Assessment of Slope Stability by Continuum and Discontinuum Methods

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***Abstract*—**The development of numerical analysis and its application to geomechanics problems have provided geotechnical engineers with extremely powerful tools. One of the most important problems in geotechnical engineering is the slope stability assessment. It is a very difficult task due to several aspects such the nature of the problem, experimental consideration, monitoring, controlling, and assessment. The main objective of this paper is to perform a comparative numerical study between the following methods: The Limit Equilibrium (LEM), Finite Element (FEM), Limit Analysis (LAM) and Distinct Element (DEM). The comparison is conducted in terms of the safety factors and the critical slip surfaces. Through the results, we see the feasibility to analyse slope stability by many methods.

***Keywords*—**Comparison, factor of safety, geomechanics, numerical methods, slope analysis, slip surfaces.

I. Introduction

SLOPE instability is a typical geotechnical phenomena in the worldwide, and causes a huge threat to the human life and property [1], [2]. The landslide will happen to a slope because of many factors such as earthquake, rainfall and manmade excavation [3]. Therefore, it is important to determine the dangerous sliding surface and safety factor of slope. The first and essential condition for assessing the stability of a slope is the understanding of the mechanical processes that lead or may lead to movements or failure. In most applications, the primary purpose of slope stability analysis is to contribute to the safe and economic design of excavations, embankments, earth dams, and landfills. The aims of many search about slope stability analysis, are to analyze landslides and to understand failure mechanisms with the influence of environmental factors; to assess the stability of slopes under short-term and long-term conditions; to study the effects of seismic loadings on slopes [3]; and many other aims.

In this paper, we will compare the results of factor of safety and critical slip surfaces obtained by distinct element method (DEM), with the limit equilibrium method (LEM), finite element methods (FEM) and limit analysis (LA).

II. Methods

Many methods exist to analysis the slope stability, as Limit Equilibrium Method is widely used by researchers and engineers conducting slope stability analysis. The most common limit equilibrium techniques are methods of slices. In addition, numerical methods have been extensively used in the past several decades due to advances in computing power such as continuum methods, Finite Difference Method; Finite Element Method, and others. For discontinuum methods, the Discontinuous Deformation Analysis; Discrete Element Method, and many others, these methods having all of the advantages and disadvantages, none is perfect. The most basic purpose of slope stability analysis, is determining a factor of safety against a potential failure, and indicates the failure slope. The mostly failure criterion used to assessment slope stability is Mohr–Coulomb failure criterion, there are several failure criterions, one of these criterions is the Generalized Hoek-Brown criterion.

A. Limit Equilibrium Method

For slope stability analysis, the (LEM) is widely used by researchers and engineers conducting slope stability analysis, because these are traditional and well established. The most common limit equilibrium techniques are methods of slices, such as the ordinary method of slices [4], and the Bishop simplified, Spencer, and Morgenstern-Price methods. The difference between variants of slices methods represented in, overall equilibrium conditions is the assumptions about inter-slice forces, and the shape of slip surface. Here we will give a brief discussion about shapes of slip surface, the LEMs can be grouped in tow: the first group is methods of analysis which use circular slip surfaces include: [4]; and [5]. The second is methods of analysis which employ non-circular slip surfaces include: [6]-[9], and others. Many authors have summarized the slice methods such as Zhu et al [10]. Usual hypotheses of the LEM are:

1) The sliding body over the failure surface is divided into a finite number of slices. The slices are usually cut vertically, but horizontal as well as inclined cuts have also been used by various researchers. In general, the differences between different methods of cutting are not common, and the vertical cut is preferred by most engineers at present.
2) The strength of the slip surface is mobilized to the same degree to bring the sliding body into a limit state. It means there is only a single factor of safety which is applied throughout the whole failure mass.
3) Assumptions regarding inter-slice forces are employed to render the problem determined; finally, the factor of safety is computed from force and/or moment equilibrium equations.

The definition of the Factor of Safety (FS) is the same for

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all these methods, is defined as:

$$FS = \frac{Shear\ strength\ of\ soil}{Shear\ stress\ required\ for\ equilibrium} \quad (1)$$

The various slice methods of limit equilibrium analysis have been well surveyed and summarized in many studies such as [11], [12].

B. Finite Elements Method

Among the continuum methods, the Finite Element Method (FEM) is largely used to analysis the solid and structural mechanics [13]-[16]. The numerical methods, and in particular the finite element method (FEM), has developed rapidly and become increasingly popular for the slope stability analysis. Literature analysis of slope stability using FEM, based on the technique of shear strength reduction was reviewed by [12], [17], and [18]. Generally, there are two approaches using the finite element method to analyze slope stability, [19]. One approach is to increase the load of gravity and the second approach is to reduce the strength characteristics. The second approach is adopted in this study using the finite element software. Generally, two major tasks coupled in the slope stability analysis: the computation of the factor of safety and the location of the critical slip surface. The definition of the factor of safety is not unique [20], [21]. The technique of strength reduction (SRM) is typically applied to calculate the factor of safety by progressively reducing or increasing the shear strength of the material to bring the slope to a state of limiting equilibrium [22]. In recent years, there have been various developments in the strength reduction method (SRM) for slope stability analysis. This method was used as early as 1975 by Zienkiewicz et al. [23], and has since been applied by Griffiths and Lane [17], and others [24]-[30]. The technique is also adopted in several well-known commercial geotechnical finite element programs. The main advantages of the SRM are as follows:

1) The critical failure surface is found automatically from the application of the gravity loads and/or the reduction of shear strength;
2) It requires no assumption on the inter-slice shear force distribution; and it is applicable to many complex conditions and can give information such as stresses, movements, and pore pressures.

The strength reduction method by Mohr–Coulomb, it is the most used in the programs of FEM and FDM, for slope stability analysis, the SRM decrease gradually the strength parameters (c, φ) of the slope until the instability of this slope. The Mohr-Coulomb failure criterion can be written as the equation for the line that represents the failure envelope. The equation of the line is given by:

$$\tau = c + \sigma_n \tan\varphi \quad (2)$$

where τ is shear stress; σ_n is normal stress; c is the cohesive strength, and φ is the friction angle.

The safety of factor by SRM is the ratio between actual strength parameters and critical strength parameters, the corresponding formula is:

$$FS = \frac{c}{c_r} = \frac{\tan\varphi}{\tan\varphi_r} \quad (3)$$

FS: Safety of factor; c: Initial cohesive strength; φ: initial internal friction angle; c_r: reduced cohesive strength; and φ_r: reduced internal friction angle.

The second strength reduction method is the gravity increase method (GIM), in this method the *gravity* forces, such weight, increase progressively until the instability of this slope, to give results more reliable its used to study during construction of embankments, Colby C. Swan, [31].

The factor of safety with GIM is the ratio between gravitational acceleration in the time of failure and actual gravitational acceleration, is defined according to the equation:

$$FS = \frac{g_0^{trail}}{g_0} \quad (4)$$

FS: Safety of factor; g_0^{trail}: trial gravitational acceleration (m/s^2); and g_0: the initial gravitational acceleration (m/s^2).

C. Limit Analysis Method

The limit theorems provide a simple and useful way of analyzing the stability of geotechnical structures, this method is a powerful mathematical tool that provides rigorous lower and upper bounds to the exact stability factor in slope stability problems. The soil is assumed to deform plastically according to the normality rule associated with the Coulomb yield condition. The applied of this method started by [32], [33], to analysis slope stability undergoing plane strain failure, with rotational and translational failure mechanisms. Method of limit analysis based on two theorems:

1) The lower bound theorem, which states that any statically admissible stress field will provide a lower bound estimate of the true collapse;
2) The upper bound theorem, which states that when the power dissipated by any kinematically admissible velocity field is equated with the power dissipated by the external loads, then the external loads are upper bounds on the true collapse load, [33].

Currently, most the slope stability evaluations based on using the limit analysis are based on the upper bound method alone, such as [34]-[39].

D. Distinct Element Method

More recently, the Distinct Element Method (DEM) has also become more and more popular, the DEM is a numerical tool devoted to the modeling of assemblies of particles, and it has been used since late seventies to study the (micro) mechanical behavior of granular materials, mainly in the field of soil mechanics. This technique originally developed for dry granular materials by [40]. There are many researchers used the Distinct Element Method to analyse discontinuous problems such as, granular mechanics [41], the anisotropy of clay [42], the strain localization [43], [44]. Moreover, in the dynamic behaviour or liquefaction of sands [45], [46]. The main advantages of the discrete approach can be summarized as follows:

1) The need for a constitutive model for the equivalent continuum is bypassed, not to mention the computational difficulties related to some particular features of the mechanical behavior of granular soils (softening, non-associativeness of the flow rule);
2) In principle, the same DEM model can be used to study in a comprehensive way the problem, including triggering, propagation, run-out, interaction with sheltering structures.

III. Numerical Example

In this study we will use the slope example of Zienkiewicz et al. [47]. This example consists of a homogeneous soil slope, with a single layer, the geometry of the slope illustrated in Fig. 1. This example has been treated by [48], and then by [49], which uses the distinct element method (PFC2D code) to analysis this problem. Table I shows the mechanical characteristics model of the linear elastic contact by the PFC2D code. Preh [48] determines these parameters, the same table shows the physical and mechanical properties of the Mohr-Coulomb failure example of [47].

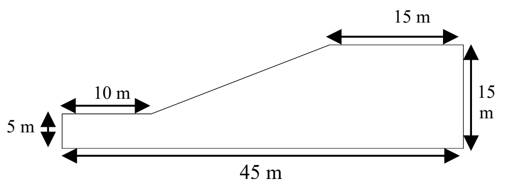

Fig. 1 Slope geometry

TABLE I
Properties Physical and Mechanical of the Slope

Symbol	Mechanical Properties [47]	Mechanical Characteristics (Micro-properties) [48], [49]	Units
E	200000	-	KPa
ν	0.25	-	-
c	3	-	KPa
φ	20	-	°
γ	20	-	KN/m^3
ρ_s	-	2381	Kg/m^3
k_s	-	1.6×10^5	KN/m
k_n	-	4×10^5	KN/m
R_{min}	-	0.08	m
R_{max}	-	0.10	m
n	-	0.16	-
μ	-	0.1317	-

IV. Result and Discussion

Our example has been treated by [41] and [42]. They used the distinct element method (PFC2D code) to analysis this problem. In this study, we took the results of Preh and Riad, and we compared the results, with the limit equilibrium method, Finite element method, and limit analysis.

A. Distinct Element Method

This problem already studied by discontinuous methods (distinct element method), unlike to classical model which is based on continuous media. The modelling of a granular media with circular elements is representative to describing the behavior of discontinuous materials for this type of problem, the PFC2D computer code was used suited to this study. We have 15133 particles in this modelling.

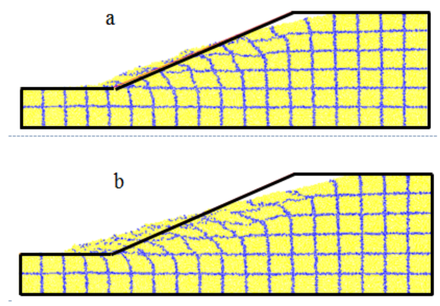

Fig. 2 Deformation of slope by Distinct Element Method

Fig. 2 shows results of deformation of the slope, through the different stages calculation cycles. Fig. 2 (a) indicates the deformation after 200000 cycles, and Fig. 2 (b) shows the deformation after1000000 cycles. These deformations are marked by the deformation of colored particles grid (vertical and horizontal lines).

B. Limit Equilibrium Method

Firstly, we use the limit equilibrium method to assess this slope, and found the factor of safety with critical slip surfaces, by four method of slices, Ordinary; Bishop; Janbu; and Morgenstern-Price method. Table II presents the safety factors obtained by these methods. All the safety factors of these methods (see Table II) indicate that the slope is an unstable. The results are smalls.

TABLE II
Factors of Safety by the Slices Methods

Method	FS
Ordinary	0.964
Bishop	1.029
Janbu	0.956
Morgenstern-Price	1.027

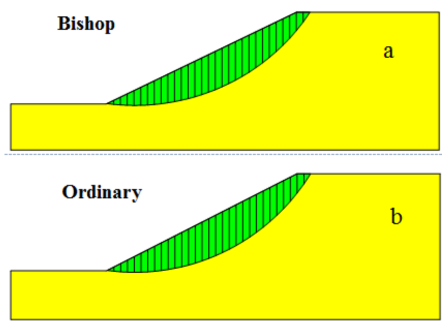

Fig. 3 Slip surfaces by slices method

Fig. 3 presents the slip surfaces of the slices method by Bishop method in Fig. 3 (a), and Ordinary method in Fig. 3 (b). These calculations have shown that for this slope the critical circles are all of Toe circles. In the limit equilibrium method, there is no displacement to compare with distinct element method.

C. Finite Element Method

After using the limit equilibrium method, now we assess the same slope by Finite Element Method. The results are carried out by strength reduction method using Mohr-Coulomb parameters to determine the factor of safety, failure surface, and deformations. The factors of safety obtained by the Finite Element Method is **FS = 0996**. Again, the factor of safety is small, that indicate the slope is unstable, this factor is similar with the factors of safety of limit equilibrium method. Figs. 4, and 5 present the failure surface and shear Strains obtained by FEM.

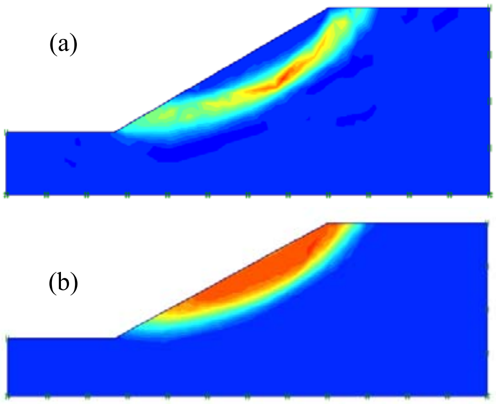

Fig. 4 Shear strains and the failure surface by the finite element method

Fig. 4 (a) shows results of shear strains of the slope, through strength reduction method calculation. Fig. 4 (b) illustrates the critical failure surface, and the results have shown the shape of this slip surface is circler, the shear strains pass to toe of the slope.

We note that this Fig. 5, present the mesh deformation of the slope by finite element method, without discontinuity as we have seen in the method of Distinct Element Method, in Fig. 2, but the deformation is similar when we see the top and the toe of the slope in the finite element method and Distinct Element Method.

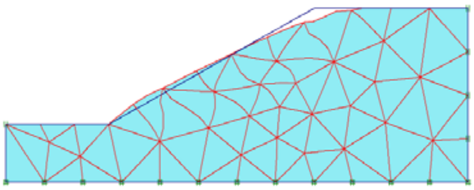

Fig. 5 Deformation mesh of slope by the Finite element method

D. Limit Analysis

Finally, we finished our assessment by Limit Analysis, the results are carried out by strength reduction method, by the Upper and Lower bound we discover the factor of safety, critical slip surfaces, and the deformation. The results of factors of safety obtained by this method are shown in Table III. The factors of safety obtained by the Limit Analysis is (**FS = 0999** ∓ 0.0315). Another time, the factor of safety is small, the slope is unstable, this factor is similar with the factors of safety of limit equilibrium method and finite element method. The Figs. 6, 7, give the failure surface and shear Strains obtained by Upper and Lower bound.

TABLE III
FACTORS OF SAFETY BY THE LIMIT ANALYSIS METHODS

Limit Analysis Method	FS
Upper	1.031
Lower	0.968
	0.999 ∓0.0315

Results of the analyses are present in Table III, the factor of safety by Upper is FS= 1.031 it is a little big compared with finite element method, and limit equilibrium method. However, is small to make this slope stable. The following Figures give us the results of the Upper bound method.

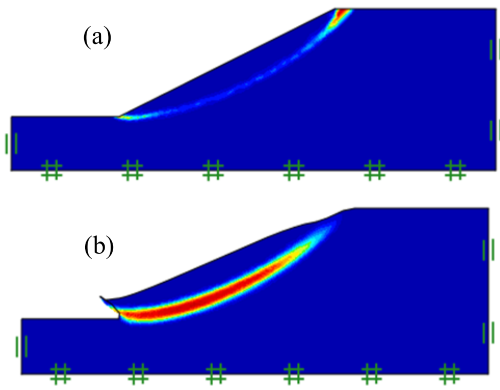

Fig. 6 Plastic multiplier and the deformation by the upper bound

Results of the upper bound, of this study are obtained by the strength reduction method to evaluate the deformations, and the failure surface, like what we see in the Fig. 6. The failure surface of this slope is circular and pass in toe of the slope like limit equilibrium method. The deformation is similar to the deformation of finite element method in the top of slope, in Fig. 6 (b), we see the deformation without discontinuity as we have seen in the method of distinct element method, in Fig. 2.

In this section, we compared the factor of safety of Lower bound method with other methods. The factor of this method is FS= 0.968 is little small compared with finite element method and the Upper bound method this method is small that make the slope is unstable, Figs. 7 (a) and (b) show the deformation and failure slope.

Fig. 7 presents the Lower bound results, it is study by the strength reduction method to evaluate the deformations, and the failure surface, like what we see in Fig. 6. The failure surface of this method is circular and passes in toe of the slope like limit equilibrium method, finite element method and like the Upper bond. The deformation is similar to the deformation of Upper bound see Fig. 6 (b).

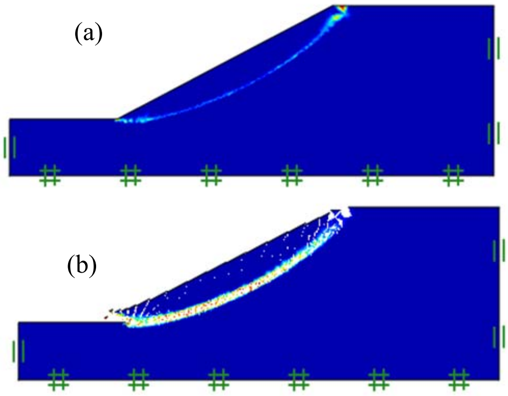

Fig. 7 Plastic multiplier and the deformation by the lower bound

V. Conclusions

The aim of this research is to compare results and observations obtained by many methods, we have noticed that there is a great similarity in the shape of failure surface, despite the difference between failure modes, the slip surfaces are circular in limit Equilibrium method, Finite element method, and Limit Analysis. The distinct element method by PFC2D gives the shape of rupture close to reality, it is also offer the discontinuity in the deformation of the slope. The soil moves, it is similar the same volume in these methods, unless the method of limit equilibrium method, it cannot give the deformation of slope, it offer just the line of critical slip surface. We note also, the displacements of the continuous methods, are low compared to those of distinct element method by PFC2D, because, the continuous methods consider the slope as a rigid and continuous media, not like the distinct element method. On the other hand, the factor of safety is similar in these methods, not a big difference between them, however, we cannot determine a safety factor for a slope by PFC2D. Finally, the simulation time in limit equilibrium method, finite element method, limit analysis, are fast, compared to the time simulation of distinct element method by PFC2D.

References

[1] Chen, Z. Y.: Stability analysis of soil slope—Theory, Method and Programs. Chinese Water Press, Beijing, 2003.
[2] Kim, J. Y., Lee, S. R.: An improved search strategy for the critical slip surface using finite element stress fields. Computers and Geotechnics, 1997.
[3] Kentli, B., Topal, T.: Assessment of rock slope stability for a segment of the Ankara-Ppzant1 motorway, Turkey. Engineering Geology, 2004, 74, 73–90.
[4] Fellenius W. Calculation of stability of earth dams. Transactions, 2nd Congress on Large Dams, Washington, DC, 1936, vo1445-462.
[5] Bishop A.W. The use of slip circle in the stability analysis of slopes. Geotechnique, 1955, 5(1):7-17.
[6] Morgenstern, N. R., and Price, V. E. The analysis of the stability of general slip surfaces. Geotechnique, London, 1965, 15(1), 79-93.
[7] Spencer, E. A method of analysis of the stability of embankments assuming parallel interslice forces. Geotechnique. London, 1967, 17(1), 11-26.
[8] Sarma S K. Stability Analysis of Embankment and Slopes". Geotechnique, 1973, 23:423-33.
[9] Janbu, N. Slope Stability Computations, Embankment-Dam Engineering, Casagrande Volume, Hirschfeld, R.C. and Poulos. J. (ed.), John Wile and Sons, New York, USA, 1973, 47-86.
[10] Zhu, D. Y., Lee, C. F., Jiang, H. D. Generalized framework of limit equilibrium methods for slope stability analysis (J). Geotechnique, 2003, 53(4): 377-395.
[11] Abramson L W; Lee T S; Sharma S; Boyce G M. Slope Stability Concepts. Slope Stabilisation and Stabilisation Methods, Second Edition, published by John Wiley & Sons, Inc., 2002, pp. 329-461.
[12] Duncan J. M. State of the art: limit equilibrium and finite-element analysis of slopes. J Geotech Eng, 1996, 122(7):577–96.
[13] O. C. Zienkiewicz, R. L. Taylor, 4th edition, The Finite Element Method, vol. 2, McGraw-Hill, London, 1991.
[14] M. A. Crisfield, Non-linear Finite Element Analysis of Solids and Structures, John Wiley, Chichester, 1991.
[15] E. Hinton, D. R. J. Owen, Finite Elements in Plasticity: Theory and Practice, Pineridge Press, Swansea, Wales, 1980.
[16] K. J. Bathe, Finite Element Procedures, Prentice-Hall, New Jersey. 1996.
[17] Griffiths D. V., Lane P. A. Slope stability analysis by finite elements. Géotechnique, 1999, (3):387–403.
[18] L.C. Li, C. A. Tang, W. C. Zhu, Z. Z. Liang; Numerical analysis of slope stability based on the gravity increase method, journal of Computers and Geotechnics, 2009.
[19] Rabie, M. «Comparison study between traditional and finite element methods for slopes under heavy rainfall» Housingand Building National Research Center, 2014, 9.
[20] Abramson LW, Lee TS, Sharma S, et al. Slope stability and stabilization methods. New York: Springer, 1996.
[21] Zheng H., Tham L. G., Liu D. F. On two definitions of the factor of safety commonly used in the finite element slope stability analysis. Comput Geotech, 2006, 33: 188–95.
[22] Dawson E. M., Roth W. H., Drescher A. Slope stability analysis by strength reduction. Geotechnique, 1999, 49(6):835–40.
[23] Zienkiewicz O. C., Humpheson C., Lewis R. W. Associated and non-associated visco-plasticity and plasticity in soil mechanics. Geotechnique, 1975, 25(4): 671–89.
[24] Naylor D. J. Finite elements and slope stability. Proceedings of the NATO Advanced Study Institute, Lisbon, Portugal, 1981. Numer. Methods Geomech, 1982, 229–44.
[25] Donald I. B., Giam S. K. Application of the nodal displacement method to slope stability analysis. In: Proceedings of the 5th Australia–New Zealand conference on geomechanics, Sydney, Australia, 1988, p. 456–60.
[26] Matsui T., San K. C. Finite element slope stability analysis by shear strength reduction technique. Soils Foundations, 1992, 32(1):59–70.
[27] Ugai K. A method of calculation of total factor of safety of slopes by elastoplastic FEM. Soils Foundations, 1989, 29(2):190–5.
[28] Song E. Finite element analysis of safety factor for soil structures. Chinese J Geotech Eng, 1997.
[29] Dawson EM, Roth WH, Drescher A. Slope stability analysis by strength reduction. Geotechnique, 1999.
[30] Zheng Y. R., Zhao S. Y., Kong W. X., Deng C. J. Geotechnical engineering limit analysis using finite element method. Rock Soil Mech 2005, 26(1):163–8.
[31] Colby, C. Swan and Young-Kyo Seo. Slope stability analysis using finite element techniques. 13 th Iowa ASCE Geotechnique Conference, USA. 1999.
[32] Drucker, D. C., Greenberg, W. and Prager, W., The safety factor of an elastic plastic body in plane strain, Transactions of the ASME, Journal of Applied Mechanics, 1951, 73, 371.
[33] Drucker D. C. and Prager W., Soil mechanics and plastic analysis or limit design, Quarterly of Applied Mathematics, 1952, 10, 157–165.
[34] Chen, J., Yin, J. H., and Lee, C. F. Upper bound limit analysis of slope stability using rigid finite elements and nonlinear programming. Can. Geotech. J., 2003, 40: 742-752.
[35] Chen, J., Yin, J. H., and Lee, C. F. A three-dimensional upper-bound approach to slope stability analysis based on RFEM. Geotechnique, 2005.
[36] Donald, I.B., and Chen, Z.Y. Slope stability analysis by the upper bound approach: fundamentals and methods. Can. Geotech. J., 1997, 34: 853-862.
[37] Michalowski, R. L. Three-dimensional analysis of locally loaded slopes. Geotechnique, 1989, 39(1): 27-38.
[38] Michalowski, R. L. Stability charts for uniform slopes. Journal of Geotechnical and Geo-environmental Engineering, ASCE, 2002, 128(4): 351-355.
[39] Viratjandr, C.., and Michalowski, R. L. Limit analysis of submerged slopes subjected to water drawdown. Can. Geotech. J., 2006, 43: 802-814.

[40] Cundall P.A. and Strack O.D.L. The distinct numerical model for granular assemblies. Géotechnique, 1979, 29: 47–65.
[41] Thorn ton C. Numerical simulation of deviatoric shear deformation of granular media. Geotechnique, 2000, 50(1): 43–53.
[42] Anandarajah A. On influence of fabric anisotropy on the stress-strain behaviour of clays.Computers and Geotechnics, 2000, 27: 1–17.
[43] Bardet J. P. and Proubet J. A numerical investigation of the structure of persistent shear bands in granular media. Géeotechnique, 1991, 41: 599–613.
[44] Iwashita K.and Oda M.. Rolling resistance at contacts in simulation of shear band development by DEM. Journal of Engineering Mechanics, ASCE, 1998.
[45] Kiyama H., Nishimura T.and Fujimura H..Adv anced distinct element model coupling with pore water. Dob oku Gakkai Rombun-Hokokushu/Proceedings of the Japan Society of Civil Engineers, 1994, 499 (III-28), pp.31–39.
[46] Ng T. T.and Dobry R., A nonlinear numerical model for soil mechanics. Journal of Geotechnical Engineering, ASCE, 1994, 120(2): 388–403.
[47] O. C. Zienkiewicz, C. Humpheson & R. W. Lewis, Associated and non-associated visco-plasticity and plasticity in soil mechanics, Géotechnique 25, No 4, 1975, pp. 671-689.
[48] A. Preh, Modellierung des verhaltens von massenbewegungen bei großen verschiebungen mit hilfe des Particle Flow Codes (PFC), Thèse de doctorat, Université de technologie, Vienne, 2004.
[49] R. Hamza-Cherif. Étude des mouvements de pentes par le code de calcul PFC2D, mémoire de Magister, Tlemcen, Algérie, 2009.