Continuous Wave Interference Effects on Global Position System Signal Quality

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Abstract—Radio interference is one of the major concerns in using the global positioning system (GPS) for civilian and military applications. Interference signals are produced not only through all electronic systems but also illegal jammers. Among different types of interferences, continuous wave (CW) interference has strong adverse impacts on the quality of the received signal. In this paper, we make more detailed analysis for CW interference effects on GPS signal quality. Based on the C/A code spectrum lines, the influence of CW interference on the acquisition performance of GPS receivers is further analysed. This influence is supported by simulation results using GPS software receiver. As the most important user parameter of GPS receivers, the mathematical expression of bit error probability is also derived in the presence of CW interference, and the expression is consistent with the Monte Carlo simulation results. The research on CW interference provides some theoretical gist and new thoughts on monitoring the radio noise environment and improving the anti-jamming ability of GPS receivers.

Keywords—GPS, CW interference, acquisition performance, bit error probability, Monte Carlo.

I. INTRODUCTION

The GPS is an excellent application example of satellite communication technology [1]. It is widely used in navigation, measurement and time service [2], [3]. Interference signals can be considered as one of the most disruptive events in the operation of a GPS receiver, because interference signals affect the operation of the automatic gain control (AGC) and the low noise amplifier (LNA) in the RF front-end, as well as the acquisition and carrier-code tracking loops [4].

The interference effects on GPS system has already been addressed by many researchers during the past years, including antennas [5], [6], the AGC [4], IF signal processing [7], and time-frequency domain techniques [8], [9]. Since interference signals have significant impact on the quality of the received GPS signal, investigations regarding some most important characteristics of GPS receivers in the presence of interference signals have been conducted in detail recently. CW interference effects on the tracking performance of the GPS receivers was investigated in [10]. The authors formulated analytical equations representing DLL error and PLL error when a CW interference was presented. Another research paper [11] presented an analysis for some of the most important characteristics in various GPS III signals. In terms of the bit error probability, the error probability of BPSK and QPSK direct sequence spread spectrum (DSSS) communication systems was developed using a simple technique in [12]-[15]. This technique clearly illustrated the same error probability between the BPSK and QPSK systems. Bit error rates for both quadrature and biphase, in conjunction with both QPSK and BPSK date, were considered in the presence of noise and jamming. However, the frequency location of the jammer was arbitrary and there is a limitation to take the spectrum characteristics of spread spectrum codes into account.

In this paper, the CW interference effects on the acquisition performance of the GPS receiver are discussed. It is shown that the frequency difference between the interference and the carrier plays an important role in the effects of CW interference. We also display three dimensional acquisition results when a CW interference is presented. As a result, it is clearly shown that the effects of CW interference are different when CW interference signal obtains different frequency for the specific PRN number.

The main contributions of this paper is to derive the mathematical expression of bit error probability of the GPS receiver under CW interference. We first take the spectrum characteristics of C/A code into account. In this way, the performance of bit error probability can be analyzed under different interference frequency. We then construct the Monte Carlo simulation model to test the validity of the mathematical expression, and the constellation diagrams of the Monte Carlo results are given in this paper. Finally, both diagrams of the mathematical expression and the Monte Carlo are displayed. Clearly, the results of the expression and the Monte Carlo simulation have the same trend, which leads us to the conclusion that the formula is feasible. The overall goal of this paper is to provide the theoretical basis and instruction for enhancing the anti-jamming ability of GPS receivers in the presence of CW interference signal.

II. THE GPS ACQUISITION PERFORMANCE UNDER CW INTERFERENCE

A. The GPS Acquisition Model

The purpose of the acquisition process is to get initial estimate of the Doppler frequency and code delay of all the satellites in view [16]. The received signal via the GPS receiver antenna can be written as

\[ r(t) = s(t) + J(t) + n(t) \]  

where \( n(t) \) is the Gauss white noise that has mean value of zero and noise spectral power density value of \( N_0 \). \( J(t) \) is
CW interference signal which can be expressed as

$$J(t) = \sqrt{2P_s} \cos(2\pi f_J + \theta_J)$$

(2)

where $P_s$ is the power of the interference signal, $f_J$ and $\theta_J$ are the frequency and phase of the interference signal, respectively.

The received signal is down-converted and filtered by the receiver front-end which is omitted in Fig. 1. That is, the acquisition operation of the receiver in Fig. 1 mainly completes the correlation processing of the GPS signals. The local reference C/A code and the reference carrier frequency are multiplied by the input signal $r(t)$ in the correlator. The resulting signals are then coherently integrated within an integration time $T_{coh}$. Finally, the signal after correlator can be expressed as

$$r_p(t) = \frac{1}{T_{coh}} \int_{t_1}^{t_1 + T_{coh}} r(t) e^{j(2\pi f_0 + \theta_0)} c(t - \tau) dt$$

(3)

where $f_0$ and $\theta_0$ are the frequency and phase of local reference carrier, respectively. $c(t - \tau)$ represents the local reference C/A code, $\tau$ is the chip delay.

Assuming the frequency of the intermediate frequency (IF) carrier signal is 3.096 MHz. According to the bandpass sampling theorem, the sampling frequency of the digital signal is 9 MHz, so that the recorded data of 1 ms contains 9000 sampling points. Figs. 3 and 4 show the correlation output under CW interference in the case of SNR = -22 dB and J/SR = 20 dB when $\Delta f_J$ are 173 kHz and 172 kHz, respectively.

The simulation results, as shown in Fig. 3, depict that the correlation output of PRN9 is chaotic when $\Delta f_J$ is 173kHz, which means that the effects of the interference are different when $\Delta f_J$ are 173 kHz and 172 kHz, respectively. $\Delta f_J = f_J - f_0$ represents the frequency difference between the reference carrier signal and received carrier signal. $c_w$ is the amplitude of the C/A code spectral line of $w$kHz. $T_d$ is the coherent integration time. $P_s$ is the GPS signal power, $P_J$ is the interference signal power. $\Delta f_J$ is the interference frequency difference between the reference carrier signal and CW interference signal.

Accordingly, we can formulate the $C/N_0$ of the GPS signal after the correlator in the presence of CW interference as:

$$\frac{C}{N_0} = \frac{(\sqrt{2P_s} T_d R_0(\tau) \sin c(\pi \Delta f_J T_d))^2}{L_n N_0 + (\sqrt{2P_s} T_d c_w \sin c(\pi \Delta f_J T_d))^2}$$

(6)

where $N_0$ is the thermal noise power, $L_n$ is the processing gain applied to the noise.

In Fig. 2, the correlator output $C/N_0$ for PRN9 is presented. $C/N_0$ is an important parameter to evaluate the effects of the interference signal to the GPS signal. The deep troughs in this graph correspond to the coincidence of CW interference with the code spectral lines. This occurs at 1kHz spacing in $\Delta f_J$. There are different values for different lines which means that the effects of the interference are different when CW interference signal obtains different $\Delta f_J$.

Fig. 1 Block diagram of the GPS acquisition

B. Analysis of the Acquisition Results

In the previous section, we can conclude that the frequency corresponding to the worst $C/N_0$ of PRN9 is 173kHz, which means that the effects of CW interference on PRN9 is best when $\Delta f_J = 173$kHz. Therefore, the acquisition performance under this case is theoretically worse than others. This assumption is supported by the following simulation results.

Assuming the frequency of the intermediate frequency (IF) carrier signal is 3.096 MHz. According to the bandpass sampling theorem, the sampling frequency of the digital signal is 9 MHz, so that the recorded data of 1 ms contains 9000 sampling points. Figs. 3 and 4 show the correlation output under CW interference in the case of SNR = -22 dB and J/SR = 20 dB when $\Delta f_J$ are 173 kHz and 172 kHz, respectively.

The simulation results, as shown in Fig. 3, depict that the correlation output of PRN9 is chaotic when $\Delta f_J$ is 173 kHz, and there is no correlation peak. In other word, the GPS receiver dose not acquire the specific satellite. However, when we just reduce $\Delta f_J$ by 1 kHz, i.e. $\Delta f_J = 172$ kHz, as shown in Fig. 4, the correlation output of PRN9 appears an obvious correlation peak which means the satellite is captured successfully.

III. THE BIT ERROR PROBABILITY PERFORMANCE OF GPS RECEIVERS UNDER CW INTERFERENCE

The bit error probability performance of GPS receivers under CW interference is discussed in this section, which divided into two parts: the mathematical expression derivation and the Monte Carlo simulation.
A. The Bit Error Probability

For simplification, we assume that there is an ideal phase, sequence and symbol synchronization, and the error of the phase and frequency in the GPS receiver can be ignored. In addition, because the frequency of the CW interference signal is close enough to the frequency of the GPS signal, it is reasonable to assume the CW interference signal is not affected by the broadband filter before the correlation [18].

To derive the bit error probability for the GPS system, we assume that the information is transmitted via BPSK. Within the bit interval \(0 \leq t \leq T_s\), the transmitted signal can be expressed as

\[
s(t) = d_0 g_T(t) C(t) \cos(2\pi f_c t + \theta_c) \quad 0 \leq t \leq T_s
\]

where \(d_0 = \pm 1\) is the information symbol, the pulse \(g_T(t)\) is defined as

\[
g_T(t) = \begin{cases} 
\sqrt{2\Delta f_j / T_s} & 0 \leq t \leq T_s \\
0 & \text{else}
\end{cases}
\]

and \(C(t)\) is the output of the PN code generator which, over s bit interval \([0 - T_s]\), is expressed as

\[
C(t) = \sum_{i=0}^{L_c-1} c_i p(t - iT_c)
\]

where \(c_i = \pm 1\) represents a chip of the spreading sequence. \(L_c\) is the ratio of the bit interval \(T_s\) to the chip interval \(T_c\), which is usually selected to be an integer in practical spread spectrum systems. We denote this ratio as

\[
L_c = T_s / T_c
\]

hence, \(L_c\) is the number of chips per bit of C/A code bit.

In order to generating the decision variable, \(L_c\) continuous signals of the output signal in the formula (3) need to add together, which has the following form:

\[
\tau_{P_i} = \sqrt{2} \int_{iT_c}^{(i+1)T_c} r(t) p(t - iT_c) e^{j(2\pi f_c t + \theta_c)} dt \quad 0 \leq i \leq L_c - 1
\]

Accordingly, the input of the decision device is

\[
H = \sum_{i=0}^{L_c-1} c_i \tau_{P_i} = d_0 \sqrt{\sigma_d^2} + H_1 + H_2
\]

where, by definition,

\[
H_1 = \sum_{i=0}^{L_c-1} c_i J_i
\]

\[
H_2 = \sum_{i=0}^{L_c-1} c_i n_i
\]

The error probability depends on the statistical characteristics of the noise component and CW interference component. Clearly, for noise component, we can easily obtain \(E[H_2] = 0\) and \(\text{var}(H_2) = N_0 / 2\), where \(E[\cdot]\) represents mean operation and \(\text{var}(\cdot)\) represents variance operation.

For CW interference component, its mean and variance value can be expressed respectively as

\[
E[H_1] = 0
\]

\[
\text{var}(H_1) = \sum_{i=0}^{L_c-1} E[J_i^2]
\]

In addition, the mean of the decision variable is

\[
E[H] = d_0 \sqrt{\sigma_d^2}
\]

To determine the variance of \(H\), we must postulate the form of the CW interference. As a result, we next analysis the effects of CW interference signal on the bit error probability for GPS system. Considering the CW interference which can be written as:

\[
J(t) = \sqrt{2P_j} \cos(2\pi f_j t + \theta_j)
\]

By substituting (18) into (11), we obtain

\[
J_i = \sqrt{2P_j} \int_{iT_c}^{(i+1)T_c} p(t) \cos(2\pi \Delta f_j t + \Delta \theta_j + i2\pi \Delta f_j T_c) dt
\]

Fig. 3 The acquisition result under CW interference when: \(\Delta f_j = 173kHz\)

Fig. 4 The acquisition result under CW interference when: \(\Delta f_j = 172kHz\)
where $\Delta f_j = f_j - f_e$ and $\Delta \theta_j = \theta_j - \theta_e$. What’s more, $J_i$ is the signal component, which has been filtered out the high frequency.

The cycle of the C/A code is 1023 chips, which repeats every 1 ms. In order to take spectral characteristics of C/A code into account in the analysis of the bit error probability, we redefine the integration time $T_o = 1023T_e = 1ms$. Here, $L_o$ will be replaced by $G = T_o/T_e$. By integrating and triangular change, we can obtain

$$J_i = \sqrt{\frac{P_f}{T_o}} |c_w| \sin c (\pi \delta f_j T_o) \cos (2\pi \Delta f_j T_o + \theta)$$

(20)

where $\theta = \pi \Delta f_j T_o + \Delta \theta_f$. By substituting (20) into (16), the variance of $H_1$ can be written as

$$\text{var} (H_1) = \frac{1}{2G} P_f T_o |c_w|^2 \sin^2 (\pi \delta f_j T_o) \left[G + \sum_{i=0}^{G-1} \cos (i4\pi \Delta f_j T_o + 2\theta)\right]$$

(21)

Using the following identity to estimate the accumulation term in (21):

$$\sum_{i=0}^{N-1} \cos (iy + x) = \cos \left(x + \frac{N - 1}{2} y\right) \sin \left(Ny/2\right) \sin \left(y/2\right)$$

(22)

then (21) becomes

$$\text{var} (H_1) = \frac{1}{2} P_f T_o |c_w|^2 \sin^2 (\pi \delta f_j T_o) \left[1 + \frac{\sin c(2\Delta f_j T_o)}{\sin c(2\Delta f_j T_o)} \cos 2\phi\right]$$

(23)

where $\phi = \theta + \pi \Delta f_j (T_o - T_e)$

$d_0 = +1$ represents logical symbol 1, an error will occur when $H < 0$; Similarly, $d_0 = -1$ represents logical symbol -1, an error will also occur when $H > 0$. Due to the symmetry of the model, we can only consider the case, for example, $d_0 = +1$ but $H < 0$. Under this assumption, the bit error probability for CW interference is

$$P_e = P_r \left[\left(\sigma^2 + \text{var} (z_1)\right) > 0\right]$$

$$= \frac{1}{\sqrt{2\pi}\sigma} \exp \left[-\frac{(\alpha - \sqrt{\pi})^2}{2\sigma^2}\right] d\alpha$$

(24)

$$= Q \left(\frac{\sqrt{\pi}}{\sigma}\right)$$

where $Q(x)$ is the Q-function. The variance of the output is the sum of the variance due to the Gaussian noise and the variance due to CW interference from (23), which can be expressed as $\sigma^2 = \text{var} (H_1) + \text{var} (H_2)$.

By substituting $\sigma^2$ into (24), we can obtain

$$P_e (\phi) = Q \left[\frac{1}{\sqrt{2\pi}} \exp \left[-\frac{(\alpha - \sqrt{\pi})^2}{2\sigma^2}\right] d\phi\right]$$

(25)

The average bit error probability is calculated by integrating over $\phi$.

$$P_r = \frac{2}{\pi} \int_0^{\pi/2} P_e (\phi) d\phi$$

(26)

### B. Monte Carlo Simulation

We next construct the Monte Carlo simulation model to test the validity of (26). Consider the model [19] given in Fig. 5, the model contains uniform random number generator that generate binary numbers as date codes, which takes the value of either -1 or 1. Each data code repeats 1023 times (length of a cycle of the C/A code), then Gauss noise and CW interference signal are added to the data codes. It also includes decision device in which we can obtain the decision variable in the case of CW interference after the processing of coherent demodulation. In addition, the bit error rate of the system can be obtained after the comparator and error statistics.

![Fig. 5 The structure diagram of the Monte Carlo simulation model](image)

We know that, as empirical formula, the quantity of samples $N$ should satisfy the condition: $N > 10/P_r$, where $P_r$ is the estimated error probability. In this paper, we set $N=1000000$. In order to enable the GPS receiver to work properly, it is necessary that the signal-to-noise ratio of the receiving channel is not less than 10dB. As a result, we set signal-to-noise ratio $\varepsilon_d/N_0 = 10$dB in the following simulation.

![Fig. 6 The point distribution before the decision device in the case of: (a) JSR=15dB and $\Delta f_j=173$kHz, (b) JSR=15dB and $\Delta f_j=172$kHz, (c) JSR=25dB and $\Delta f_j=173$kHz, (d) JSR=25dB and $\Delta f_j=172$kHz](image)

A comparison of the point distribution before the decision device under different circumstances are shown in Fig. 6, the
brown points on the left represent the date code with a value of -1 while the blue points represent the date code which take the value of 1. Here, as we have discussed in the previous subsection, the decision threshold is 0, and an error will occur if the brown points cross over the decision threshold into the right region as well as the blue points into left region. Clearly, it can be concluded that in the case of low JSR (i.e. JSR=15 dB), the effect of CW interference on the performance of the GPS system is very small. With the increase of JSR, the power of the interference component is increased, and it causes more bit error. Meanwhile, the effect of $\Delta f_J = 173$ kHz on the performance of bit error probability is far greater than that of $\Delta f_J = 172$ kHz.

According to (26) the bit error probability performances of the GPS receiver under CW interference are plotted in Fig. 7. It can be inferred from Fig. 7 that in the range of error rates of interest, i.e., $P_e < 10^{-2}$, the effects of CW interference are significant. Since both $\Delta f_J = 172$ kHz and $\Delta f_J = 173$ kHz have the same performance under CW interference, the advantage of $\Delta f_J = 172$ kHz over $\Delta f_J = 173$ kHz against CW interference tends toward approximately 4 dB as $P_e \to 10^{-2}$. As the worst spectral line of PRN9 is 173 kHz, it can be concluded that CW interference provides the worst bit error probability performances when $\Delta f_J = 173$ kHz. However, the two curves gradually coincide when JSR over 30 dB. It should be noted that these conclusions are corresponding to averaging over $\phi$.

![Fig. 7 The contrast diagram of the formula and the Monte Carlo simulation results](image)

In order to compare these two results of our expression and Monte Carlo simulation, we also plot the Monte Carlo simulation results on the same figure. Clearly, the simulation curves of these two results are almost coincident, which show that our mathematical expression is effective. It should be noted that the different trend of these two types lines (i.e. solid and dashed lines) under low JSR, for example, JSR vary from 10 dB to 15 dB, is mainly result of finite number of Monte Carlo simulation points.

IV. CONCLUSION

In this paper, we make more detailed analysis for CW interference effects on the performance of GPS receivers. The influence of CW interference on the acquisition performance and bit error probability of GPS receivers is further analysed, and the simulation results are coincidence with the assumption. It is shown that CW interference has a significant effect on the GPS receiver performance. In addition, using the result of the characterization of the effect presented in this paper, we can further research the advantage of a preventative mitigation technique for CW interference.

With the development of new GPS signals and other satellite navigation systems, such as Beidou Navigation System, the research on the interference and anti-interference technology in satellite communication system become increasingly important. In future work, we plan to explore different types of interference signals. On one hand, in order to affect more than one satellite simultaneously, the sweep interference signals need to be generated. On the other hand, the integration of GPS receivers and micro-miniature inertial sensors is another popular research topic. These two aspects are inextricably linked, and that will be discussed in our future work.

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