Application of the Total Least Squares Estimation Method for an Aircraft Aerodynamic Model Identification

Zaouche Mohamed, Amini Mohamed, Foughali Khaled, Aitkaid Souhila, Bouchiha Nihad Sarah

Abstract—The aerodynamic coefficients are important in the evaluation of an aircraft performance and stability-control characteristics. These coefficients also can be used in the automatic flight control systems and mathematical model of flight simulator. The study of the aerodynamic aspect of flying systems is a reserved domain and inaccessible for the developers. Doing tests in a wind tunnel to extract aerodynamic forces and moments requires a specific and expensive means. Besides, the glaring lack of published documentation in this field of study makes the aerodynamic coefficients determination complicated. This work is devoted to the identification of an aerodynamic model, by using an aircraft in virtual simulated environment. We deal with the identification of the system, we present an environment framework based on Software In the Loop (SIL) methodology and we use Microsoft® Flight Simulator (FS-2004) as the environment for plane simulation. We propose The Total Least Squares Estimation technique (TLSE) to identify the aerodynamic parameters, which are unknown, variable, classified and used in the expression of the piloting law. In this paper, we define each aerodynamic coefficient as the mean of its numerical values. All other variations are considered as modeling uncertainties that will be compensated by the robustness of the piloting control.

Keywords—Aircraft aerodynamic model, Microsoft flight simulator, MQ-1 Predator, total least squares estimation, piloting the aircraft.

I. INTRODUCTION

In literature, the aeronautical system identification is widely explored. Particularly in avionics, in this field, the aerodynamic coefficients identification from the flight data was successfully done by the physical model interlud [1], [2]. The parameters of an aircraft aerodynamic model are variable because the aerodynamic coefficients changes. The techniques employed to obtain the different values of the aerodynamic coefficients derivatives are based on the four information sources:

1. Handbook methods
2. Computational fluid dynamics
3. Wind tunnel
4. Flight testing

In the first two sources, the aerodynamic coefficients derivatives are based on theoretical calculations. In wind tunnel experiments, the aerodynamic derivatives are more or less directly measurable through systematically varying one model state at a time and observing the resulting changes in the aerodynamic forces and moments obtained from the wind tunnel balance measurements.

Unfortunately, the straightforward procedure applied in wind tunnel experiments does not carry over to flight testing. In free flight, the aircraft is not attached to a balance and the aerodynamic forces and moments cannot be measured directly. Measuring all aircraft state elements, which is relatively simple in wind tunnel testing, becomes a non-trivial problem in flight testing. Certain state elements may be too cumbersome to be measured directly. Moreover, systematically changing one state variable at a time is not feasible (for example, changing the elevator deflection angle \( \delta_e \) during flight, will immediately result in a pitch rate and angle-of-attack response as well). As a result of these difficulties, the extraction of aerodynamic derivatives from flight data is a complicated matter which is most efficiently solved with the application of the system identification techniques [3].

In this paper, we have to identify the aerodynamic aircraft model. The dynamic model of this system is nonlinear, MIMO and coupled. It’s composed by six states variables: the roll, pitch and yaw rate \( p, q, r \) around body axes in \( \text{rad/s} \) and the airspeed components \( u, v, w \) along the body axes in \( \text{m/s} \).

The first three components are provided by the gyro meter, after their processing we can use them in the control algorithms, in the opposite, the last three components are unknown which forces us to estimate them.

II. PROBLEM STATEMENT

Through a methodology based on the confrontation between the real and the simulated world, the main objective of the present work is to identify the aerodynamic model of an aircraft flying in a virtual environment (Fig. 1). To achieve this objective, we use Flight Simulator FS2004 as simulated world environment, coupled to a hardware and software development platform. Flight Simulator FS2004 is developped by Microsoft and it has a worthy simulated aircrafts library. To identify the aerodynamic aircraft model, we propose the following approach:

- Implementation of a real time interface between the flight simulator FS2004 and the real time Windows target module of Simulink/Matlab.

M. Zaouche is with the Technology Department, Centre de Recherche et Développement Réghaia, Algeria (phone: 00213551391389, e-mail: zaouchemohamed@yahoo.fr).

M. Amini, K. Foughali, S. Aitkaid and N. S. Bouchiha are researchers at Centre de Recherche et Développement Réghaia.
- Description and analysis of the aerodynamic model.
- Development and implementation of the identification techniques based on TLSE for the identification of the aerodynamic parameters.

- Flight tests.
  We choose the MQ-1 Predator airplane which is used in reconnaissance or attack.

III. CHARACTERISTICS OF THE UNMANNED AERIAL VEHICLE MQ-1 PREDATOR

Airwrench tool gives access to flight dynamic characteristics (mudpond.org/Air Wrench/main.htm). This tool allows creating and tuning flight dynamics files description of simulated planes models. This software uses aerodynamics formulas and equations described on the Mudpond Flight Dynamics Workbook. It calculates aerodynamic coefficients based on the physical characteristics and performance of the aircraft (Table I).

IV. IMPLEMENTATION OF A REAL-TIME INTERFACE BETWEEN MICROSOFT FLIGHT SIMULATOR AND THE “REAL TIME WINDOWS TARGET” MODULE OF SIMULINK/MATLAB

We design our Software to interface the simulated aircraft in Flight Simulator environment (read and/or write many sensors, actuators data and parameters).

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Fixed Pitch propeller</th>
<th>Moments of inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length: 11.88 M</td>
<td>Prop Diameter: 1.92 M</td>
<td>Pitch: 1800.00</td>
</tr>
<tr>
<td>Wingspan: 14.84 M</td>
<td>Prop Gear Ratio: 1.00</td>
<td>Roll: 3700.00</td>
</tr>
<tr>
<td>Wing Surface Area: 11.43 M2</td>
<td>Tip Velocity: 1.478 Mach</td>
<td>Yaw: 1800.00</td>
</tr>
<tr>
<td>Wing Root Chord: 1.55 M</td>
<td>Prop Blades: 2</td>
<td>Cross: 0.00</td>
</tr>
<tr>
<td>Aspect Ratio: 19.28</td>
<td>Beta Fixed Pitch: 20.00deg</td>
<td></td>
</tr>
<tr>
<td>Taper Ratio: 0.10</td>
<td>Prop Efficiency: 0.870</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Design Altitude: 1524.0 M</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1 Real trajectory

Fig. 2 Aircraft and environment visualization

Fig. 3 Block diagram of the software environment design
We communicate with FS2004 by using a dynamic link library called FSUIPC.dll (Flight Simulator Universal Inter Process Communication). This library created by Peter Dowson is downloadable from his website [20] and can be installed by being copied in the directory (module) of FS2004. It allows external applications to read and write in and from Microsoft Flight Simulator MSFS by exploiting a mechanism for IPC (Inter-Process Communication) using a buffer of 64 Kb. The organization of this buffer is explained in the documentation given with FSUIPC, from which Table II is taken.

To read or write a variable, we need to know its address in the table, its format and the necessary conversions. For example, the indicated air speed is read as a signed long S32 at the address 0x02BC.

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>FLIGHT PARAMETERS IN THE BUFFER FSUIPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adress</td>
<td>Name</td>
</tr>
<tr>
<td>6010</td>
<td>Latitude (λ)</td>
</tr>
<tr>
<td>6018</td>
<td>Longitude (µ)</td>
</tr>
<tr>
<td>6020</td>
<td>Altitude (h)</td>
</tr>
<tr>
<td>057C</td>
<td>Bank angle (φ)</td>
</tr>
<tr>
<td>0578</td>
<td>Elevation angle (θ)</td>
</tr>
<tr>
<td>0578</td>
<td>Head angle (ψ)</td>
</tr>
<tr>
<td>30B0</td>
<td>Rotation rate (p)</td>
</tr>
<tr>
<td>30A8</td>
<td>Rotation rate (q)</td>
</tr>
<tr>
<td>30B8</td>
<td>Rotation rate (r)</td>
</tr>
<tr>
<td>0842</td>
<td>Acceleration (a x)</td>
</tr>
<tr>
<td>02BC</td>
<td>Speed IAS (V)</td>
</tr>
<tr>
<td>057C</td>
<td>Elevation angle (θ)</td>
</tr>
<tr>
<td>0578</td>
<td>Head angle (ψ)</td>
</tr>
<tr>
<td>0842</td>
<td>Vertical speed (Vz)</td>
</tr>
<tr>
<td>02BC</td>
<td>Speed IAS (V)</td>
</tr>
<tr>
<td>057C</td>
<td>Elevation angle (θ)</td>
</tr>
<tr>
<td>0578</td>
<td>Head angle (ψ)</td>
</tr>
<tr>
<td>0BB2</td>
<td>Elevator deflection (δe)</td>
</tr>
<tr>
<td>0BB6</td>
<td>Aileron deflection (δa)</td>
</tr>
<tr>
<td>0BB8</td>
<td>Rudder deflection (δr)</td>
</tr>
<tr>
<td>088C</td>
<td>Thrust control (δx)</td>
</tr>
</tbody>
</table>

V. IDENTIFICATION PROCEDURE

Structuring the Aerodynamic Data Base Aerodynamic Models

The aerodynamic data are expressed in terms of three forces (drag \(X\), lift \(Z\), side force \(Y\)) and three moments (pitching moment \(M\), rolling moment \(L\), yawing moment \(N\)) that act on the aircraft. Next, the effect of dynamic pressure \(2V^2/\rho\) and aircraft size (expressed in terms of wing area \(S\), and mean aerodynamic chord \(c\) or wing span \(b\)) is eliminated through working with dimensionless aerodynamic forces and moments coefficients \(C_x\)–\(C_n\) [8], [9], [13]–[15].

\[
C_x = \frac{X}{\frac{1}{2}\rho V^2 S} \\
C_y = \frac{Y}{\frac{1}{2}\rho V^2 S} \\
C_z = \frac{Z}{\frac{1}{2}\rho V^2 S} \\
C_i = \frac{L}{\frac{1}{2}\rho V^2 S b} \\
C_m = \frac{M}{\frac{1}{2}\rho V^2 S c} \\
C_n = \frac{N}{\frac{1}{2}\rho V^2 S b}
\]

where; \(V\) : true airspeed \(\frac{m}{s}\), \(\rho\) : air density \(Kg/m^3\).

The aerodynamic coefficients \(C_x\)–\(C_n\), in (1) and (2) are functions of the time histories of the aircraft state, i.e. angle-of-attack \(\alpha\), angle-of-sideslip \(\beta\), the aircraft rotation rates \(p\), \(q\), \(r\), as well as the control surface deflections \(\delta_x\), \(\delta_y\), \(\delta_z\). The functional relationship between the aerodynamic coefficients and the state variables are expressed in terms of Taylor’s series expansions about a reference state. A representative example is [7], [8], [14], [15], [18], [19]:

\[
C_x = C_{x_{\alpha}} + C_{x_{\beta}} \delta_x + C_{x_{q}} \delta_q + C_{x_{r}} \delta_r + C_{x_{\alpha\beta}} \delta_x \delta_y + C_{x_{\alpha q}} \delta_x \delta_q + C_{x_{\alpha r}} \delta_x \delta_r + C_{x_{\beta q}} \delta_y \delta_q + C_{x_{\beta r}} \delta_y \delta_r + C_{x_{q r}} \delta_q \delta_r + C_{x_{\alpha q r}} \delta_x \delta_q \delta_r \tag{3}
\]

where,

\[
\delta_i = (x_i - x_{i0}) \quad \delta = (z_i - z_{i0})
\]
\(x_{\mu}, y_{\mu}, z_{\mu}\) are the positions of the specific force sensors.

**TABLE III**

<table>
<thead>
<tr>
<th>Subject</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerodynamic Coefficients</td>
<td>(m(\alpha_{o} \cos \alpha \cos \beta + a_x \sin \beta + a_y \sin \alpha \cos \beta) - F_{e} \cos \alpha_{o} \cos \beta_{o} = \frac{1}{2} \rho SV^2 )</td>
</tr>
<tr>
<td></td>
<td>(C_{x_s} + C_{x_{\alpha}} \alpha + C_{x_{\beta}} \beta^2 + C_{x_{\delta}} \delta_{s} + \frac{C_{x_{\theta_{\delta}}}}{V} + C_{x_{\dot{\delta}}} \delta_{s} )</td>
</tr>
<tr>
<td></td>
<td>(m(-a_x \sin \alpha + a_y \cos \alpha) - F_{e} \sin \alpha_{o} = C_{m_{\alpha}} + C_{m_{\alpha}} \alpha + \frac{C_{m_{\alpha_{\delta}}}}{V} \alpha \cos \delta_{s} )</td>
</tr>
<tr>
<td></td>
<td>(m(-a_y \cos \alpha \sin \beta + a_y \cos \beta - a_x \sin \alpha \sin \beta) + F_{e} \cos \alpha_{o} \sin \beta_{o} = \frac{1}{2} \rho SV^2 )</td>
</tr>
<tr>
<td></td>
<td>(C_{y_s} + C_{y_{\alpha}} \alpha + C_{y_{\beta}} \beta^2 + C_{y_{\delta}} \delta_{s} + \frac{C_{y_{\theta_{\delta}}}}{V} + C_{y_{\dot{\delta}}} \delta_{s} )</td>
</tr>
<tr>
<td></td>
<td>(\dot{m}<em>{T</em>{\alpha}} + q_{p}(I_{\alpha} - I_{\beta}) - (p^2 - r^2)I_{\alpha} - \Delta F_{a} = \frac{1}{2} \rho SV^2 )</td>
</tr>
<tr>
<td></td>
<td>(C_{m_{\alpha}} + C_{m_{\alpha}} \alpha + \frac{C_{m_{\alpha_{\delta}}}}{V} \alpha \cos \delta_{s} + C_{m_{\delta}} \delta_{s} )</td>
</tr>
<tr>
<td></td>
<td>(\dot{m}<em>{T</em>{\beta}} + q_{p}(I_{\beta} - I_{\gamma}) - (r^2 - p^2)I_{\beta} = \frac{1}{2} \rho SV^2 )</td>
</tr>
<tr>
<td></td>
<td>(C_{m_{\beta}} + C_{m_{\beta}} \beta + \frac{C_{m_{\beta_{\delta}}}}{V} \beta + C_{m_{\delta}} \delta_{s} + C_{m_{\dot{\delta}}} \delta_{s} )</td>
</tr>
</tbody>
</table>

\(a_x, a_y, a_z\) are the linear acceleration components.

In *Airwrench*, the aircraft engine position has a pitch and a yaw offset orientation angles. In the case of our Unmanned Aerial Vehicle “UAV” (MQ-1 Predator), the pitch setting is \(\alpha_{o} = 20\) degree = 0.349 radian, and the yaw setting is \(\beta_{o} = 0\). The engine propulsion force is written in the body frame reference [5]:

\[
F = F_{e} \left( \frac{\cos \beta_{o} \cos \alpha_{o}}{\cos \beta_{o} \sin \alpha_{o}} \right) \sigma_{e} = \frac{K_{a} \rho}{V_{e}} \tag{5}
\]

The modulus of the aerodynamic velocity is represented by \(V_{e}\) with \(K_{a}\) as a constant and \(\sigma_{e}\) representing the position of the throttle, between 0 and 1 inclusive. This verification model allows a linear formulation of the aerodynamic coefficients identification problem presented by the (9).

**TABLE IV**

<table>
<thead>
<tr>
<th>sensor</th>
<th>measured value</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerometer</td>
<td>(a_{x}, a_{y}, a_{z})</td>
<td>Acceleration in body coordinates.</td>
</tr>
<tr>
<td>Rate gyro</td>
<td>(p \cdot q \cdot r)</td>
<td>Angular velocity in body coordinates.</td>
</tr>
<tr>
<td>Alpha vane</td>
<td>(\alpha)</td>
<td>Angle of attack sensor.</td>
</tr>
<tr>
<td>Beta vane</td>
<td>(\beta)</td>
<td>Measures the flanks angle.</td>
</tr>
<tr>
<td>Pitot-static tube</td>
<td>(Q)</td>
<td>Measures dynamic pressure.</td>
</tr>
<tr>
<td>Aileron sensor</td>
<td>(\delta_{a})</td>
<td>Aileron deflection angle.</td>
</tr>
<tr>
<td>Elevator sensor</td>
<td>(\delta_{e})</td>
<td>Elevator deflection angle.</td>
</tr>
<tr>
<td>Rudder sensor</td>
<td>(\delta_{r})</td>
<td>Rudder deflection angle.</td>
</tr>
</tbody>
</table>
The estimation of the aerodynamic coefficients derivatives of an aircraft requires the data processing of flight. Consequently, these data are recorded in real-time and treated. Between the flights, measured variables can be traced to make sure that at least the sensors answered the movements of the aircraft. However, the estimated aerodynamic coefficients derivatives start as soon as the test routine of flight is finished. This after flight procedure of analysis of data does not modify the flight test results.

\[
Y_i = m[a, \cos \alpha \cos \beta + a, \sin \beta + a, \sin \alpha \cos \beta] - F_x \cos \alpha \cos \beta \\
Y_2 = m(-a, \sin \alpha + a, \cos \beta) - F_x \sin \alpha \sin \beta \\
Y_3 = m(a, \cos \alpha \sin \beta + a, \sin \beta - a, \sin \alpha \sin \beta) + F_x \cos \alpha \sin \beta
\]

\[
\Sigma = \begin{bmatrix} 0.5 \rho S \Sigma^2 & 0 & 0 \\ 0 & 0.5 \rho S \Sigma^2 & 0 \\ 0 & 0 & 0.5 \rho S \Sigma^2 \end{bmatrix}
\]

\[
\rho \Sigma = \begin{bmatrix} \rho \Sigma_{11} & \rho \Sigma_{12} & \rho \Sigma_{13} \\ \rho \Sigma_{21} & \rho \Sigma_{22} & \rho \Sigma_{23} \\ \rho \Sigma_{31} & \rho \Sigma_{32} & \rho \Sigma_{33} \end{bmatrix}
\]

\[
\begin{bmatrix} p \delta I_{xx} + q \delta I_{yy} - r \delta I_{zz} \\ q \delta I_{xx} - r \delta I_{yy} + p \delta I_{zz} \\ r \delta I_{xx} + p \delta I_{yy} - q \delta I_{zz} \end{bmatrix}
\]

\[
\begin{bmatrix} p \delta I_{xx} + q \delta I_{yy} - r \delta I_{zz} \\
q \delta I_{xx} - r \delta I_{yy} + p \delta I_{zz} \\
-r \delta I_{xx} + p \delta I_{yy} - q \delta I_{zz} \end{bmatrix}
\]

\[
\begin{bmatrix} p \delta I_{xx} + q \delta I_{yy} - r \delta I_{zz} \\
q \delta I_{xx} - r \delta I_{yy} + p \delta I_{zz} \\
-r \delta I_{xx} + p \delta I_{yy} - q \delta I_{zz} \end{bmatrix}
\]

A level of (4) and (6), we note that there is no sensors which provide the angular accelerations \( \tilde{\beta} \), \( \tilde{\gamma} \) and \( \tilde{\rho} \). We used a differentiator presented by [5]–[7], [12]:

\[
\dot{z}_i = z_{i+1} + \lambda_i \left[ y_i - f_i(t) \right] \left( \frac{y_i - f_i(t)}{n} \right)
\]

where, \( i = 1, 2, 3, \lambda_i > 0 \), \( i = 1, 2, 3, z_i = [p \quad q \quad r] \).

The modified aerodynamic coefficients are presented in Table V [7].

**VI. TOTAL LEAST-SQUARES METHOD**

### A. Introduction

The Total Least Squares (TLS) method has been devised as a more global fitting technique than the ordinary least squares technique for solving over determined sets of linear equations \( A\Theta = Y \) when errors occur in all data. This method, introduced into numerical analysis by Golub and Van Loan [4], is strongly based on the Singular Value Decomposition (SVD). TLS is an extension of the usual Least Squares method: it allows dealing also with uncertainties on the sensitivity matrix. In this paper the TLS method is analyzed with a robust use of the SVD decomposition technique, which gives a clear understanding of the sense of the problems and provides a solution expressed in closed form in the cases where a solution exists. We discuss its relations with the LS problem [16], [17] and give the expression for the parameters governing the stability of the solutions. At the end we present the algorithm for computing \( \Theta_{TLS} \), solution of the estimated aerodynamic coefficients problem.

### B. Algorithm

The following theorem gives conditions for the existence and uniqueness of a TLS solution.

**Theorem 1 [4], [10], [11]** (Solution of the classical TLS problem):

Let: \( C := [A \quad Y] = U \Sigma V^T \), where \( \Sigma = diag(\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{n-3}) \) is the Singular Value Decomposition (SVD) of \( C \) with \( \sigma_1 \geq \ldots \geq \sigma_{n-3} \) are the singular values of \( C \). The partitioning is defined as:
A TLS solution exists if and only if $V_{22}$ is non-singular. In addition, it is unique if and only if $\sigma_s \neq \sigma_{s+1}$.

In the case when TLS solution exists and is unique, it is given by:

$$\hat{\Theta}_{TLS} = -V_{22}^{-1}$$

and the corresponding TLS correction matrix is:

$$\Delta C_{TLS} = [A_{TLS}] \Delta Y_{TLS} = -U \cdot \text{diag}(0, \Sigma) V^T$$

In the generic case when a unique TLS solution $\hat{\Theta}_{TLS}$ exists, it is computed from the right singular vectors corresponding to the smallest singular values by normalization. This gives Algorithm 1 as a basic algorithm for solving the classical TLS problem. Note that TLS correction matrix $\Delta C_{TLS}$ is such that TLS data approximation

$$\hat{C}_{TLS} = C + \Delta C_{TLS} = U \cdot \text{diag}(\Sigma, 0) V^T$$

is the best rank-n approximation of $C$.

**Algorithm 1** Basic total least squares algorithm

**Input:** $A \in R^{n \times m}, Y \in R^{n \times d}$

1. Compute the singular value decomposition

   $$A = \begin{bmatrix} \alpha & 0 & 0 & 0 & 0 & \alpha^2 \frac{q_c}{V} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 & 0 & 0 & 0 & \frac{pb}{2V} & \frac{rb}{2V} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & \frac{qc}{V} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & \frac{pb}{2V} & \frac{rb}{2V} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & \frac{qc}{V} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & \frac{pb}{2V} & \frac{rb}{2V} \end{bmatrix}$$

   $$A_z = \begin{bmatrix} \delta_e & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta_a & \delta_r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_e & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_e & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_a & \delta_r \end{bmatrix}_{6 \times 9}$$

2: if $V_{22}$ is not singular then
3: Set $\hat{\Theta}_{TLS} = -V_{22}^{-1}$

4: else
5: Output a message that the problem (TLS) has no solution and stop.
6: end if
7: Output: $\hat{\Theta}_{TLS}$ a total least squares solution of $A\Theta = Y$

VII. APPLICATION OF THE TLSE METHOD FOR MQ-1 PREDATOR

**A. Problem Formulation**

The equations can be represented using vector and matrix notation,

$$Y = AX$$

where $Y$ is $(16 \times 1)$ dimensional vector of the variable-to-be-explained, $A$ is the $(37 \times 37)$ dimensional matrix of explanatory variables, and $X$ is the $(37 \times 1)$ dimensional vector of system parameters where,

$$Y = \begin{bmatrix} C_X & C_Y & C_Z & C_i & C_w & C_n \end{bmatrix}^T_{6 \times 1}$$

$$A = \begin{bmatrix} A_1 & A_2 & A_3 \end{bmatrix}_{6 \times 37}$$

$\Theta = \begin{bmatrix} \Theta_1, \Theta_2, \Theta_3, \Theta_4 \end{bmatrix}^T_{1 \times 4}$
\[ \Theta = \begin{bmatrix} C_{x_\theta} & C_{y_\theta} & C_{z_\theta} & C_{\mu_\theta} & C_{\nu_\theta} & C_{\kappa_\theta} \\ C_{x_\phi} & C_{y_\phi} & C_{z_\phi} & C_{\mu_\phi} & C_{\nu_\phi} & C_{\kappa_\phi} \end{bmatrix} \]

\[ \Theta_1 = \begin{bmatrix} C_{x_\gamma} & C_{y_\gamma} & C_{z_\gamma} & C_{\mu_\gamma} & C_{\nu_\gamma} & C_{\kappa_\gamma} \end{bmatrix} \]

\[ \Theta_2 = \begin{bmatrix} C_{x_\delta} & C_{y_\delta} & C_{z_\delta} & C_{\mu_\delta} & C_{\nu_\delta} & C_{\kappa_\delta} \end{bmatrix} \]

\[ \Theta_3 = \begin{bmatrix} C_{x_\epsilon} & C_{y_\epsilon} & C_{z_\epsilon} & C_{\mu_\epsilon} & C_{\nu_\epsilon} & C_{\kappa_\epsilon} \end{bmatrix} \]

This is accomplished through rewriting the linear model of (3) as,

\[ \begin{bmatrix} A \\ Y \end{bmatrix} \begin{bmatrix} \Theta \\ -1 \end{bmatrix} = 0 \]  

(17)

Both the vector of the variable-to-be-explained \( \hat{Y} \) and certain columns of the matrix of explanatory variables \( A \) stem from measurements which are subject to measurement errors. Under these circumstances, a clear distinction between true values and measured data must be made,

\[ \begin{bmatrix} A_m \\ \hat{Y}_m \end{bmatrix} = \begin{bmatrix} A_0 \\ \hat{Y}_0 \end{bmatrix} + \begin{bmatrix} \Delta A_0 \\ \Delta \hat{Y}_0 \end{bmatrix} \]  

(18)

where, index \( m \) is used to indicate the measurements, index 0 is used to indicate the true values, and prefix \( A \) is used to indicate measurement errors.

Notice that the linear relation (13) is valid for the true data but will, in general, not be valid for the measured data,

\[ \begin{bmatrix} A_0 \\ \hat{Y}_0 \end{bmatrix} \begin{bmatrix} \Theta_0 \\ -1 \end{bmatrix} = 0 \]

\[ \begin{bmatrix} A_m \\ \hat{Y}_m \end{bmatrix} \begin{bmatrix} \Theta_0 \\ -1 \end{bmatrix} \neq 0 \]  

(19)

Define the Singular Value Decomposition of the compound data matrix according to Theorem 1 as:

\[ \begin{bmatrix} \sigma_1 & & & & & \\ & \sigma_2 & & & & \\ & & \ddots & & & \\ & & & \sigma_m & & \\ & & & & 0 \end{bmatrix} = U \begin{bmatrix} \hat{Y} \\ Y \end{bmatrix} \]  

(20)

An estimated of the (most probable) extended and transformed parameter vector \( \hat{\Theta} \) must satisfy,

\[ \begin{bmatrix} \sigma_1 & & & & \end{bmatrix} \begin{bmatrix} \sigma_2 & & & & \\ \ddots & & & & \end{bmatrix} \begin{bmatrix} \sigma_m & & & & \end{bmatrix} = \begin{bmatrix} \sigma_1 & \sigma_2 & \ddots & \sigma_m & \end{bmatrix} \]  

(21)

or,

\[ \begin{bmatrix} \hat{\Theta} \\ 0 \end{bmatrix} = \lambda \ker \begin{bmatrix} \hat{A} \\ \hat{Y} \end{bmatrix} \]  

(22)

in which \( \lambda \) is a scalar multiplier used to make the last element of \( \sum_{m+1} \) equal to -1. The kernel of matrix \( \lambda \begin{bmatrix} \hat{A} \\ \hat{Y} \end{bmatrix} \) equals the last right singular vector \( \sum_{m+1} \).

B. Simulation Results

We note that the aircraft’s takeoff operation is executed manually (by keyboard and / or joystick). Then we make the real-time acquisition of sensor response signals developed by our code. The piloting controls are sent by using the PPJoy (virtual joystick).

Several flight tests were conducted by changing the simulation parameters (season, time of day, weather). We present some recorded signals taken from the Inertial Measurement Unit (IMU).

We present some results of aerodynamic coefficients derivatives. They are function of the time and their values are around the intrinsic values.

The mean values of aerodynamic coefficients derivatives are given in Table VI.
TABLE VI

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{x_1}$</td>
<td>-0.0314</td>
</tr>
<tr>
<td>$C_{x_2}$</td>
<td>132e - 06</td>
</tr>
<tr>
<td>$C_{y_1}$</td>
<td>-37.82</td>
</tr>
<tr>
<td>$C_{y_2}$</td>
<td>-0.93</td>
</tr>
<tr>
<td>$C_{z_1}$</td>
<td>-1.041</td>
</tr>
<tr>
<td>$C_{z_2}$</td>
<td>0.078</td>
</tr>
<tr>
<td>$C_{x_3}$</td>
<td>-3.23</td>
</tr>
<tr>
<td>$C_{y_3}$</td>
<td>0.9982</td>
</tr>
<tr>
<td>$C_{z_3}$</td>
<td>-1.852</td>
</tr>
<tr>
<td>$C_{x_4}$</td>
<td>-0.8129</td>
</tr>
<tr>
<td>$C_{y_4}$</td>
<td>-0.265</td>
</tr>
<tr>
<td>$C_{z_4}$</td>
<td>-3.104</td>
</tr>
<tr>
<td>$C_{x_5}$</td>
<td>-0.07652</td>
</tr>
<tr>
<td>$C_{y_5}$</td>
<td>-1</td>
</tr>
</tbody>
</table>

In the frame of this paper, an identification procedure based on free flight measurements was developed for the aerodynamic coefficients determination and tested for a piloting application of a UAV. Moreover, to increase the probability that the coefficients define the system’s aerodynamics over the entire range of test conditions and to improve the accuracy of the estimated coefficients, a multiple fit strategy was considered. This approach provides a common set of aerodynamic coefficients that are determined from multiple data series simultaneously analyzed, and gives a more complete spectrum of the system’s motion.

We have presented TLSE applied to the aerodynamic identification problem. This method is based on the use of the SDV decomposition. It has the interesting propriety of giving the best approximation of the augmented measurement matrix, by another matrix with the same dimension, but with a lesser range, in the sense of the least squares.

In addition to the dimension reducing propriety, the SDV has the advantage of being able to estimate the invert of any matrix, whether it is square or rectangular, and most of all, whether it is singular or not.

The SDV interpretation key is the weights distribution exam (singular values). The decreasing order of those weights allows us to say that the first modes contain the main proprieties of the considered data, more exactly, they are the modes that will catch the major part of the global variance of the data.

The obtained results $C_{x_1} ... C_{y_5}$ by TLSE, are defined as the mean values of those aerodynamic coefficients derivatives. All parametric variations $\Delta C_{x_1} ... \Delta C_{y_5}$ will be compensated by the robustness proprieties of the piloting law to be elaborated.

VIII. Conclusion

In addition to the decreasing order of those weights allows us to say that the first modes contain the main proprieties of the considered data, more exactly, they are the modes that will catch the major part of the global variance of the data.

The obtained results $C_{x_1} ... C_{y_5}$ by TLSE, are defined as the mean values of those aerodynamic coefficients derivatives. All parametric variations $\Delta C_{x_1} ... \Delta C_{y_5}$ will be compensated by the robustness proprieties of the piloting law to be elaborated.
REFERENCES


