Low Complexity Hybrid Scheme for PAPR Reduction in OFDM Systems Based on SLM and Clipping

V. Sudha, D. Sriram Kumar

Abstract—In this paper, we present a low complexity hybrid scheme using conventional selective mapping (C-SLM) and clipping algorithms to reduce the high peak-to-average power ratio (PAPR) of orthogonal frequency division multiplexing (OFDM) signal. In the proposed scheme, the input data sequence (X) is divided into two sub-blocks, then clipping algorithm is applied to the first sub-block, whereas C-SLM algorithm is applied to the second sub-block in order to reduce both computational complexity and PAPR. The resultant time domain OFDM signal is obtained by combining the output of two sub-blocks. The simulation results show that the proposed hybrid scheme provides 0.45 dB PAPR reduction gain at CCDF value of 10^{-2} and 52\% of computational complexity reduction when compared to C-SLM scheme at the expense of slight degradation in bit error rate (BER) performance.

Keywords—CCDF, Clipping, OFDM, PAPR, SLM.

I. INTRODUCTION

OFDM is a method of encoding digital data over a large number of narrow band orthogonal sub-carriers (N). Each symbol modulates a separate sub-carrier frequency and these modulated sub-carrier frequencies are multiplexed. The bandwidth of each sub-carrier is less than the coherence bandwidth of the channel in order to reduce the multipath fading effects. The OFDM system is highly spectral efficient, robust to impulse interference and channel variations. Therefore, it has been used in many wireless standards such as digital audio broadcasting/digital video broadcasting (DAB/DVB), WLAN and also considered as an important candidate in 4G systems. However, the system performance is limited by PAPR of OFDM signal. This high peak occurs when all the sub-carriers add up constructively at the inverse fast Fourier transform (IFFT) block.

To reduce PAPR, a number of solutions are provided in the literature [1], [2] which include clipping [3], clipping and filtering [4]-[6], coding method i.e. Golay complementary sequences [7] and Reed Muller codes [8]. Other approaches include tone injection [9], tone reservation [10], modified Muller companding [11], active constellation extension [12] and multiple signaling techniques such as SLM [13]-[15], partial transmit sequence (PTS) [16]-[18] and so on. SLM and PTS provide PAPR reduction without introducing distortion and SLM is most widely used method due to its simple structure. However, it suffers by more computational complexity.

Recently, hybrid schemes based on PTS and clipping [19], serial combination of SLM and clipping [20], clipping and SLM [21], [22], SLM and PTS [23] algorithms are applied to enhance the PAPR reduction performance of OFDM signal. However, these algorithms do not show any improvement in terms of computational complexity.

In this paper, to achieve both reduction in computational complexity and PAPR, a hybrid scheme comprising of the parallel combination of SLM and clipping algorithms is proposed, where the first half of the input data sequence is processed by nonlinear signal processing (clipping) algorithm and the second half of the input data sequence is processed by linear (SLM) algorithm. This method combines the advantages of both SLM and clipping algorithms. Simulation results demonstrate the effectiveness of the proposed scheme.

The rest of the paper is organized as follows. In Section II, the definition for PAPR of OFDM signal is given. Conventional clipping and SLM algorithms are discussed in Section III. In Section IV, the proposed hybrid scheme is presented. In Section V, computational complexity of the proposed scheme is analyzed. Simulation results are discussed in Section VI. Finally, in Section VII the concluding remarks are given.

II. PAPR OF OFDM SIGNAL

For one OFDM symbol with \( N \) number of sub-carriers, the continuous time OFDM signal can be expressed as

\[
x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X(n) e^{j2\pi n \Delta f t}, 0 \leq t \leq T
\]  

where \( X(n) \) is the data symbol of \( n^{th} \) sub-carrier, \( \Delta f \) is the frequency spacing between the adjacent sub-carriers and \( T \) is one OFDM symbol duration.

The PAPR of OFDM signal is defined as the ratio of the instantaneous peak power to the average power. For the continuous time signal \( x(t) \), it can be expressed as

\[
PAPR = 10 \log_{10} \max_{0 \leq t \leq NT} \frac{|x(t)|^2}{P_{av}}
\]
where \( P_{av} \) is the average power of the transmitted symbol.

In general, PAPR of a continuous time signal is not necessarily same as discrete time signal. However, it is closely approximated by over sampling the time domain signal with the factor \( L=4 \). The discrete time OFDM signal can be expressed as

\[
x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k)e^{j2\pi kn/N}, 0 \leq n \leq N - 1
\]

where

\[
X(k) = [X(0), X(1), ..., X(N-1)]^T
\]

PAPR of discrete time signal is defined as

\[
PAPR = 10 \log_{10} \left( \frac{\max_{0 \leq n \leq N-1} |x(n)|^2}{P_{av}} \right)
\]

According to the central limit theorem, for the larger value of \( N \), the real and imaginary parts of \( x(n) \) follow a Gaussian distribution with zero mean and variance of 0.5. The amplitude of OFDM signal follows a Rayleigh distribution.

PAPR of OFDM signal is characterized by the complementary cumulative distribution function (CCDF), which is defined as the probability that the PAPR of OFDM signal exceeds a given threshold \( PAPR_o \). CCDF of PAPR is given by

\[
CCDF = Pr(PAPR > PAPR_o) = 1 - (1 - e^{-PAPR_o})^N
\]

PAPR value exponentially increases for large value of \( N \).

### III. EXISTING PAPR REDUCTION TECHNIQUES

#### A. C-SLM [2]

![Fig. 1 Block diagram of C-SLM](image)

In C-SLM, \( U \) statistically independent data sequences are generated from the input data sequence \( X \) by multiplying it with \( U \) unique random phase sequences and then PAPR is calculated for each data sequence. Among \( U \) data sequences, one with minimum PAPR is selected for transmission as shown in Fig. 1.

The input data sequence \( X \) is expressed as an \( N \) dimensional vector i.e.

\[
X = [X_0, X_1, ..., X_{N-1}]^T
\]

and the phase rotation vector is denoted as

\[
B^u = [b_{u,0}, b_{u,1}, ..., b_{u,N-1}]^T, 1 \leq u \leq U
\]

As mentioned earlier, oversampling of each data sequence is essential for having a better approximation of the true PAPR and it can be done by inserting \((L-1)/N\) zeros in the middle of the frequency domain input data sequence.

\( U \) candidates are generated by performing component-wise multiplication of the input sequence \( X \) with the phase rotation vector \( B^u \) and is given by

\[
X^u = X \cdot B^u
\]

\[
X^u = [X_0^u, X_1^u, ..., X_{L=N-1}, X_{L=N-1}]^T, u = 1, 2, ..., U
\]

The elements in the phase vector consist of \{1, -1, j, -j\} and each symbol in the phase vector has unit magnitude in order to preserve the power. The \( U \) time domain data sequences are generated by taking IFFTs of the frequency domain vector represented as (10)

\[
x^u = \text{IFFT}(X^u)
\]

\[
x^u = \frac{1}{\sqrt{LN}} \sum_{n=0}^{LN-1} X_n \cdot b_{u,n} e^{j2\pi n\phi(n)}, 0 \leq n \leq LN - 1, u = 1, 2, ..., U
\]

Among the \( U \) data sequences, one with lowest PAPR is selected for transmission and the corresponding phase sequence index is transmitted to the receiver as side information (SI) to recover the transmitted signal.

#### B. Clipping Algorithm

Let \( x(n), n = 0, 1, ..., LN - 1 \) denote the output of \( LN \)-point IFFT. Clipping method limits the peak amplitude of complex OFDM symbol \( x(n) \) whenever it exceeds a certain threshold value (\( A \)), otherwise it allows the input signal for transmission as,

\[
x(n) = \begin{cases} 
  x(n), & |x(n)| \leq A \\
  A e^{j\phi(n)}, & |x(n)| > A
\end{cases}
\]

where \( \phi(n) \) is the phase of the input signal, \( A \) is the maximum amplitude of the clipped signal.

Clipping of OFDM signal introduces out-of-band radiation and it can be minimized by filtering of clipped signal. Many
clipping methods have been proposed [24] namely MIN clipping, ZERO clipping, MAX clipping, suboptimal MIN/MAX clipping. Among them, suboptimal MIN/MAX clipping provides good amount of PAPR reduction.

IV. PROPOSED LOW COMPLEXITY HYBRID SCHEME

In [21], Byung Moo Lee et al., theoretically analyzed BER performance of clipping with SLM over flat and frequency selective fading channels and concluded that the combined scheme based on clipping with C-SLM can achieve a good amount of PAPR reduction. However, no change in computational complexity.

In [23], C-SLM is applied for half of the input data sequence, whereas C-PTS algorithm is applied for the remaining half of the input data sequence. Though, their method achieves PAPR reduction, computational complexity lies between C-SLM and C-PTS.

In this paper, to reduce computational complexity and retain implementation simplicity, clipping and C-SLM schemes are utilized simultaneously. Clipping is applied for the first half of the input data sequence (0 to N/2-1) and C-SLM for the remaining half of the input data sequence (N/2 to N-1) to construct the new candidate with minimum PAPR as shown in Fig. 2. Hence, the overall performance of proposed algorithms does not interfere with individual algorithm performance. In addition, PAPR reduction performance of the proposed hybrid scheme is analyzed for different input data segmentation namely adjacent segmentation and interleaving segmentation [25].

A. Algorithm 1 (Algorithm with Adjacent Segmentation)

Step 1: The input data sequence $X$ is partitioned into two sub-blocks, namely $X_1$, $X_2$ and can be expressed as

$$X_1 = [X(0), X(1), ..., X(N/2 - 1)]^T$$ (13)

$$X_2 = [X(N/2), X(N/2 + 1), ..., X(N - 1)]^T$$ (14)

Step 2: LN/2 point IFFT is taken for the frequency domain input vector $X_1$, then clipping algorithm is applied to the obtained time domain signal $x_1$, by using (12) and the resultant signal is represented as $\overline{x}_1$.

Step 3: Second sub-block data sequence $X_2$ is multiplied with the $U$ number of phase sequences to generate $U$ time domain vectors all representing the same information $x_2^u$, ($1 \leq u \leq U$) by using (11). One candidate from $U$ candidates with minimum PAPR is chosen for transmission.

Step 4: The resultant time domain OFDM signal with minimum PAPR is obtained by combining the first sub-block clipped signal $\overline{x}_1$ with a second sub-block candidate $x_2^u$ as

$$x(n) = [\overline{x}_1 : x_2^u]$$ (15)

B. Algorithm 2 (Algorithm with Interleaving Segmentation)

Step 1: The input data block $X$ is partitioned into two sub-blocks, namely $X_{odd}, X_{even}$ respectively. $X_{odd}$ consists of odd indexed input symbols and $X_{even}$ consists of even indexed input symbols denoted as

$$X_{odd} = [X(1), X(3), ..., X(N - 1)]^T$$ (16)

$$X_{even} = [X(0), X(2), ..., X(N)]^T$$ (17)

Step 2: LN/2 point IFFT is taken for the input vector $X_{odd}$, then clipping algorithm is applied. The resultant time domain signal $\overline{x}_{odd}$ is obtained by using (12).

Step 3: Generate $U$ time domain vectors for the input vector $X_{even}$ by using (11) and one candidate from $U$ candidates is selected and combined with clipped signal as

$$x(n)' = [\overline{x}_{odd} : x_{even}^u]$$ (18)

V. ANALYSIS OF COMPUTATIONAL COMPLEXITY

In this section, computational complexity of the proposed hybrid scheme is analyzed and compared with C-SLM. In general, the total number of complex multiplications ($Nmul$) and additions ($Nadd$) required for one $LN$ point IFFT operation is given by

$$Nmul = (LN/2) \log_2 LN$$ (19)

$$Nadd = LN \log_2 LN$$ (20)

In C-SLM, $U$ number of LN-Point IFFT blocks are required to generate $U$ candidate signals. In the proposed scheme, since only half of the input sequence (N/2) is applied to the C-SLM
algorithm, it requires only the $U$ number of $LN/2$ point IFFT blocks. Therefore, the total number of IFFT blocks required for the proposed method is $(U+1) LN/2$ and the total number of complex multiplications ($Nmul$) and additions ($Nadd$) required for the proposed method are given by

$$Nmul = LN / 4 \times \log_2 LN / 2 + ULN / 4 \times \log_2 LN / 2 \quad (21)$$

$$Nadd = LN / 2 \times \log_2 LN / 2 + ULN / 2 \times \log_2 LN / 2 \quad (22)$$

The computational complexity reduction ratio (CCRR) [16] of the proposed algorithm over C-SLM is defined as,

$$CCRR = \left(1 - \frac{\text{Complexity of Proposed algorithm}}{\text{Complexity of C-SLM}}\right) \times 100\% \quad (23)$$

<table>
<thead>
<tr>
<th>S. No</th>
<th>C-SLM</th>
<th>Proposed Algorithm</th>
<th>CCRR%</th>
</tr>
</thead>
<tbody>
<tr>
<td>When $U=4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Nmul$</td>
<td>20,480</td>
<td>11,520</td>
<td>43.75</td>
</tr>
<tr>
<td>$Nadd$</td>
<td>40,960</td>
<td>23,040</td>
<td>43.75</td>
</tr>
<tr>
<td>When $U=16$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Nmul$</td>
<td>81,920</td>
<td>39,168</td>
<td>52.18</td>
</tr>
<tr>
<td>$Nadd$</td>
<td>163,840</td>
<td>78,336</td>
<td>52.18</td>
</tr>
<tr>
<td>When $U=64$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Nmul$</td>
<td>327,680</td>
<td>149,760</td>
<td>54.29</td>
</tr>
<tr>
<td>$Nadd$</td>
<td>655,360</td>
<td>299,520</td>
<td>54.29</td>
</tr>
</tbody>
</table>

The CCRR of the proposed algorithm over C-SLM is given in Table I. Here, we assumed that $N=256$ and $U=4, 16, 64$ respectively. When $U=16$, the CCRR of the proposed algorithms is 52% when compared to C-SLM.

VI. SIMULATION RESULTS AND DISCUSSION

The performance of the proposed algorithms is evaluated in terms of CCDF of PAPR, BER and computational complexity. For the purpose of simulation, 10000 randomly generated data symbols are modulated using Quadrature Amplitude Modulation (QAM) and the number of sub-carriers $N=256$, oversampling factor $L=4$ and the number of candidates $U=16$ are examined. The PAPR reduction performance of proposed algorithms 1 and 2 is analyzed through simulations.

Fig. 3 shows the CCDF comparison of the proposed algorithms 1 and 2 with respect to OFDM, clipping with clipping ratio $CR=1.4$ & 1.6, C-SLM with $U=16$ and a hybrid scheme with the serial combination of SLM and clipping. The simulation result shows that the proposed algorithms 1 and 2 provide same PAPR reduction performance. The PAPR at CCDF $= 10^{-2}$ is improved by 0.25 dB, 0.45 dB than C-SLM for the clipping ratio (CR) 1.4 and 1.6 respectively, and is poorer than applying clipping alone and hybrid scheme with a serial combination of SLM and clipping.

Fig. 4 shows the BER performance comparison of proposed algorithms with conventional algorithms over Additive White Gaussian Noise (AWGN) channel. The BER performance of the proposed algorithm slightly degrades with respect to OFDM, SLM and serial combination of SLM & clipping, but better than clipping as the clipping noise of the combined scheme would be less than the clipping technique applied to the whole data.
VII. CONCLUSION

In this paper, we proposed a low complexity hybrid scheme based on clipping and SLM algorithms for PAPR reduction in OFDM systems. To reduce the computational complexity, both algorithms are applied simultaneously. Simulation results show that our algorithm achieves improved PAPR reduction performance compared to other hybrid schemes and C-SLM algorithm. Hence, the proposed hybrid scheme is promising to reduce PAPR of OFDM signal with much reduced complexity at the cost of slight degradation in BER performance.

REFERENCES


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