World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:10, No:8, 2016

Tracking Trajectory of a Cable-Driven Robot for Lower Limb Rehabilitation

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***Abstract*—This paper investigates and presents a cable-driven robot to lower limb rehabilitation use in sagittal plane. The presented rehabilitation robot is used for a trajectory tracking in joint space. The paper covers kinematic and dynamic analysis, which reveals the tensionability of the used cables as being the actuating source to provide a rehabilitation exercises of the human leg. The desired trajectory is generated to be used in the control system design in joint space. The obtained simulation results is showed to be efficient in this kind of application.

***Keywords*—Cable-driven multibody system, computed-torque controller, lower limb rehabilitation, tracking trajectory.

I. Introduction

A Person's functional mobility (measured as the ability to walk) is necessary to ensure the Activities of Daily Living (ADL). However, several neurological disorders including neuromuscular diseases, stroke, and spinal cord injury cerebellar disorders, or the impaired functions of the member musculature after a serious injury, illness or surgery, lead to the problems in the lower limb joint and generate atypical gait movement, thereby reducing patient's quality of life. To remedy these problems, the rehabilitation process is used, based on physical therapy to restore patient's strength, mobility and fitness. Traditionally, with assistance of therapists, the physical therapy sessions of limb (lower or upper) were carried out manually. However, due to the poor performances which have been reached in this case [1], many researchers thought to develop the robotic devices for rehabilitation process. Indeed, the automation of rehabilitation processes offers several potential advantages, including good repeatability, a precisely controllable assistance, and thereby the required effort of physical therapists can be greatly reduced [2].

In general, the robotic systems used in assistance applications can broadly be divided as prosthesis and orthosis. Prosthesis is an artificial substitute of a missing body part. Orthosis is an orthopedic apparatus that can be used to support and correct deformities of a person or to improve functionality of movable parts of the body [3]. The orthoses can further be divided into two major types: Devices that have been made to mechanically align its joints with the joints of the user, and devices that are made to align the end effector with the user. The first group known as Exoskeleton robot is generally worn by the patient where the robot have direct correspondence with the human joints and limbs respectively. The robot axes, in this case, are expected to be aligned with the anatomical axes of the part of body. For lower limb, there are several exoskeleton robots recently developed, evaluated, and in some cases commercialized, [4]. However, most of these devices offer fixed trajectory tracking, which are limited than those with variable trajectory training patterns that ensure the better recovery [5]. Generally, these devices still have an extra cost of its complicated mechanical structure [6], [7], despite the improvement of studies in this respect [8].

For the second group of orthoses, the position and orientation of the lower limb are changed indirectly by the movement of the end-effector. In comparison with the others, this group has some extra advantages, as simpler structure, less risk of injury to the patient, and easier set up accordingly to control algorithm. However, it is difficult to isolate the motion of the specific joint, because the structure of anatomical movement of a subject's lower limbs is complex. Hence, we must seek the best solution that meets the desired requirements.

Currently, the cable robots with their remarkable advantages are increasingly used, especially in physical rehabilitation, where they can be classified as end-effector-based devices. In this case, one may talk about multi-body cable-driven mechanisms, which represent a cable robot with the moving platform is the human limbs, considered as being Multi-Body System (MBS) [9]. Indeed, the cables are attached between the winches and the links of the platform to reach the desired motion, and thereby ensure to this type of robots a low inertia, ability to reach high speeds, relatively a large workspace, low fabrication costs, a possibility to reconfiguration etc., needed in rehabilitation exercises. However, among challenges with such mechanisms are the risk of interference between the cables and the links of the multi-body mechanism, or between the cables themselves [10], appearing more frequently for the redundant cable robot. Though, the major challenge is ensuring their ability to have a best force distribution in all cables (positive tension), known as tensionability condition (wrench closure, or force closure) [11]. In multi-body cables driven, it is proved that one can both reducing the risk of cable interference and lowering the cost, respecting a minimum necessary and a sufficient number of cables [12].

In review of the literature, several researchers are motivated recently to develop and design cable robot to physical

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rehabilitation use, by adopting a specific configuration and geometry to satisfy a rehabilitation exercise desired, but few of them have developed the lower one. In this paper, we analyze a robotic lower limb rehabilitative device driven by cable with the specific design, in order to develop the appropriate control algorithm. The paper adopted the following structure: In Section II, the kinematic analysis is described. The equations of motion of lower limb cable driven robot in joint space is developed, in Section III, the tension distribution of used cables is discussed. The aim of Section IV is to develop and simulate the control system with some restriction, using computed torque controller.

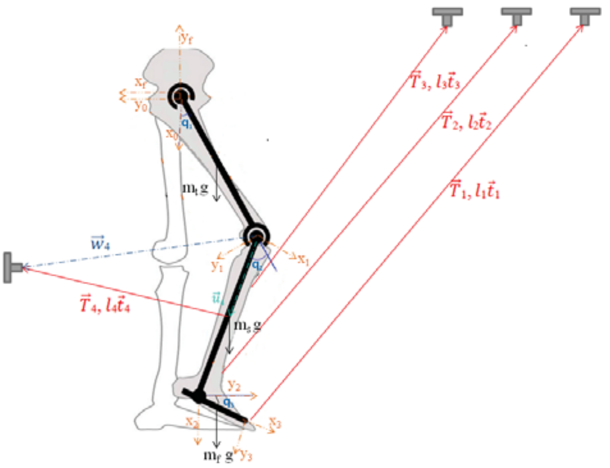

Fig. 1 Lower limb cable driven (x_f, y_f); (x_0, y_0); (x_1, y_1); (x_2, y_2); (x_3, y_3) defined coordinates of the fixed frame; the frame (0), (1), (2), and (3)

II. Kinematic Analysis

A. Robot Description

The designed robotic lower limb rehabilitative device driven by cable uses four cables attached to the fixed winches, then, to the each shank and ankle, in order to control the degree of freedom of right and left human leg in sagittal plane. Due to similarity, in this paper, the right one is investigated, as depicted in Fig. 1, where each cable, supposed as the mass-less inextensible in this work, is connected to motorized winch which ensure the lower limb mobility.

The motion provided describes the human leg motion having three rotational degrees of-freedom in sagittal plane (as hinge joints): A hip extension-flexion degree-of-freedom (with $+q_1$ in the direction of flexion), a knee extension-flexion degree-of-freedom (with $+q_2$ in the direction of flexion), an ankle dorsiflexion-plantar flexion degree-of-freedom (with $+q_3$ in the direction of dorsiflexion), as illustrated in the Fig. 1. These joint angles are indexed such that at pose $(q_1; q_2; q_3) = (0; 0; 0)$ the leg is fully extended and vertical.

To ensure users security during rehabilitation session, physiological safety of humans lower limb must be considered in the design of robot. Indeed, the suspension of the human leg according to cable use, allows maintain of the patients balance during training, when he is unable to do it. In the other hand, consider the limitation of joints flexion, defined by the Maximum Range Of Motion (MROM) and Maximum Speed (MS), help to increase the physiological safety. These specifications are used as constraints of the desired trajectory, ensuring that MROM and MS will not be exceeded during motion exercise. The designed robot, can assist patients to perform joint training, gait training, or other desired trajectory tracking in hip, knee and ankle joints, according to the controller's performance.

B. Kinematic Equation

The motion manipulation of human leg is performed by variation length of cables connected to the shank and ankle, as illustrated in Fig. 1. For $i = 1 \cdots 4$, l_i denoted the i^{th} cable length, $\vec{t}_i$ the i^{th} cable unit vector, $\vec{w}_i$ the i^{th} position of the winch, $\vec{u}_i$ the i^{th} position vector of cable attachment point, and $\vec{T}_i$ the force vector of the i^{th} cable. Using geometrical notation, the loop-closure equation for each i^{th} cable can be written as:

$$\vec{u}_i - \vec{w}_i + l_i\vec{t}_i = \vec{0} \tag{1}$$

The relationship between the joint angular velocities and the cable velocities can be deducted from (1) by:

$$\begin{pmatrix} \dot{l}_1 \\ \dot{l}_2 \\ \dot{l}_3 \\ \dot{l}_4 \end{pmatrix} = -\begin{pmatrix} \vec{t}_1^T \cdot \frac{\partial \vec{u}_1}{\partial q_1} & \vec{t}_1^T \cdot \frac{\partial \vec{u}_1}{\partial q_2} & \vec{t}_1^T \cdot \frac{\partial \vec{u}_1}{\partial q_3} \\ \vec{t}_2^T \cdot \frac{\partial \vec{u}_2}{\partial q_1} & \vec{t}_2^T \cdot \frac{\partial \vec{u}_2}{\partial q_2} & \vec{t}_2^T \cdot \frac{\partial \vec{u}_2}{\partial q_3} \\ \vec{t}_3^T \cdot \frac{\partial \vec{u}_3}{\partial q_1} & \vec{t}_3^T \cdot \frac{\partial \vec{u}_3}{\partial q_2} & \vec{t}_3^T \cdot \frac{\partial \vec{u}_3}{\partial q_3} \\ \vec{t}_4^T \cdot \frac{\partial \vec{u}_4}{\partial q_1} & \vec{t}_4^T \cdot \frac{\partial \vec{u}_4}{\partial q_2} & \vec{t}_4^T \cdot \frac{\partial \vec{u}_4}{\partial q_3} \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix} \tag{2}$$

$$= -J_T \dot{q} \tag{3}$$

where J_T is the Jacobian matrix, and "." depicts the scalar product.

In general case, for each segment of the multi-body platform, the angular velocity vectors and angular acceleration vectors are related to angular position vectors as expressed in the following expressions.

For the thigh:

$$\vec{\omega}_t = \dot{\vec{q}}_1; \quad \dot{\vec{\omega}}_t = \ddot{\vec{q}}_1$$

$$\vec{a}_t = \vec{r}_t \wedge \dot{\vec{\omega}}_t + (\vec{\omega}_t \wedge \vec{r}_t) \wedge \vec{\omega}_t$$

where $r_t = {}^F R_1 {}^1 r_t$ is the thigh mass center vector expressed in fixed frame, ${}^F R_i$ is the transformation matrix from fixed frame to i^{th} frame of reference; ${}^1 r_t$ is the thigh mass center vector expressed in frame 1 (position vector from the origin of frame 0 to the origin of frame 1), and a_t is the linear acceleration of the thigh mass center of links.

With the proposed design, the hip joint angle "q_1" does not admit the control constraints, it is not supported by cable.

For the shank:

$$\vec{\omega}_s = \dot{\vec{q}}_1 + \dot{\vec{q}}_2; \quad \dot{\vec{\omega}}_s = \ddot{\vec{q}}_1 + \ddot{\vec{q}}_2$$

$$\vec{a}_s = \vec{a}_t + \vec{r}_s \wedge \dot{\vec{\omega}}_s + (\vec{\omega}_s \wedge \vec{r}_s) \wedge \vec{\omega}_s$$

where $r_s = {}^F R_2 {}^2 r_s$ is the shank mass center vector expressed in fixed frame, ${}^2 r_s$ is the shank mass center vector expressed in frame 2 (position vector from the origin of frame 1 to the origin of frame 2), and a_s is the linear acceleration of the shank mass center of links.

For the foot:

$$\vec{\omega}_f = \dot{\vec{q}}_1 + \dot{\vec{q}}_2 + \dot{\vec{q}}_3$$
$$\dot{\vec{\omega}}_f = \ddot{\vec{q}}_1 + \ddot{\vec{q}}_2 + \ddot{\vec{q}}_3$$
$$\vec{a}_f = \vec{a}_s + \vec{r}_f \wedge \dot{\vec{\omega}}_f + (\vec{\omega}_f \wedge \vec{r}_f) \wedge \vec{\omega}_f$$

where: $r_f = {}^F R_3 {}^3 r_f$ is the foot mass center vector expressed in fixed frame, and ${}^3 r_f$ is the shank mass center vector expressed in frame 3 (position vector from the origin of frame 2 to the origin of frame 3), and a_f is the linear acceleration of the foot mass center of links.

III. Dynamic Analysis

In order to control the tension of cables which actuate human leg in sagittal plane during rehabilitation exercises, the dynamic equations have to be developed based on rigid body mechanic theory.

A. Equation of Motion

There are several methods for dynamic modeling of multi-rigid-body robot, such as Lagrange equation, Newton-Euler equation, Newton-motional theory and Hamilton theory. However, Lagrange formulation and Newton-Euler formulation are more widely used for controller design [13]. In order to describe the motion equation of designed robot, the Newton-Euler approach is used. In classical mechanics, the two Newton-Euler laws relate the motion of the rigid body's links (center of gravity) with the sum of forces and moments acting on the rigid body. With respect to a coordinate frame, general formulation can be expressed in matrix form (without vectorial notation) as:

$$\begin{pmatrix} F \\ M \end{pmatrix} = \begin{pmatrix} mI_3 & 0 \\ 0 & I_{mc} \end{pmatrix} \begin{pmatrix} a \\ \dot{\omega} \end{pmatrix} + \begin{pmatrix} 0 \\ \omega \times I_{mc}\omega \end{pmatrix} \quad (4)$$

where F the total force acting on body links; m the mass of the segment body, I_3 the 3×3 identity matrix, a the linear acceleration of the mass center of link, M the total moment acting about the mass center of link, I_{mc} the moment of inertia about the mass center of link, ω the angular velocity of the body, $\dot{\omega}$ the angular acceleration of the body. Using this Newton-Euler formulation, the dynamic equations for the segment of the human leg has been developed, taking into account the cables forces as well as constraint forces and moments, as illustrated in Fig. 1. Therefore, the dynamic equation firstly of the thigh segment can be written as:

$$\begin{cases} \vec{F}_{bt} - \vec{F}_{ts} + m_t\vec{g} - m_t\vec{a}_t = \vec{0} \\ \vec{u}_{t1} \wedge \vec{F}_{bt} - \vec{u}_{t2} \wedge \vec{F}_{ts} + \vec{M}_{bt} - \ldots] \\ \ldots - \vec{M}_{ts} - I_t\dot{\vec{\omega}}_t - \vec{\omega}_t \wedge (I_t\vec{\omega}_t) = \vec{0} \end{cases} \quad (5)$$

Similarly, for the shank segment:

$$\begin{cases} \vec{T}_2 + \vec{T}_3 + \vec{T}_4 + \vec{F}_{ts} - \vec{F}_{sf} + m_s\vec{g} - m_s\vec{a}_s = \vec{0} \\ \vec{u}_{s2} \wedge \vec{T}_2 + \vec{u}_{s3} \wedge \vec{T}_3 + \vec{u}_{s4} \wedge \vec{T}_4 - u_{s5} \wedge \vec{F}_{sf} + \ldots] \\ \ldots + \vec{u}_{s1} \wedge \vec{F}_{ts} + \vec{M}_{ts} - \vec{M}_{sf} - I_s\dot{\vec{\omega}}_s - \vec{\omega}_s(I_s\vec{\omega}_s) = \vec{0} \end{cases} \quad (6)$$

Finally, for the foot segment:

$$\begin{cases} \vec{T}_1 + \vec{F}_{sf} + m_f\vec{g} + \vec{F}_{ext} - m_f\vec{a}_f = \vec{0} \\ \vec{u}_{f1} \wedge \vec{T}_1 + \vec{u}_{f5} \wedge \vec{F}_{sf} + \vec{u}_{f6} \wedge \vec{F}_{ext} + \vec{M}_{sf} + \ldots \\ \ldots + \vec{M}_{ext} - I_f\dot{\vec{\omega}}_f - \vec{\omega}_f \wedge (\vec{I}_f\omega_f) = \vec{0} \end{cases} \quad (7)$$

where $\vec{F}_{sf}$ and $\vec{M}_{sf}$ are respectively the force and moment vectors exerted from shank to the foot in sagittal plane; $\vec{F}_{ts}$ and $\vec{M}_{ts}$ are the force and moment vectors exerted from thigh to the shank in sagittal plane; $\vec{F}_{bt}$ and $\vec{M}_{bt}$ are respectively the force and moment vectors exerted from human body to the thigh; $\vec{F}_{ext}$ and $\vec{M}_{ext}$ are the given external force and moment vectors from the ground to the foot during walking; m_f, m_s and m_t are the mass of the foot, the shank and the thigh respectively; $\vec{u}_{t1}, \vec{u}_{t2}$ ($\vec{u}_{s1}, \vec{u}_{s2}, \vec{u}_{s3}, \vec{u}_{s4}, \vec{u}_{s5}$, and $\vec{u}_{f1}, \vec{u}_{f5}, \vec{u}_{f6}$) are position vectors from the mass center of the thigh (shank, and foot) to the point of application of the applied forces expressed in the fixed frame. I_t , I_s and I_f are the inertia tensors of the thigh, the shank and the foot which are expressed in the fixed frame of reference.

For simplification use, consider all used force and moment as the multiplication of magnitude term and associated unit vector:

$$\vec{X} = \|\vec{X}\|.\frac{\vec{X}}{\|\vec{X}\|} = \|\vec{X}\|.\vec{x} = X.\vec{x}$$

Therefore, Newton-Euler formulation for shank, foot and thigh expressed in (5)-(7), can be written briefly as:

$$J_t T_t = F_t \quad (8)$$
$$J_s T_s = F_s \quad (9)$$
$$J_f T_f = F_f \quad (10)$$

where

$$J_t = \begin{bmatrix} -\vec{t}_{ts} & \vec{t}_{bt} & 0 & 0 \\ -\vec{u}_{t2} \wedge \vec{t}_{ts} & \vec{u}_{t1} \wedge \vec{t}_{bt} & -\vec{m}_{ts} & \vec{m}_{bt} \end{bmatrix}$$
$$T_t = \begin{bmatrix} F_{ts} & F_{bt} & M_{ts} & M_{bt} \end{bmatrix}^T$$
$$F_t = \begin{bmatrix} m_t\vec{g} - m_t\vec{a}_t \\ -I_t\dot{\vec{\omega}}_t - \vec{\omega}_t \wedge (I_t\vec{\omega}_t) \end{bmatrix}$$
$$J_s = \begin{bmatrix} \vec{t}_2 & \vec{t}_3 & \vec{t}_4 & -\vec{t}_{sf} & \vec{t}_{ts} & 0 & 0 \\ \vec{u}_{s2} \wedge t_2 & \vec{u}_{s3} \wedge \vec{t}_3 & \vec{u}_{s4} \wedge \vec{t}_4 & -\vec{u}_{s5} \wedge \vec{t}_{sf} & \vec{u}_{s1} \wedge \vec{t}_{ts} & -\vec{m}_{sf} & \vec{m}_{ts} \end{bmatrix}$$
$$T_s = \begin{bmatrix} T_2 & T_3 & T_4 & F_{sf} & F_{ts} & M_{sf} & M_{ts} \end{bmatrix}^T$$
$$F_s = \begin{bmatrix} m_s\vec{g} - m_s\vec{a}_s \\ -I_s\dot{\vec{\omega}}_s - \vec{\omega}_s \wedge (I_s\vec{\omega}_s) \end{bmatrix}$$
$$J_f = \begin{bmatrix} \vec{t}_1 & \vec{t}_{sf} & \vec{0} \\ \vec{u}_{f1} \wedge \vec{t}_1 & \vec{u}_{f5} \wedge \vec{t}_{sf} & \vec{m}_{sf} \end{bmatrix}$$
$$T_f = \begin{bmatrix} T_1 & F_{sf} & M_{sf} \end{bmatrix}^T$$
$$F_f = \begin{bmatrix} -m_f\vec{g} - \vec{F}_{ext} + m_f\vec{a}_f \\ \vec{u}_{f6} \wedge \vec{F}_{ext} - \vec{M}_{ext} + I_f\dot{\vec{\omega}}_f + \vec{\omega}_f \wedge (I_f\vec{\omega}_f) \end{bmatrix}$$

where $\vec{t}_{bt}$ and $\vec{m}_{bt}$ are unit vectors of F_{bt} and M_{bt}. $\vec{t}_{ts}$ and $\vec{m}_{ts}$ are unit vectors of F_{ts} and M_{ts}; finally, $\vec{t}_{sf}$ and $\vec{m}_{sf}$ are the unit vectors of F_{sf} and M_{sf}.

B. Tension Distribution

1) Formulation of Tensionability: Equations (5)-(7) of independent action/reaction moments and forces applied at the joint during motion of lower limb driven by cable, specially in sagittal plane, can be represented in the compact form, by assembling (8)-(10) as:

$$\mathbf{JT} = \mathbf{F} \quad (11)$$

where

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_f^1 & 0 & 0 & 0 & \mathbf{J}_f^2 & 0 & 0 & \mathbf{J}_f^3 & 0 & 0 \\ 0 & \mathbf{J}_s^1 & \mathbf{J}_s^2 & \mathbf{J}_s^3 & \mathbf{J}_s^4 & \mathbf{J}_s^5 & 0 & \mathbf{J}_s^6 & \mathbf{J}_s^7 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{J}_t^1 & \mathbf{J}_t^2 & 0 & \mathbf{J}_t^3 & \mathbf{J}_t^4 \end{bmatrix}$$

$$T = \begin{bmatrix} T_1 & T_2 & T_3 & T_4 & F_{sf} & F_{ts} & F_{bt} & M_{sf} & M_{ts} & M_{bt} \end{bmatrix}^T$$

$$F = \begin{bmatrix} F_f^T & F_s^T & F_t^T \end{bmatrix}^T$$

where, J_t^i , J_s^i and J_f^i denoted the ith columns of the J_t, J_s and J_f respectively.

The obtained Jacobian matrix J, called also structure matrix of the multibody, can be used to evaluate tensionability condition. However, the size of the Jacobian matrix is large, which lead to numerical complexities in its analysis.

To simplify the tensionability analysis, according to Lagrange's approach and the notion of generalized forces, the equations related to the force cables can be isolated (as will be explained in Section III-C). In this order, consider the following expression:

$$F_c = J_c T_c \tag{12}$$

where, T_c is the part of generalized force related to magnitude of cable force, J_c is a Jacobian matrix which maps cable tensions into the joint torques as defined.

In order to ensure the tensionnability condition of system, different approach can be used following the system configuration. The proposed system is considered as multi-bodies system supported by four cables as explained previously. We can talk about the Multiple Redundant Cables. Therefore, it is important to note that cable forces must be defined as:

$$T_{cmin} \leq T_c \leq T_{cmax} \tag{13}$$

where the lower bound of the cable tension is T_{cmin}, and the upper bound of the cable tension is T_{cmax}.

In review of the literature, we can find several approaches required for the condition tensionability according to redundant cable driven robot. However, when controlling such cable driven robot, the number of tension distributions is infinite. Therefore, the problem here become well-posed as being the optimization problem, where the optimal tension distribution should be feasible and determined with respect to some useful objective function.

C. Closed Form Dynamic Equation

For control application, the closed form dynamic equations in joint space are required to be developed, by using Lagrange theory, where the redundant constraint and all the forces/moments at joint from the dynamic equations are eliminated. The dynamic equations of motion take the following form [14]:

$$M(q)\ddot{q} + C(\dot{q}, q)\dot{q} + G(q) = \tau \tag{14}$$

where $q = [q_1 \; q_2 \; q_3]^T$ is the vector of generalized coordinates which represent the angle of hip flexion, knee flexion and ankle dorsiflexion, respectively; $M(q)$ is the symmetric positive definite inertia matrix; $C(\dot{q}, q)$ is the Coriolis and centripetal matrix; and $G(q)$ is the gravitational vector; τ is the torque vector including all generalized force corresponding to the generalized coordinates q.

The M matrix is intrinsic to the robot; therefore, it is expressed in terms of the linkages parameters. Moreover, since these parameters can be adjusted to fit for the different heights of patients, the system parameters, especially M, have to be changed correspondingly.

In order to establish the relationship between the cable tension T_c which define the actuating source, and the torque vector τ defined as ideal up to now, the principle of virtual torque can be used as:

$$\begin{aligned} \delta W &= T_c \delta l = \tau \delta q \\ \tau^T \delta q &= F_e^T \delta P + \sum_{i=1}^{4} T_{ci} t_i^T \delta X_{Ti} \\ &= F_e^T J_e \delta q + \sum_{i=1}^{4} T_{ci} t_i^T \frac{\partial u_i}{\partial q} \delta q \end{aligned}$$

where J_e is the Jacobian matrix which maps end effector output force into n-dimensional joint torques; $F_e = [F_{ext}, M_{ext}]^T$; δP and δX_{Ti} denote the virtual displacement vector of the end effector and cable attachment point on each segment of lower limb, then they are substituted by the expression with q, as mentioned. Finally, one can express the torque vector τ, as:

$$\tau = J_T^T T_c + J_e^T F_e \tag{15}$$

where J_T^T is the transpose of the Jacobian matrix which maps cable tensions into the joint torques as defined in (2).

Indeed, from (12) and (15), we can deduce that:

$$Jc = J_T^T F_T = \tau - J_e^T F_e \tag{16}$$

The aim of this section is to present the relationship between the lower limb cable-driven robot and ideal joint torque. However, to better describe actuating model including motor model, the output torque motor is given by [15]:

$$\tau = k_g \tau_m - \tau_f \tag{17}$$

where k_g is the gear ratio, τ_m is the output torque motor, τ_f the friction torque.

According to use DC motor for actuating winches, and since the output torque of DC motor is proportional to current, the motor torque τ_m can be obtained by:

$$\tau_m = K_m i_m \tag{18}$$

where K_m is torque sensitivity of DC motor; i_m is the current of DC motor. Thereby, the cable force can be directly expressed by motor current or motor voltage.

IV. Control System and Simulation Results

A. Control System

Independently of the actuating source, in this contribution, the control design, allows trajectory tracking for lower limb cable driven robot, described by its dynamic equation.

To control this nonlinear robot system, the feedback linearization is used as being a nonlinear design methodology. The basic idea is to first transform a nonlinear system into a (fully or partially) linear system, and then use the linear design techniques to complete the control design.

In this order, the *Computed Torque Controller* (CTC) is proposed, to ensure the passive training, where the method of position control is used, so that the ankle, hip, and knee, can be exercised according to the trajectory planning in joint space. Generally, CTC is characterized to be special application of linearising controller of nonlinear systems, which includes several a broad range of designs.

The presented controller is provided to carry out the trajectory tracking in joint space. Suppose that the desired trajectory is $q_d(t)$ and the generated joints variable is $q(t)$. To ensure trajectory tracking, the tracking error is defined as:

$$q_e(t) = q_d(t) - q(t) \tag{19}$$

The computed-torque control law is defined as [15] which linearise the dynamic of the system in joint space [16]:

$$\tau = M\ u + C(\dot{q}, q) + G(q) \tag{20}$$

The effectiveness of this control law, depend of the choice of the auxiliary control signal $u(t)$ to ensure the stability and the low tracking error.
Among the used approaches in this context, the signal $u(t)$ is provided to be the Proportional-Derivative (PD) feedback:

$$u = \ddot{q}_d + K_p q_e + K_d \dot{q}_e \tag{21}$$

where:

$$K_p = \begin{pmatrix} kp_1 & 0 & 0 \\ 0 & kp_2 & 0 \\ 0 & 0 & kp_3 \end{pmatrix}$$

and

$$K_d = \begin{pmatrix} kd_1 & 0 & 0 \\ 0 & kd_2 & 0 \\ 0 & 0 & kd_3 \end{pmatrix}$$

are the gains of PD controller. According to:

$$\ddot{q}_d + K_p q_e + K_d \dot{q}_e = 0 \tag{22}$$

the stability is ensured for any choice of the gains K_d and K_p, as long as these are positive definite matrices.

To have a low tracking error, the choice of the gains controller is provided using method explained in [15].

According to the last formulations, the control architecture is illustrated in Fig. 2. The computation of linearising control law (20) is performed in an inner loop, with τ the output, and u the input. Then the computation of the input term u according to (21) is performed in the outer loop of PD feedback.

Note that the inner loop control allows linearising the nonlinear system robot, which illustrated by the plant represented by the dotted lines in Fig. 2, and the outer loop control is used to complete control design system.

B. Trajectory Planning

In order to apply the control approach explained previously, the trajectory is generated to be used as signal input (trajectory planner) as depicted in Fig. 2. Indeed, during patients rehabilitation process, desired moving trajectories should be taken according to an individual patients characteristics. However, up to now it is consider as a critical problem of rehabilitation robots for most researchers.

In this paper, the desired trajectory is developed in joint space according to the control system, where the motion of each joint of human leg is defined for a duration time. The desired trajectory is obtained from the synthetic data, taking into account the biomechanics constraints of lower limb as presented in [17].

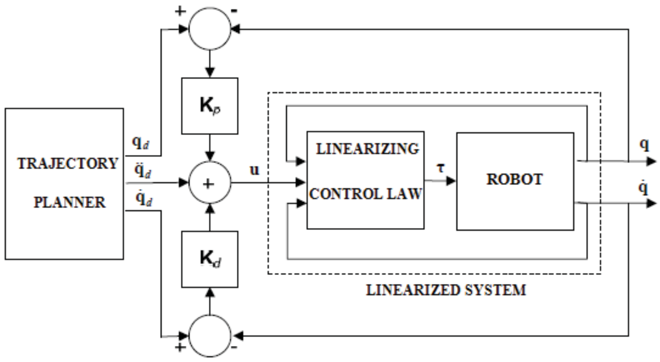

Fig. 2 Control Architecture

C. Simulation Results

In order to illustrate the effectiveness of the proposed controller on the rehabilitation system, the Matlab software is used. The complete simulation model of the control system following the control architecture in Fig. 2, includes the desired trajectory (trajectory planner) in joint space obtained from the synthetic data, the robot model consisting of the kinematics and dynamics parameters of the system, obtained using Denavit-Hartenberg (D-H) convention [14], and the controller bloc while CTC controller as explained previously.

The used controller implementation, requires the computation at each sample instant of the inertia matrix $M(q)$ and the vector of Coriolis and centrifugal $C(\dot{q}, q)$, and gravitational $G(q)$.

Adjusting the controller gains (K_p,K_d) is achieved by making a compromise with the generated torque, which has a direct influence on the actuator power. The simulation results are shown in Figs. 3 and 4. The first one illustrates the desired trajectory and the computed trajectory where the low difference between them, indeed the good efficient of the used controller. Also, it is seen from the second one that the tracking error is acceptable in this application.

V. Conclusion

According to the proposed system for rehabilitative exercises of the human leg, in this paper, the kinematic and dynamic modeling was performed based on the rigid body approach, using in the first the Newton-Euler formulation in order to show the cables tension of used cable-driven multibody system, as being its actuating source, and thereby the tensionability condition was discussed. To ensure a trajectory tracking of the used system in joint space, independently of actuating source, and using the synthetic data

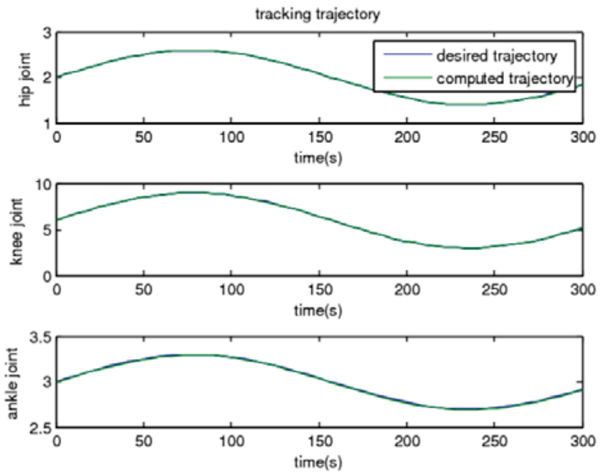

Fig. 3 Trajectory tracking

joints as desired trajectory, the computed torque controller was applied. Simulation results were present an acceptable performances in this kind of application.

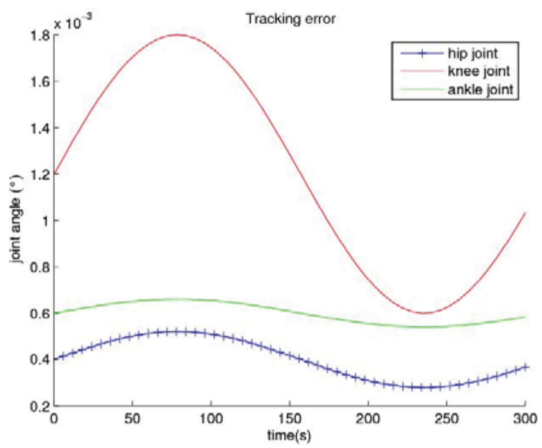

Fig. 4 Tracking Error

The current study has some limitations which are currently being developed. According to the system modeling, it should take into account the muscular parameters of human leg. Moreover, the effect of the system's intrinsic and dynamic parameters uncertainty should be introduced. To better simulate the patient's rehabilitation process, the desired trajectory will be generated in task space according to Clinical Gait Data (CGD) analysis.

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