Abstract—In this paper, we study the Minimum Latency Broadcast Scheduling (MLBS) problem in wireless sensor networks (WSNs). The main issue of the MLBS problem is to compute schedules with the minimum number of timeslots such that a base station can broadcast data to all other sensor nodes with no collisions. Unlike existing works that utilize the traditional omni-directional WSNs, we target the directional WSNs where nodes can collaboratively determine and orientate their antenna directions. We first develop a 7-approximation algorithm, adopting directional WSNs. Our ratio is currently the best, to the best of our knowledge. We then validate the performance of the proposed algorithm through simulation.

Keywords—Broadcast, collision-free, directional antenna, approximation, wireless sensor networks.

I. INTRODUCTION

Wireless Sensor Networks (WSNs) consist of wireless sensor devices whose power source is usually in the form of a battery. These sensor devices are called wireless nodes, and they are scheduled to turn on their power to emit radio signals, or to turn it off to conserve energy for specific time duration.

One important task of a WSN is to disseminate data from a base station to all other nodes in the network periodically. This type of application is commonly known as broadcasting in the literature. During the process of broadcasting, a node sends (forwards) data to other nodes by emitting its radio signal. However, a collision can occur at a node if the data transmission is interfered by signals concurrently sent by other nodes. In this case, the data should be re-transmitted. As the small-sized sensors still have limited energy resources, it is desirable to reduce the unnecessary retransmissions in order to prolong the network’s lifetime.

An interesting approach for broadcasting is to assign timeslots to sensor nodes to obtain a good schedule through which data can be disseminated without any collisions. As the broadcasting occurs periodically, reducing the latency of the schedule, that is, constructing a schedule with a minimum number of timeslots, becomes a fundamental problem. Such problem, whose objective is to construct a collision-free broadcasting schedule with minimum latency, is called the Minimum Latency Broadcast Scheduling (MLBS) problem.

In the literature, the MLBS problem has been actively investigated with two different power models: non-uniform power model, where each node can have various transmission power levels, and uniform power model, where every node is assigned a uniform transmission power level. With the non-uniform power model, Gandhi et al. [1] showed the first result of the NP-hardness of the problem, and a $O(1)$-approximation algorithm with latency bounded by $648R$, where $R$ is the network radius. Later, they improved the latency bound to be less than $400R$ in [2]. With the uniform-power model, Dessmar et al. [3] proposed a 2400-approximation algorithm with latency bounded by $2400R$, and later Huang et al. [4] proposed two approximation algorithms with the ratios of 51 and 24, whose latencies are bounded by $51R$ and $24R$, respectively. Then they improved the ratio to be 16 in [5]. An et al. [6] also proposed another approximation algorithm with the same ratio of 16. Huang et al. [7] again proposed a cell-based algorithm, which partitions a network into several hexagonal cells, and showed that the latency of the algorithm is bounded by $24R$ with the ratio of 24. Later, Krzywydzinski [8] proposed a distributed algorithm with latency bounded by $255R$ with the ratio of 258. Currently, Gandhi et al. [9], [10] achieved the best approximation ratio of 12 for the MLBS problem with uniform-power model.

Note that these existing works have studied the MLBS problem in the traditional omni-directional WSNs, where every node is equipped with omni-directional antenna with a beam-width $\theta = 360^\circ$. The omni-directional WSNs are commonly modeled as undirected graphs, where any two nodes are connected via an undirected communication edge if they are covered by each other's broadcasting range. Let us assume that a sender node $v$ sends data to its receiver node $u$. If there is any other simultaneously sending node $w$ covering $u$ in its broadcasting range as well, then a collision occurs at $u$, and the data sent from $v$ is not delivered to $u$ successfully and should be re-transmitted (See Fig. 1 (a)).

Recently, advances in networks have led to the development of new wireless sensor devices equipped with directional antenna with a beam-width $\theta \in (0, 360^\circ]$. WSNs, consisting of such nodes that can collaboratively determine and orientate their antenna directions, are called directional WSNs. The directional WSNs are commonly modeled as directed graphs, where a directed edge exists from nodes $v$ to $u$ if $u$ resides in the broadcasting sector of $v$. In such networks, if we orient the antennas of nodes $v$ and $w$ as seen in Fig. 1 (b), then the two nodes $v$ and $w$ can send data simultaneously without causing any collisions.

In this paper, we study the MLBS problem in the directional wireless sensor networks with the uniform power model. We first develop an approximation algorithm with $7$-approximation ratio whose latency is bounded by $7R$, and then validate the performance of the proposed algorithm through simulation.

The rest of this paper is organized as follows. In Section II, we introduce the two main classes of techniques to equip directional antennas in wireless sensor networks, describe our network models, and then define the MLBS problem. In Section III, we describe our constant factor approximation algorithm for the MLBS problem, and analyze the algorithm.
In Section IV, we evaluate the latency performance of the algorithm with simulated networks. Finally, we conclude with some remarks in Section V.

II. PRELIMINARIES

In this section, we introduce the two main classes of techniques to equip directional antennas in wireless sensor networks. Then, we describe our network model, and define the MLBS problem.

A. Directional Antenna Models

First, we review the two classes of techniques to equip directional antennas in wireless sensor networks: steer beam directional antenna system and switch beam directional antenna system.

- Steer beam directional antenna model [11]: In this model, each node is equipped with directional sending antenna and omni-directional receiving antenna. The sensing range sec (i.e., a broadcasting sector) with a beam-width θ ∈ (0, 360°] is steered to some direction for transmission.

- Switch beam directional antenna model [11]: In this model, each node is equipped with switch beam directional sending antenna and omni-directional receiving antenna. Each node has K fixed transmission directions (i.e., a broadcasting sector, denoted by seck, 1 ≤ k ≤ K, whose central angle is θ), and it can switch on one sector for transmission (i.e., the node is scheduled to be active to work to the secK direction.) Commercially available sectored antennas are typically designed for beam-widths of 180°, 120°, 90°, 60° and 45° [12].

B. Network Model

In this paper, we consider a wireless sensor network (WSN) that consists of a set V of sensor nodes deployed in a plane. Each node u ∈ V is assigned a transmission power level p(u) and equipped with a switch beam directional antenna with a fixed beam-width θ and omni-directional receiving antenna. Accordingly, a directed edge (u, v) exists from node u to node v, if v resides in secK(u), where secK(u) denotes a broadcasting sector with an angle of θ centered at u with radius p(u). Let CsecK(u) = {v | v ∈ V and v resides in secK(u)} denote the set of nodes that reside in secK(u). Then, the collision is said to occur at node w if there exist concurrently sending nodes u and u’ such that w ∈ CsecK(u) and w ∈ CsecK(u’).

C. Problem Definition

The MLBS problem is defined as follows. Given a set of nodes in a plane, we assign each node a timeslot and activate its one of antenna sectors. The goal is to compute a schedule with the minimum number of timeslots such that a base station can broadcast data to all other sensor nodes with no collisions. A schedule is defined as a sequence of such timeslots at each of which several sender nodes sij are scheduled to successfully broadcast data to their neighbors that reside in secK(sij) using transmission power p(sij) with no collisions. Formally, at each timeslot t, we have an assignment set πt = {(sij, secK(sij), p(sij)), · · · , (sM, secK(sM), p(sM))}, 1 ≤ i ≤ m, where m denotes the number of nodes scheduled at timeslot t.

A broadcast schedule is a sequence of assignment sets Π = (π1, π2, ..., πM), where M is the length of the schedule, also called latency. A broadcast schedule Π is successful if data sent from a base station c ∈ V is broadcasted to every node v ∈ V \ {s}. The MLBS problem is formally defined as follows:

**Input:** A set V of nodes, and transmission power level p(v) for every node v ∈ V

**Output:** A successful minimum latency schedule Π

III. CONSTANT FACTOR APPROXIMATION ALGORITHM

In this section, we describe our new constant factor approximation algorithm for the MLBS problem. We assume the uniform power model, where all nodes are initially assigned a uniform power level r, i.e., p(u) = r, for every u ∈ V. We also assume that every node u ∈ V \ {c}, i.e., except the base station c, is equipped with a switch beam directional antenna with a fixed beam-width θ = 60°.
Accordingly, broadcasting sector of each node $u \in V \setminus \{s\}$ is assumed to be partitioned into 6 sectors, each of which is identified as $sec_k$, $k = 1, \cdot \cdot \cdot , 6$, and the central angle of each sector is $60^\circ$, as shown in Fig. 2. Note that the base station has much more energy and communication resources than other sensor nodes. Therefore, we assume the base station is exceptionally equipped with the omni-directional antenna with beam-width $60^\circ$.

See Table I for notations.

### A. Algorithm

We start this section by introducing some standard notations that are used subsequently [13] (cf. [14]):

- **Graph Center**: Given a communication graph $G = (V, E)$, we call a node $c$ a *center* node if the hop distance from $c$ to the farthest node from $c$ is minimum.

- **Maximal Independent Set (MIS)**: A subset $V' \subseteq V$ of the graph $G$ is said to be *independent* if for any vertices $u, v \in V'$, $(u, v) \notin E$. An independent set is said to be *maximal* if it is not a proper subset of another independent set.

- **Connected Dominating Set (CDS)**: A dominating set (DS) is a subset $V' \subseteq V$ such that every vertex $v$ is either in $V'$ or adjacent to a vertex in $V'$. A DS is said to be *connected* if it induces a connected subgraph.

In our proposed algorithm shown in Algorithm 1, we assume that the initial communication graph, modeled as a bidirectional unit disk graph $G = (V, E)$, whose $E = \{(u, v) \mid d(u, v) \leq r\}$, is connected. We further assume that a center node of $G$ is the base station $c$ in order to obtain a latency bound in terms of the network radius $R$ rather than its diameter $D$ as the lower bound of the MLBS problem is $R$. (See Lemma 3 in Section III-B.)

### Algorithm I MLBS-Dir

**Input**: A set $V$ of nodes in a plane, $p(u) = r$ for every node $u \in V$, and a base station $c \in V$.

**Output**: Schedule $\Pi$.

1. Construct a UDG $G = (V, E)$, where $E = \{(u, v) \mid d(u, v) \leq r\}$.
2. Broadcast tree $T(V, E_T) \leftarrow$ Data aggregation tree of $G$ rooted at $c$ as constructed in [15].
3. Starting timeslot $t \leftarrow 1$
4. $\pi_t \leftarrow \pi_{t-1} \cup (c, \text{broadcasting range of } c, r)$
5. $t \leftarrow t + 1$
6. for $i = 1$ to $R$ do
7. $S_i \leftarrow \{u \mid u \text{ is a dominator or connector, } \ell(u) = i, \text{ and } u \in V\}$
8. if $S_i \neq \emptyset$ then
9. for $k = 1$ to $6$ do
10. $\text{isTimeslotUsed} \leftarrow \text{false}$
11. for every $u \in S_i$ do
12. if $\{\{(u, v) \mid v \text{ resides in } \text{sec}_k(u)\}, \ell(v) = \ell(u) + 1, \text{ and } (u, v) \in E_T\} > 0$ then
13. $\text{isTimeslotUsed} \leftarrow \text{true}$
14. $\pi_t \leftarrow \pi_{t-1} \cup (u, \text{sec}_k(u), r)$
15. end if
16. end for
17. if $\text{isTimeslotUsed}$ then
18. $t \leftarrow t + 1$
19. end if
20. end for
21. end if
22. end for
23. $M \leftarrow t - 1$
24. return $\Pi \leftarrow (\pi_1, \pi_2, \cdot \cdot \cdot , \pi_M)$

Algorithm I describes the details of the Minimum Latency Broadcast Scheduling with Directional Antennas (MLBS-Dir) algorithm which works as follows.

The algorithm starts by constructing *broadcast tree* $T$ (Step 2), which is a data aggregation tree constructed as in [15]. The details of the construction of $T$ is as follows. We first construct a breadth-first-search (BFS) tree (cf. [14]) on $G$ rooted at the center node, i.e., base station, $c$. Based on the BFS tree obtained, $T$ is constructed by first finding an MIS level by level on the BFS tree using the algorithm in [16]. Let us call the nodes in the MIS *dominators*, and the remaining nodes *dominatees*. In Fig. 3 (a), the dominators in the MIS are denoted by black nodes, and the dominatees are denoted by gray nodes. Here, the constructed MIS guarantees that the shortest hop-distance between any two complementary pairs,
A and MIS \( A \), where \( A \subseteq MIS \), is exactly two hops. For example, in Fig. 3 (a), consider one pair \( A = \{v_1, v_2\} \) and \( B = MIS \setminus A = \{v_3, v_4, v_5, c\} \). The shortest hop-distance between the sets \( A \) and \( B \) on \( G \) is exactly two hops. Then, as Li et al. [15] proposed, we obtain a CDS of \( G \) by connecting the dominators using dominatees. The dominatees used to connect dominators are renamed connectors. If there exist some remaining dominatees that are not connected to the CDS, then each of the dominatees is connected to its neighboring dominator. For example, in Fig. 3 (b), the bolded edges represent the CDS, and in Fig. 3 (c), white nodes represent dominatees and the bolded edges represent dominators. We then use this newly constructed tree \( T \) as the broadcast tree to guide to find a minimum latency schedule \( \Pi \) in our algorithm.

Then, the root \( c \) broadcasts its data to its neighbors at the timeslot 1 (Step 4).

Next, Steps 6-22 are iterated \( R \) times (from level 1 to level \( R \)) to schedule the nodes at each level, as follows.

- **Scheduling sender nodes at level \( i \) at iteration \( i \) - Connector to Dominator:**
  During the iteration, every node in the sender set \( S_i \) is scheduled, where the set \( S_i \) consists of connectors at level \( i \). For every connector \( u \in S_i \), only the lower level dominators of \( u \) need to receive data. So, if any connectors have their dominators at \( sec_1 \), then the algorithm assigns the same timeslot, say \( t \), to them to activate \( sec_1 \). Next, if any connectors have their dominators at \( sec_2 \), then the algorithm assigns the next timeslot, \( t+1 \), to them to activate \( sec_2 \), and repeat until all sectors of connectors are examined thereby assigning appropriate timeslots (Steps 9-20).

  - **Scheduling sender nodes at level \( i+1 \) at iteration \( i+1 \) - Dominator to Connectors and Dominatees:**
    During the next iteration, every node in the sender set \( S_{i+1} \) is scheduled, where the set \( S_{i+1} \) consists of dominators at level \( i+1 \). So, a dominator \( u \) at level \( i+1 \) is assigned the timeslot, say \( t \), to activate \( sec_1 \) if there exist any lower level neighbors in \( sec_1 \), the timeslot \( t+1 \) to activate \( sec_2 \) if there exist any lower level neighbors in \( sec_2 \), and so on. Lastly, \( u \) is assigned the timeslot \( t+5 \) if there exist any neighbors in \( sec_6 \) (Steps 9-20).

In order to avoid using timeslots to activate sectors in which no neighbors reside, the algorithm activates timeslots only for the sectors in which at least one receiver resides (Step 12). The algorithm repeats the above iterations until the last level’s nodes are scheduled (Steps 6-22).

Figs. 3 (d)-(h) illustrates the steps of scheduling nodes at each level.

- **Base station scheduling (Fig. 3 (d)):**
  At the very first step, the base station \( c \) is assigned
timeslot 1 for its receivers $v_1, v_2, v_3, v_4$, and $v_5$.

- **Iteration $i = 1$ (Fig. 3 (e)):**
  The sender set $S_1 = \{u_1, u_2, u_3\}$ consists of only the connectors whose level is 1. First, $u_1$ is assigned timeslot 2 for its receiver $v_1$ in $sec_1(u_1)$, and timeslot 3 for $v_2$ in $sec_3(u_1)$. Next, $u_2$ is assigned the timeslot 4 for its receiver $v_3$ in $sec_4(u_2)$. Then, $u_3$ is assigned the next timeslot 5 for receiver $v_4$ in $sec_5(u_3)$.

- **Iteration $i = 2$ (Fig. 3 (f)):**
  The sender set $S_2 = \{u_1, u_2, u_3, u_4\}$ consists of only the dominators whose level is 2. First, $u_1$ and $u_2$ are assigned the same timeslot 6 for their receivers $v_1$ and $v_2$ in $sec_1(u_1)$ and $sec_1(u_2)$, respectively. Next, $u_2, u_3$ and $u_4$ are assigned the same timeslot 7 for their receivers $v_3$ in $sec_4(u_2)$, $v_5$ in $sec_4(u_3)$, and $v_6$ and $v_7$ in $sec_4(u_4)$, respectively. Then, $u_4$ is assigned the next timeslot 8 for receiver $v_8$ in $sec_8(u_4)$.

- **Iteration $i = 3$ (Fig. 3 (g)):**
  The sender set $S_3 = \{u_1\}$ consists of a connector whose level is 3. $u_1$ is assigned the next timeslot 9 for $v_1$ in $sec_9(u_1)$.

- **Iteration $i = 4$ (Fig. 3 (h)):**
  The sender set $S_4 = \{u_1\}$ consists of a dominator whose level is 4. $u_1$ is assigned the next timeslot 10 for $v_1$ in $sec_1(u_1)$.

B. Analysis

In this section, we analyze the MLBS-Dir algorithm (Algorithm 1) and bound the latency of the resulting broadcasting schedules.

**Lemma 1:** No collision occurs when two dominators $u$ and $v$, whose level $\ell(u) = \ell(v) = i$, simultaneously send their data to their lower level receivers, which reside in $sec_k(u)$ and $sec_k(v)$, respectively.

**Proof:** Consider two dominators $u$ and $v$ whose level $\ell(u) = \ell(v) = i$, and their receivers which are lower level dominatess and connectors in $sec_k(u)$ and $sec_k(v)$, respectively. Here, a collision occurs only when

- $u$ and $v$ send data simultaneously (i.e., $sec_k(u)$ and $sec_k(v)$ are activated at the same time),
- $sec_k(v)$ covers $u$ and $sec_k(u)$ covers $v$.

As illustrated in Fig. 4, the above conditions imply that $d(u', v') \leq r$, which is contraction because the distance between two dominators must be greater than $r$. Therefore, no collision occurs at $u'$ and $v'$ when $u$ and $v$ send data simultaneously.

**Lemma 2:** No collision occurs when two connectors $u$ and $v$, whose level $\ell(u) = \ell(v) = i$, simultaneously send their data to their lower level dominators, which reside in $sec_k(u)$ and $sec_k(v)$, respectively.

**Proof:** Consider two connectors $u$ and $v$, and their receivers which are lower level dominators, $u'$ in $sec_k(u)$ and $v'$ in $sec_k(u)$, respectively. Here, a collision occurs only when

- $u$ and $v$ send data simultaneously (i.e., $sec_k(u)$ and $sec_k(v)$ are activated at the same time),
- $sec_k(v)$ covers $u'$ and/or $sec_k(u)$ covers $v'$.

**IV. SIMULATION**

In this section, we examine the latency performance of our proposed algorithm, the MLBS-Dir (Algorithm 1) in terms of the number of nodes, network (graph) radius, and transmission power level.

In this simulation, we generated a set $G = \{G_n | n = 50, 100, 150, 200, \ldots, 500\}$, where $G_n = \{G_i^k | 1 \leq i \leq 100\}$ consists of 100 different networks, $G_i^k$, $G^k_{2i}$, $G^k_{100}$, each of which has $n$ nodes. All networks were generated randomly in the Euclidean plane of dimension $500 \times 500$. For each $G_n \in G$, we averaged the latencies produced by the MLBS-Dir algorithm over all the 100 networks. For the initial power assignment of nodes in the networks, we first computed a minimum spanning tree $T_{MST}$ using edge weights defined as the distance between every two nodes. Then, we set the maximum transmission range $r$ to be the distance of the longest edge in $T_{MST}$, and the uniform power level $p(u)$ for every $u \in V$ to be $r$. Notice that $r$ is the minimally required power level to get a connected graph.

First, the Fig. 5 (a) shows the latency performance of the MLBS-Dir algorithm, in which as the network size increases the latency also increases. In the boxplot, red circles represent
The average latency for each node size $n$, and the horizontal line in the middle in each box represents the median latency.

The $+$ symbols represent outliers.

Next, the Fig. 5 (b) shows our theoretical latency bound $7R$ (black dotted diagonal line) and the latencies obtained for the randomly generated 2000 different networks (red dots) in $G$. For every network, we computed the ratio of the latency to the lower bound $R$. The minimum ratio computed is 1.1667, the mean ratio is 1.6044, and the maximum ratio is 3.8889. Also, the slope of the simple regression line (the blue line over the red dots) is only 1.5352. Here, it can be observed that the ratios of the latencies to $R$ in the simulation are much lower than our theoretical ratio 7 (See Theorem 4).

Lastly, in order to evaluate the effect of the transmission power level $p(u)$, $u \in V$, we fixed $n = 500$, and varied $p(u)$. Note that as $p(u)$ increases, the network radius $R$ decreases. It also implies that as $p(u)$ increases, the network becomes denser. Fig. 5 (c) with a single network shows the latency performance of the MLBS-Dir algorithm with the fixed $n = 500$ and various $p(u) = \{r, 2r, \ldots, 20r\}$, $u \in V$. Here, as $p(u)$ increases, $R$ decreases, and thus the latency $M$ also decreases as it can be observed from the Lemma 3 and the Theorem 4.

V. CONCLUSION

In this paper, we focused on the Minimum Latency Broadcast Scheduling (MLBS) problem in the directional wireless sensor networks. We developed a 7-approximation algorithm yielding schedules, whose latency is bounded by $7R$, where $R$ is the network radius. We then evaluated the performance of the proposed algorithm with simulated networks and discussed experimental results. For future work, we plan to study other related problems such as data collection and data aggregation for directional wireless sensor networks.

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