World Academy of Science, Engineering and Technology
International Journal of Economics and Management Engineering
Vol:10, No:5, 2016

Currency Exchange Rate Forecasts Using Quantile Regression

Yuzhi Cai

Abstract—In this paper, we discuss a Bayesian approach to quantile autoregressive (QAR) time series model estimation and forecasting. Together with a combining forecasts technique, we then predict USD to GBP currency exchange rates. Combined forecasts contain all the information captured by the fitted QAR models at different quantile levels and are therefore better than those obtained from individual models. Our results show that an unequally weighted combining method performs better than other forecasting methodology. We found that a median AR model can perform well in point forecasting when the predictive density functions are symmetric. However, in practice, using the median AR model alone may involve the loss of information about the data captured by other QAR models. We recommend that combined forecasts should be used whenever possible.

Keywords—Exchange rate, quantile regression, combining forecasts.

I. Introduction

KOENKER [1] gives an excellent introduction to the quantile regression method. A general quantile regression model may be defined by

$$q^{\tau}_{Y|\mathbf{x}} = h(\beta_\tau, \mathbf{x}),$$

where Y is a response variable, $\mathbf{x}$ a vector of covariates, β_τ is a vector of model parameters, depending on τ, $0 < \tau < 1$ such that $P(Y \le h(\beta_\tau, \mathbf{x}) \mid \mathbf{x}) = \tau$. A special quantile regression is given by

$$q^{\tau}_{Y|\mathbf{x}} = \beta_{0\tau} + \beta_{1\tau} x_1 + \cdots + \beta_{p\tau} x_p \,.$$

The parameters of the model may be estimated by solving the minimization problem

$$\min_{\beta_\tau} \sum_{i=1}^{n} \rho_\tau(u_i)$$

where

$$\rho_\tau(u) = u(\tau - I_{[u<0]})$$

and

$$u_i = y_i - h(\beta_\tau, \mathbf{x}_i).$$

This model has also been generalized to deal with time series. The QAR is defined by

$$q^{\tau}_{y_t|\mathbf{y}_{t-1}} = \beta_{0\tau} + \beta_{1\tau} y_{t-1} + \cdots + \beta_{p\tau} y_{t-p} = \mathbf{y}_{t-1}^\top \beta_\tau, \quad (1)$$

where p is the order of the model, $\mathbf{y}_{t-1} = (1, y_{t-1}, \ldots, y_{t-p})^\top$, and $\beta_\tau = (\beta_{0\tau}, \beta_{1\tau}, \ldots, \beta_{p\tau})^\top$ is a vector of the model parameters.

Y. Cai is with the School of Management, Swansea University, United Kingdom (e-mail: y.cai@swansea.ac.uk).

Note that β_τ can also be estimated by minimizing

$$\sum_{t=p+1}^{T} \rho_\tau(y_t - \mathbf{y}_{t-1}^\top \beta_\tau). \quad (2)$$

So, if y_t represents the log return on the exchange rates of interest, then for $\tau = 0.5$, we have an estimated median of y_t conditional on $\mathbf{y}_{t-1}$. However, in practice, we are also interested in predicting the value of y_t or the distribution of y_t. Some work can be found in the literature on forecasting with quantile regression models. For example, the work of [3] used exponentially weighted quantile regression to forecast daily supermarket sales; [4] considered the quantile forecasting for credit risk management; and [5] studied Bayesian time-varying quantile forecasting for value-at-risk in financial markets. In this paper, we explain how to forecasting by using the QAR model based on the work of [2].

II. The Estimation and Forecasting Method

The method of [2] is based on a Bayesian approach. Let $\mathbf{y}_p = (y_1, y_2, \ldots, y_p)$. Let $\mathbf{y} = (y_{p+1}, y_{p+2}, \ldots, y_T)$. Let $\mathbf{y}_{T+M,\tau} = (y_{T+1,\tau}, \ldots, y_{T+M,\tau})$, i.e. the unknown future values of the process coming from model (1). Then the Likelihood of $(\mathbf{y}, \mathbf{y}_{T+M,\tau})$:

$$L(\mathbf{y}, \mathbf{y}_{T+M,\tau} \mid \beta_\tau, \mathbf{y}_p)$$

$$= \{\tau(1-\tau)\}^{T+M-p} \exp\left\{-\sum_{t=p+1}^{T+M} \rho_\tau(y_t - \mathbf{y}_{t-1}^\top \beta_\tau)\right\}, \quad (3)$$

where $y_t = y_{t,\tau}$ if $t > T$.

It is seen that if we can estimate both β_τ and $\mathbf{y}_{T+M,\tau}$ then we will have an estimated model and we will also have forecasts of the process up to time $T + M$. Reference [2] proposed a Bayesian method to estimate them. However, the estimated quantities depend on τ. That is, we have a sequence of models and a sequence of forecasts. So, which model should we use and which forecasts should we take? We summarize the combining method proposed by [2] below and will explain how their method is used for the exchange rate data in next section.

Let

$$A_{\tau_i} = \left\{(y^{(\ell)}_{T+1,\tau_i}, \ldots, y^{(\ell)}_{T+M,\tau_i}), \ell = 1, \ldots, L\right\}$$

be the posterior sample of $\mathbf{y}_{T+M,\tau_i}$ obtained from the τ_ith QAR model, where $i = 1, \ldots, I$, let $\alpha_{\tau_i} \in (0,1)$ be combining weights such that $\alpha_{\tau_i} \ge 0$ and $\sum_{i=1}^{I} \alpha_{\tau_i} = 1$. Then the combining forecasting method consists the following several steps:

(1) Constructing a combined sample:

(i) obtain a sub-sample of size $[L\alpha_{\tau_i}]$ from A_{τ_i}, where $[\cdot]$ stands for the integer part of a number and $i = 1, \dots, I$;

(ii) put all sub-samples together to form a combined sample A.

(2) Using the combined samples in A to estimate the predictive density functions and/or any other predictive quantities.

For the combining weight, we consider two cases.

(a) $\alpha_{0.05} = \alpha_{0.95} = 0.05$, $\alpha_{0.25} = \alpha_{0.75} = 0.20$ and $\alpha_{0.5} = 0.5$.

(b) Use all posterior samples from all fitted QAR models.

So for (a), for example, the median QAR model contributes 50% of their samples to the combined forecasts, while for (b), each fitted QAR model makes the same contribution to the final forecasts,

III. Application

We consider the log returns, denoted by y_t, of exchange rate USD/GBP for the period from 2 January 1997 to 21 November 2000. The length of the log-return time series is 974. The last 6 values were used to allow comparisons with the forecast results. The observed time series is shown in Fig. 1.

A sequence of models with different orders and τs were estimated, and we found that, according to BIC, model QAR(1) was the best. So we only discuss this model below. The estimated parameters are shown in Fig. 2 by the vertical lines for $\tau = 0.05, 0.25, 0.5, 0.75$ and 0.95. For each τ, we can also estimate the conditional quantiles of the returns at each time t. Fig. 3 shows these conditional quantiles, where the grey curve represents the observed return y_t. However, how do we know whether the estimated models are good or not? One way to check this is to check the global coverage of the estimated conditional quantiles. Table I shows the results. It is seen that estimated coverage is very close to the true value of τ, suggesting that the estimated models are very good, hence, could be used for forecasting.

Now, we consider the 1-step ahead forecasts. For fixed t, we have a sequence of y_t values when τ changes. So we actually have a discrete probability density function (pdf) of y_t. Then smoothing spline technique could be used to obtain a smoothed pdf. Fig. 4 presents the estimated 1-step ahead predictive conditional density functions for the last 10 days' log-returns. This shows clearly that the conditional distributions of the log-returns changes across time. This also illustrates that QAR models can allow us to study any specific feature of a time series at any time point.

For out of sample forecasts, We used the weights (a) and (b) to combine posterior samples obtained from the models for different quantiles. Fig. 5 shows the predictive density functions up to 6-steps ahead. It is seen that weights (a) produces much shorter predictive credible intervals than those using weights (b). As in practice a change of log-return of more than 20% is very rare, forecasts obtained from weights (a) are more reasonable than those obtained from weights (b).

We may also obtain point forecasts for y_t. Table II shows the observed log-returns and predicted log-returns based on individual QAR models and combined forecasts using weights

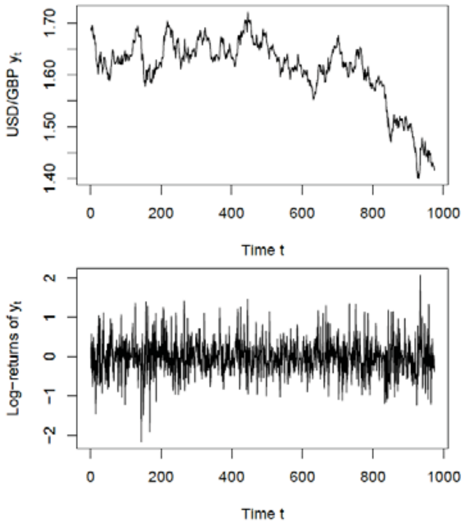

Fig. 1 Time series plots of the USD to GBP daily currency exchange rates and the associated log-return series

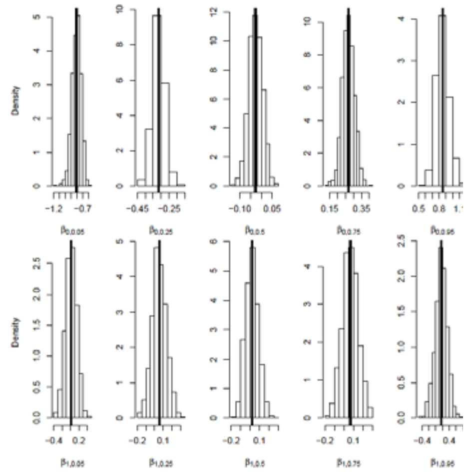

Fig. 2 Bayesian estimates of model parameters

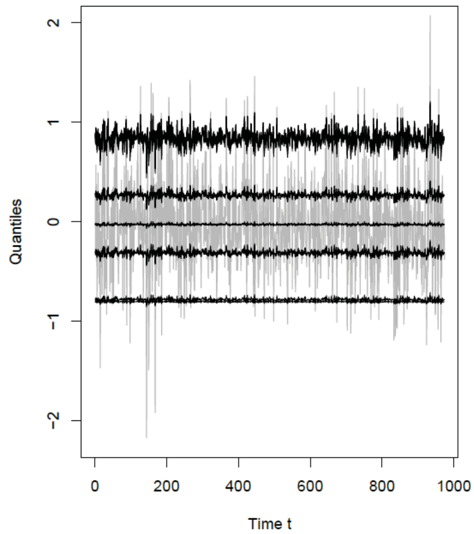

Fig. 3 Estimated conditional quantiles of y_t

TABLE I
EMPIRICAL COVERAGE OF THE ESTIMATED CONDITIONAL QUANTILE CURVES FOR THE QAR(1) MODEL FITTED TO THE USD TO GBP DAILY CURRENCY EXCHANGE DATA

τ	0.05	0.25	0.5	0.75	0.95
Number out of 968	45	236	483	724	922
Proportion	0.046	0.24	0.50	0.75	0.95

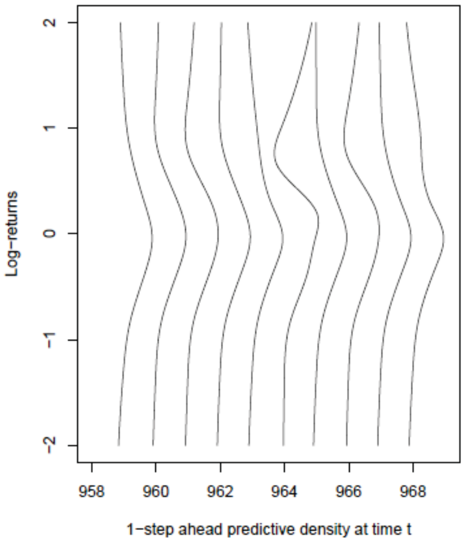

Fig. 4 1-step ahead predictive conditional density functions at times $t = 959, \ldots, 968$

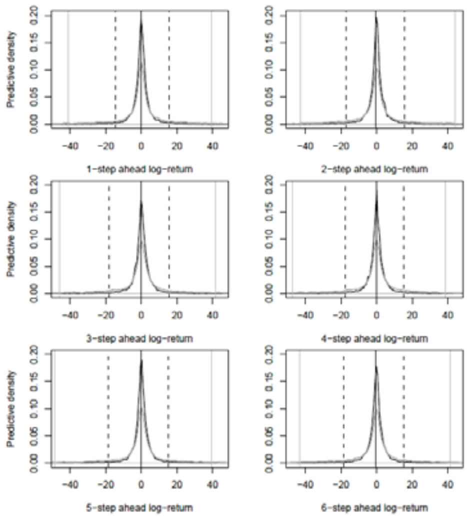

Fig. 5 Predictive conditional density functions up to 6-steps ahead. Darker (lighter) curves for weights (a) and (b) respectively

(a) and (b). MSEs between observed and predicted log-returns are also given. These results also show that the combined forecasts obtained from weights (a) are better than those from weights (b).

IV. CONCLUSIONS

We showed how to use a QAR model to analyze returns on currency exchange rates. We discussed a combining forecast technique to improve the quality of the forecasts obtained from a sequence of fitted QAR models. We also showed the density forecasts of the exchange rates. We found that the distribution of the exchange rates depends on time t. We also showed the point forecasts of the exchange rates. We found that a median AR model can perform well in point forecasts when the predictive density functions are symmetric. Our results suggest that combined forecasts are better than those obtained from individual models. We therefore recommend that combined forecasts should be used whenever possible in practice. We also found that combining weights (a) performs better than weights (b). However, the optimal weights for QAR models need further investigation in the future.

TABLE II
OBSERVED LOG-RETURNS AND PREDICTED LOG-RETURNS BASED ON INDIVIDUAL QAR MODELS AND USING WEIGHTS (A) AND (B): MSES BETWEEN OBSERVED AND PREDICTED LOG-RETURNS ARE ALSO GIVEN

m	1	2	3	4	5	6	MSE
Observed	0.000	−0.308	−0.253	0.028	−0.070	−0.38	
$\tau = 0.05$	18.508	19.907	18.510	18.267	17.706	18.87	
$\tau = 0.25$	2.379	2.538	2.696	2.607	2.430	2.609	
$\tau = 0.5$	−0.080	0.064	0.083	0.086	0.095	−0.16	
$\tau = 0.75$	−2.192	−2.601	−2.411	−2.392	−2.468	−2.60	
$\tau = 0.95$	−18.789	−21.351	−22.269	−23.381	−23.277	−22.674	
Weights (a)	−0.045	−0.087	−0.078	−0.160	−0.244	−0.271	0.027
Weights (b)	−0.035	−0.289	−0.678	−0.963	−1.103	−0.791	0.400

REFERENCES

[1] Koenker, R. (2005). *Quantiles Regression*. Cambridge University Press.
[2] Cai, Y., Stander, J. and Davies, N. (2012). A new Bayesian approach to quantile autoregressive time series model estimation and forecasting. *Journal of Time Series Analysis*, 33, 684-698.
[3] Taylor, J. W. (2007). Forecasting daily supermarket sales using exponentially weighted quantile regression. *European Journal of Operational Research*, 178, 154-167.
[4] Banachewicz, K. and Lucas, A. (2008). Quantile forecasting for credit risk management using possibly misspecified hidden Markov models. *Journal of Forecasting*, 27, 566-586.
[5] Gerlacha, R. H., Chen, C. W. S. and Chan, N. Y. C. (2011). Bayesian time-varying quantile forecasting for value-at-risk in financial markets. *Journal of Business and Economic Statistics*, 29(4), 481-492.