in the region because fitting of probability distributions can be reduced to only once per region. In fact, the regional probability distribution can be used to find IDF curves for areas with no rainfall stations.

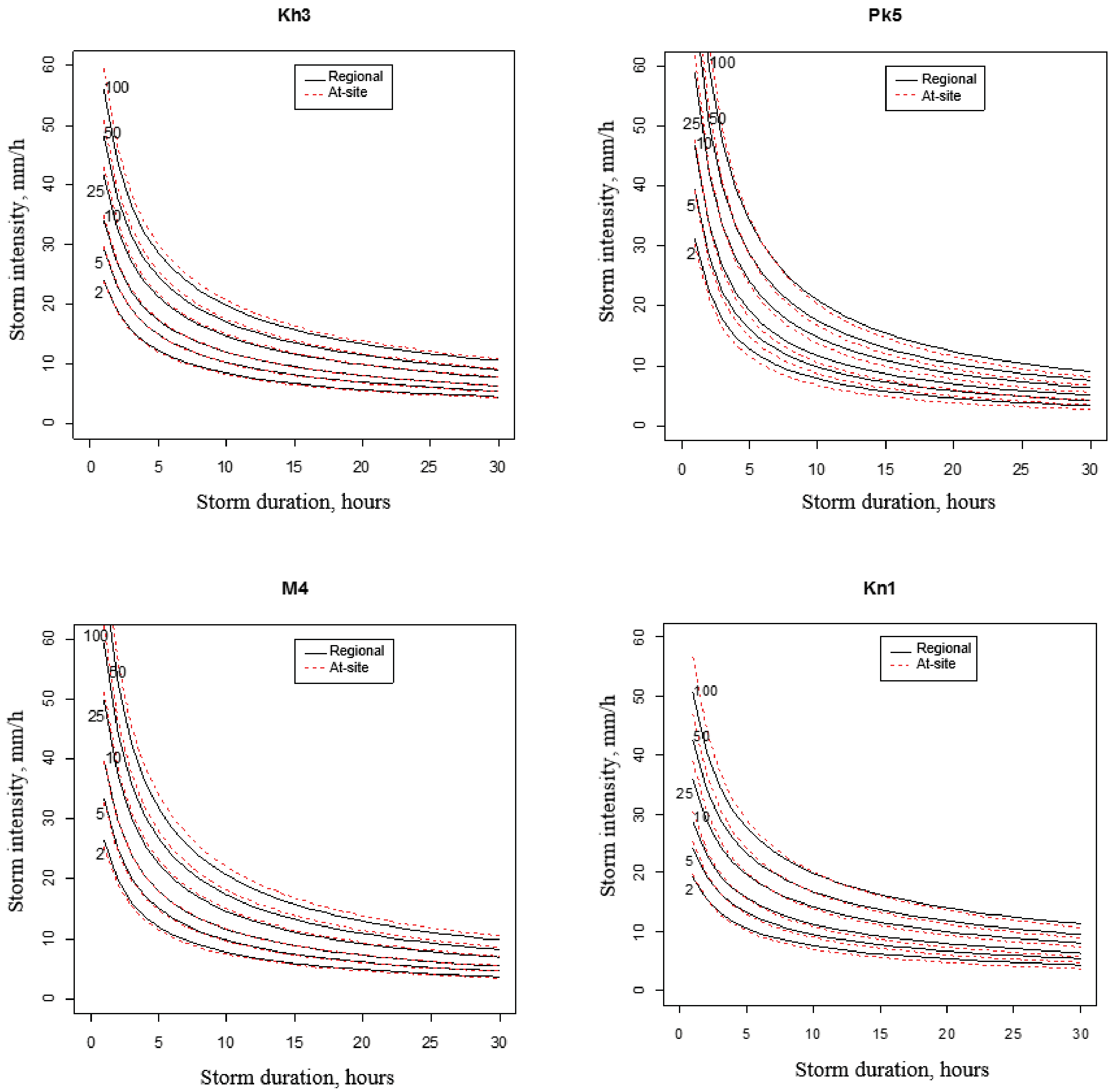

Fig. 2 IDF curves for four rainfall stations in Peninsular Malaysia (Note: The values at the end of each curve is the return period *T* for the curve)

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