Islanding Detection Techniques for Synchronous Distributed Generation

Bharti B. Parmar, Vivek J. Pandya

Abstract—The issue of unintentional islanding detection of grid connected synchronous distributed generation (SDG) remains the most challenging task faced by the distributed generation (DG) industry as SDG is highly capable of prolonging an island. This paper gives an insight of anti-islanding detection techniques mainly applied for SDG. Different techniques conclude that it is challenging to point out a generic method for a distinct purpose as the application of particular practice depends on nature of the end use and system dependent elements. Also, the setup and operational cost affect the selection of anti-islanding technique to achieve minimal compromising between cost and system quality. A test bench is created in the MATLAB/Simulink® to demonstrate the results of a 33 kV system. The results are highly satisfactory and they are according to the current practices.

Keywords—Synchronous distributed generation, islanding, point of common coupling, loss of grid.

I. INTRODUCTION

In last few years, due to advancement in technologies, electricity market deregulation, customer’s demand for better power quality, reliability, and environmental concern have resulted in increased use of small scale generation that is known as DG [1]. DG is customarily connected to the local distribution system. Penetration of DG in the local distribution system results in epitome changes in the centralized power generation [2], [3]. DG is a concept of installation and operation of small electrical generators. Synchronous/asynchronous machine (driven by internal combustion engines, combustion turbines, steam turbines, water turbines, wind turbines or electric motors), inverter based machines (supplied by dc storage sources, dc generating sources), or an ac generating source along with converter (such as a high or variable speed combustion or wind turbine) connected directly or indirectly, via power electronic converters to the distribution network are usually used for DG [4], [5]. Choice of the rotating machine, whether synchronous machine or induction machine, depends on some technical issues like steady state voltage profile, electrical power losses, voltage stability, transient stability, voltage sags during unbalanced faults and short circuit currents. Normally choice is done by considering main factors that limit the amount of DG in the system. If steady state voltage profile, voltage stability, and transient stability are considered, then the use of constant voltage synchronous generator is advantageous. It also permits to increase allowable penetration level of DG. On the other hand, if the system in which it is required to increase short circuit level, it is advantageous to use induction generator [6] [7].

DG improves the flexibility of the system by matching generation resources to system need as it can be located at numerous locations within utility service area. Ordinarily, DGs are placed near to load that improves the reliability of grid by minimizing impacts of transmission and distribution system disturbances. At the same time, it also reduces peak period congestion on the local load. The inclusion of DG in utility grid also improves security, reduce loading on T&D equipment, and reduce the necessity to build new transmission and distribution lines or up gradation of existing once. It also reduces transmission and distribution line losses and improves power quality and voltage profile of the system [8]-[11].

Traditionally, the radial distribution system is planned as a passive network, carrying the power as well as fault current from the high voltage level to medium or low voltage level (source to load) and no generation is expected in the network [12]. With DG, the distribution network gets active. Based on penetration level, placement and capacity of DGs, the amplitude and even the directions of power flow and fault currents may change which affects the existing protection related problems. Some of them are [10].

- out of phase reclosing of DG due to instantaneous recloser which results in substantial mechanical torques and currents which can damage the generators or prime movers
- Change in system earthing when islanding takes place
- Prohibited/delayed operation of protection technique when DG is islanded and not tripped
- Power quality problem during islanding operation
- Safety hazard to network personal during island operation
- Risk of accidents increases due to multiple sources which make the system more complicated
- Nuisance tripping of DG
- Possibility of false tripping of healthy feeders
- Up-gradation of primary substation bus bar protection
- Change in network fault level
- Increase in network voltage variations
- Prevention of the operation of auto recloser
- Indefinite fault clearing time in case of earth fault

Due to above-listed problems, DGs are typically not designed for feeding the public network alone. To avoid the possible hazard, it is a standard practice to disconnect the DG units as fast as possible during islanding. DG also remain
disconnected until the connection to the central grid is restored.

In this paper, the concept of islanding, hazards due to islanding, current scenario of islanding detection, different islanding detection techniques used for the SDG is briefly discussed in Sections II, III, IV and V, respectively. Simulation result of a typical system along with system data is given in Section VI, which shows the need for islanding detection of DG. In last, in Section VII conclusion is given.

II. CONCEPT OF ISLANDING

According to IEEE STD 1547-2003 [5], an island is a state of the electrical power system in which a portion of a grid is energized solely by DG when that portion of the grid is electrically isolated from the rest of the power system. System switching operation caused by fault clearance, scheduled/unscheduled load shedding and maintenance outages are some reasons of islanding. DGs are typically not designed for feeding the public network alone due to certain reasons such as hazards to the network personnel, poor power quality, and chances of damage to equipment, etc. because islanded grid cannot effectively control its frequency and voltage. Hence, the islanding situation must be detected as soon as possible. The islanding detector detects this condition and the inter-tie circuit breaker separates the DG and the utility network after the loss of grid.

Interconnection system responses to abnormal frequencies and voltages, as per IEEE 1547 – 2003, are shown in Tables I and II. According to IEEE standard 1547-2003 [5], the maximum allowable time to detect islanding conditions is 2 seconds.

### TABLE I

INTERCONNECTION SYSTEM RESPONSE TO ABNORMAL FREQUENCIES AS PER IEEE 1547 – 2003 [5]

<table>
<thead>
<tr>
<th>DR Size</th>
<th>Frequency Range (Hz)</th>
<th>Clearing Times (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>≤ 30 kW</td>
<td>&gt; 60.5</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>&lt; 59.3</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>&gt; 60.5</td>
<td>0.16</td>
</tr>
<tr>
<td>&gt; 30 kW</td>
<td>&lt; [59.8 to 57.0]</td>
<td>Adjustable</td>
</tr>
<tr>
<td></td>
<td>(adjustable set point)</td>
<td>0.16 to 300</td>
</tr>
<tr>
<td></td>
<td>&lt; 57.0</td>
<td>0.16</td>
</tr>
</tbody>
</table>

DR ≤ 30 kW, maximum clearing times; DR > 30 kW, default clearing times.

### TABLE II

INTERCONNECTION SYSTEM RESPONSE TO ABNORMAL VOLTAGES AS PER IEEE 1547 – 2003 [5]

<table>
<thead>
<tr>
<th>Voltage Range (% of Base Voltage)</th>
<th>Clearing Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V &lt; 50</td>
<td>0.16</td>
</tr>
<tr>
<td>50 ≤ V ≤ 88</td>
<td>2.00</td>
</tr>
<tr>
<td>110 ≤ V ≤ 120</td>
<td>1.00</td>
</tr>
<tr>
<td>V &gt; 120</td>
<td>0.16</td>
</tr>
</tbody>
</table>

a Base voltages are the nominal system voltages stated in ANSI C 84.1
b DR ≤ 30 kW, maximum clearing times; DR > 30 kW, default clearing times.

III. HAZARDS DUE TO ISLANDING

A. Power Quality

Power system operator is obliged to supply quality power to the customer. When islanding takes place, the voltage and frequency provided to the customer can vary significantly due to a mismatch in load and generation, and it may cross the constitutional limits in some cases that may cause damage to customer’s equipment because the utility has no control over them [13].

B. Personal Safety

The distribution network is designed as a passive network, carrying power from the high voltage level to medium or low voltage level and no generation is anticipated in the network. With DG, the distribution network gets active. Based on penetration level, placement and capacity of DG, the amplitude and even the direction of power flow may change. When the utility is disconnected, and formation of the island takes place, a section of the network that is assumed to be dead can remain energized by DG units. Utility personnel sent out for maintenance work may get in contact with the live part of the equipment that pretends a safety hazard to utility maintenance workers and the general public which are considered as the most severe safety hazard caused by islanding [12].

C. Out of Synchronism Recloser

With the help of reclosing, service can be restored in a distribution network after fault events. As 85 to 90% of overhead line faults are temporary, auto reclosing is adequate to reduce the minute customer loss [14]. When auto recloser reconnects the island to the supply system, DG damage may take place because the generators are not in synchronism with the system at the instant of reconnection. During out of synchronism, the recloser may cause overcurrent, overvoltage and severe transients, which may put the rotating machines and other equipment connected to the system at risk. [15].

D. Interference with Electric Power System Protection

If DG continues to run after utility breaker opens due to any temporary fault within the local section of the utility; the fault current contribution from the DG may cause miscoordination of utility protection functions. As a result, the temporary fault may become a permanent outage.

E. Stability Concern

Increased DG penetration in the system affects the system voltage and frequency stability. Hence, it is required that DG should not be tripped during system disturbances but continue to provide real and reactive power support to the utility. The anti-islanding algorithm must be able to differentiate between particular islanding situations for which fast disconnection from the grid is necessary and the other system disturbance for which DG disconnection is not required. Due to these reasons, it is remarkably important to detect the islanding quickly and accurately.
IV. CURRENT SCENARIO OF ISLANDING DETECTION

As stated in IEEE 1547-2003 [5], the DG must be de-energized when unintentional islanding takes place due to unfavorable conditions (as mentioned in Section III) which may take place. The DG protection must detect the islanded condition and within two seconds, the associated circuit breaker must be tripped. If auto reclosers are provided, then this detection time may be less than two seconds depending on protection coordination reclosing time of auto recloser. The common protection used for synchronous DG for unintentional islanding are 51 V - overcurrent, 87- differential, 51 N – earth fault time delayed overcurrent, 59/27 – over/under voltage, 32 – reverse power, 81 O/81 U – over/under frequency, 40 – loss of excitation.

As islanded operation of DG can improve service reliability to the customer, effort is made for intentional islanded operation of DG [10]. IEEE 1547 – 4 is a drafted regulation that will guide utilities or independent power producer to design and operate islanded DG and its grid reconnection. However, islanded operation is prohibited, although if technical issues as mentioned in Section III are resolved, DG can be operated in islanded mode.

To operate the DG in islanded mode, proper planning is the prime requirement that includes suitable islanding detection technique, an operation of DG in grid connected mode and islanded mode and reconnection of DG with the grid. Along with this, to transfer data and control information, fast and reliable communication link is required. Moreover, generator controller plays an essential role in islanding operation. The controller can be designed and modeled for two modes of operation – the grid connected mode and the islanded mode.

V. DIFFERENT ISLANDING DETECTION TECHNIQUES FOR SDG

The islanding detection technique must detect all islanding conditions. At the same time, it must not respond to disturbances in the system other than islanding. The technique must detect the islanding condition within required timeframe. The typical time delay of an auto-recloser is about 3 to 30 s hence the anti-islanding technique must be able to trip the associated DG units before reclosing take place. The typical recommended time for islanding detection is 0.5 s [16]. Moreover, islanding detection technique depends on the type of DG unit. As mentioned in Section I, there are mainly two types of DG – rotating type (synchronous generator and asynchronous generator) and inverter type. Both types of machine have different characteristics and operational principle due to which the developed techniques can be used either for synchronous generator or inverter based DG. The following section gives the review of islanding detection techniques commonly used for synchronous based DG.

Islanding detection techniques are principally sorted into two categories: Remote technique and local technique. The local technique is further divided into two main categories: Passive technique and active technique. Nowadays, the hybrid technique is also developed in which active and passive both techniques are used.

A. Remote Techniques

The remote technique depends on communication between DG and utility and give better reliability as compared to local techniques but are expensive to implement and hence uneconomical. In this technique, disconnection decisions are to be made by the utility company.

Communication-based transfer trip technique is proposed in [17] where supervisory control and data acquisition (SCADA) system observe the status of the circuit breaker and reclosers. The principle of SCADA system is to monitor the state (e.g. voltage, frequency, and other characteristics) of entire distribution system for unintentional islanding [18]. This information is then sent through to a central station communication links. After the disconnection of utility, if the parameter (voltage or frequency) can be detected from the disconnected area, the occurrence of islanding can be detected. The transfer trip technique is highly efficient to detect unintentional islanding and the problem of non-detection zone (NDZ) is eliminated, but the main drawback is its high cost and potential complexity due to the requirement of the signal transmitter which will be required at all possible disconnection points in the system. Moreover, if the feeder topology varies and there are many reclosers, this technique can become more complicated.

In [19], [20], communication-based power line signaling technique is conferred for islanding detection. In this technique, a signal generator at the transmission system continuously broadcasts a signal to all DG and/or distribution feeder using the power line as the signal path. The main advantages of this technique are no NDZ, DG inverter’s output power quality is not degraded, and some inverters on the system do not affect the performance of the technique and effectiveness at any penetration level. But when only few DG units are in service, high cost associated with the signal generator and its installation may make this technique unattractive.

Local Techniques

Local techniques are meant on local measurements and are mainly of three types: Active technique (directly interacting with the power system operation by introducing small perturbations), Passive technique (based on locally available measurements) and Hybrid technique (combination of active and passive techniques). In local techniques, disconnection decision is made by the owner of DG [21]

a) Passive Techniques

In [22]-[24] Freitas et al. proposed an islanding detection technique based on the rate of change of frequency (ROCOF), but it gives satisfactory operation only when islanding takes place when there is high (15%) active power mismatch. Moreover, this technique is highly susceptible to nuisance tripping that can be eluded by setting a high threshold value which in turn increases the NDZ. In [25], for islanding detection, a technique based on the ROCOF over power (df/dP) is presented. The test result shows that when there is a small power mismatch between DG and local loads ROCOF
over active power is much more sensitive than the ROCOF over time. After that, [26] presented a technique based on the rate of change of voltage, which efficiently detects islanding events when reactive power mismatch is very high. But when reactive power mismatch is quite small, this technique fails to provide adequate discrimination between islanding and non-islanding situation. In [27], D. Bejmont presented capacitor insertion strategy to enhance the efficiency of islanding detection with the use of standard passive anti-islanding relays. The main advantage is that NDZ can be lessened even if there is a small mismatch between powers. But this technique can be used only when over voltage/under voltage, over/under frequency and ROCOF function is used for islanding detection. With other functions, this technique cannot be used. Techniques employing under/over voltage and under/over frequency, which operate according to the relative changes of voltage and frequency, are described in [28]-[30]. The main advantage of this technique is that whenever there is a large mismatch between generation and load, an islanding condition is easily detected. But when the generation and load are almost matched, this technique fails to detect the islanding condition. Moreover, they have a large NDZ in which the scheme fails to detect the islanding condition.

b) Active Techniques

In [31], reactive power export error based islanding detection technique is proposed. This technique is based on the set value of reactive power flow between the DG site and grid or at the point of common coupling. The main disadvantage of this technique is that it is slow and cannot be used in a system where DG has to generate active power only. In [32] is presented impedance measurement based islanding detection technique that used switching inverter devices as the injection source of non-harmonic frequency. These techniques are powerful in the laboratory environment but have a drawback of multi-generator signal corruption, higher interfacing cost, and signal measurability. In [33], a technique based on utilization of impedance between phase and earth is presented to detect islanding condition, but the result of this technique is similar to that of ROCOF technique.

Afterward, fuzzy rule based approach and wavelet transform based islanding detection techniques have been presented by different researchers [34]-[37]. However, these techniques fail to detect islanding condition during lower values of active and reactive power mismatch. Gaonkar et al. [38] presented islanding detection technique by estimation of phase angle using the phase-locked loop that quickly detects islanding events under matching DG and load power rating. Jing et al. [39] proposed islanding detection technique based on the positive feedback of voltage harmonics distortion. Moreover, all of the above techniques have not considered various other non-islanding events such as switching (ON/OFF) of the capacitor bank; short-circuit on an adjacent feeder, energization/de-energization of medium transmission lines and switching (ON/OFF) of the transformer. Samantaray et al. [40] proposed a probabilistic neural network (PNN) based islanding detection technique for DG considering multiple parameters to secure the detection of islanding for any possible network topology. However, large training sets, tedious training process, and a large number of neurons are the several disadvantages of the neural network (NN) based techniques. Also, the said technique takes longer time (more than 0.85 s) to detect islanding conditions in a large power distribution network with multiple DG. Later on, [41] proposed singular wavelet entropy based islanding detection technique for lower power mismatches. However, real-time implementation of the said technique is extremely complicated due to hardware limitations. Moreover, the above technique has not considered various other non-islanding events such as switching (ON/OFF) of the capacitor bank, energization/de-energization of medium transmission lines; short-circuit on an adjacent feeder and switching (ON/OFF) of the transformer. Further, aforementioned non-islanding events may produce noise due to which said Wavelet based technique does not provide defective discrimination between islanding and non-islanding conditions [42]. Hence, none of these techniques are proficient in identifying islanding condition with various non-islanding situations irrespective of active and reactive power inequality.

VI. SIMULATION RESULT

Fig. 1 shows line diagram of a system used to carry out simulation work. This system is comprised of a 132 kV, 60 Hz sub-transmission system with a 1500 MVA short circuit level (represented by Thevenin’s equivalent) which feeds a 33 kV distribution system through 132/33 kV transformer. At bus 5, a 30 MVA synchronous generator is connected which is connected to the network via 33/0.69 kV transformer. The lines are modeled as RL series impedances and transformers are modeled using the T circuit. The synchronous generator is represented by a sixth order three phase model in a DQ rotor reference frame and is equipped with an AVR given by the IEEE type 1 model. Table III gives complete data for the test system [43]. With these data, load – generation scenario as given in Table IV is considered to verify system stability.

Fig. 1 Single line diagram of system under study

For scenario – 1, the system is analyzed and its root locus is developed as shown in Fig. 2. It shows that when the system is grid-connected the roots of the characteristic equation lies on the left side of S – plane which indicates that the system is stable. Details of equations required to carry out this study are given in the appendix.

Islanding detection technique must detect all possible conditions for which the DG should be islanded. For that, all possible combination of active and reactive power generation for different load conditions should be considered. The
possible combinations are a surplus of active and reactive power, a shortfall of active power and surplus of reactive power, a shortfall of active and reactive power, and surplus of active power and shortfall of reactive power. Table IV shows how different situation can be simulated. A test bench is created in the MATLAB/Simulink® and response of SDG for different load – generation scenario is observed when the SDG is islanded from the grid.

TABLE III
SYSTEM DATA

<table>
<thead>
<tr>
<th>Thevenin’s Equivalent (Sub) Data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Voltage (kV)</td>
<td>132</td>
</tr>
<tr>
<td>Short-Circuit Power (MVA)</td>
<td>1500</td>
</tr>
<tr>
<td>Resistance (Ω)</td>
<td>0</td>
</tr>
<tr>
<td>Inductance (mH)</td>
<td>30.80</td>
</tr>
</tbody>
</table>

| Transformers Data |
|-------------------|-----------------|
| Transformer       | Nominal Power (MVA) | Transformer       |
| 132/33 kV         | 100             | 33/0.69 kV       |
|                   | 50              |                  |
|                   |                  |                  |
|                   |                  |                  |
| Primary Winding   | Δ                | Δ                |
| Primary Voltage   | 132              | 33               |
| Secondary Winding | Z_s,             | Z_e,             |
| Secondary Voltage | 33               | 0.69             |
| Resistance (pu)   | 0                | 0                |
| Inductive Reactance (pu) | 0.04 | 0.04 |

TABLE IV
LOAD GENERATION SCENARIO

<table>
<thead>
<tr>
<th>Scenario</th>
<th>SDG Generation</th>
<th>Load at Bus - 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario - 1</td>
<td>1 pu</td>
<td>30 MW + j10 MVAR</td>
</tr>
<tr>
<td>Scenario - 2</td>
<td>0.8 pu</td>
<td>20 MW + j7 MVAR</td>
</tr>
<tr>
<td>Scenario - 3</td>
<td>0.4 pu</td>
<td>10 MW + j3 MVAR</td>
</tr>
</tbody>
</table>

Fig. 2 Root locus of the system under study with scenario - 1

Fig. 3 shows results of the simulation. Here, three different load-generation scenarios, as shown in Table IV, are simulated and response of DG frequency is observed.

From Fig. 3, it is observed that when load – generation balance is maintained in the system, the system frequency remained constant and no change is frequency has been observed. But when SDG is islanded from the grid, the SDG frequency initially starts to reduce and then continue to increase. To prevent such kind of operation and possible damage of SDG, it must be tripped. At the same time, to have possible advantages of distribution generation, a technique should be developed with help of which, load – generation balance can be obtained even in islanded mode of SDG.
of active distribution networks with DG. Simulation result shows that even for different load-generation condition, frequency violation takes place when the DG is islanded from the grid. Anti-islanding protection significantly prevents utilization of DG for enhancing power quality and reliability. But to have benefits of DG, a technique should be developed for disconnecting DGs from the utility in case of islanding situation and allowing them to operate as power islands in all possible load generation condition. These techniques should be fast, effective, easy to implement and as maintenance-free as possible. Research should also aim at developing low-cost techniques which should be cheap enough to attract the attention of the DG owners.

**APPENDIX [44]**

\[ \frac{H}{\pi f_0} \frac{d^2 \delta_0}{dt^2} = P_m - P_{max} \sin \delta_0 \]  

(1)

Here

\[ P_{max} = \frac{|E|}{|V|} \]

For small disturbances, the above equation can be written as

\[ \frac{H}{\pi f_0} \frac{d^2 (\delta_0 + \Delta \delta)}{dt^2} = P_m - P_{max} (\sin \delta_0 + \Delta \delta) \]

or

\[ \frac{H}{\pi f_0} \frac{d^2 \delta_0}{dt^2} + \frac{H}{\pi f_0} \frac{d^2 \Delta \delta}{dt^2} = P_m - P_{max} (\sin \delta_0 \cos \Delta \delta + \cos \delta_0 \sin \Delta \delta) \]

for small \( \Delta \delta, \cos \Delta \delta \approx 1 \) and \( \sin \Delta \delta \approx \Delta \delta \). So,

\[ \frac{H}{\pi f_0} \frac{d^2 \delta_0}{dt^2} + \frac{H}{\pi f_0} \frac{d^2 \Delta \delta}{dt^2} = P_m - P_{max} \sin \delta_0 - P_{max} \cos \delta_0 \Delta \delta \]  

(2)

Considering initial operating state, (2) reduces to linearized equation in terms of incremental changes in power angle i.e.

**VII. CONCLUSION**

Rapid augmentation of DG and its penetration in the grid has been increased across the world to a very high level. This paper presents a comprehensive survey of islanding protection
The quantity $P_{max} \cos \delta_0$ is the slope of power angle curve at $\delta_0$ and is equal to $P_s$, which plays an important role to determine system stability and is given by

$$P_s = \frac{dP}{d\delta}|_{\delta_0} = P_{max} \cos \delta_0$$  \hspace{1cm} (4)

Substituting (4) into (3),

$$\frac{H}{\pi f_0} \frac{d^2 \Delta \delta}{dt^2} + P_s \Delta \delta = 0$$  \hspace{1cm} (5)

The roots of (5) is given by

$$s^2 = -\frac{\pi f_0}{H} P_s$$  \hspace{1cm} (6)

For $P_s < 0$, the roots are in the right half s-plane, and the response is exponentially increasing and stability is lost. When $P_s > 0$, two roots are on the j-w axis, the motion is oscillatory and un-damped. The system is marginally stable with a natural frequency of oscillation given by

$$\omega_n = \sqrt{\frac{\pi f_0}{H} P_s}$$  \hspace{1cm} (7)

Induction motor action will take place as long as there is a difference in angular velocity between the rotor and the resultant rotating air gap field. A torque, known as damping torque, will be developed on the rotor which will minimize the difference between the two angular velocities. The damping power is proportional to the speed deviation and is given by

$$P_d = D \frac{d\delta}{dt}$$  \hspace{1cm} (8)

where $D$ can be obtained from design data or by test. When $P_s$ is positive, because of damping power, oscillations will damp out and the operation at the equilibrium angle will be restored. No loss of synchronism occurs and the system is stable.

If damping is accounted for, the linearized swing equation becomes

$$\frac{H}{\pi f_0} \frac{d^2 \Delta \delta}{dt^2} + D \frac{d\Delta \delta}{dt} + P_s \Delta \delta = 0$$  \hspace{1cm} (9)

or

$$\frac{d^2 \Delta \delta}{dt^2} + \frac{\pi f_0}{D} \frac{d\Delta \delta}{dt} + \frac{\pi f_0}{H} P_s \Delta \delta = 0$$  \hspace{1cm} (10)

or in terms of the standard second order differential equation,

$$\frac{d^2 \Delta \delta}{dt^2} + 2\zeta \omega_n \frac{d\Delta \delta}{dt} + \omega_n^2 \Delta \delta = 0$$  \hspace{1cm} (11)

where $\omega_n$ is the natural frequency of oscillation and $\zeta$ is damping ratio given by

$$\zeta = \frac{D}{2} \sqrt{\frac{\pi f_0}{HP_s}}$$  \hspace{1cm} (12)

The characteristic equation is

$$s^2 + 2\zeta \omega_n s + \omega_n^2 = 0$$  \hspace{1cm} (13)

For normal operating conditions, $\zeta < 1$, and roots of the characteristic equation are complex which is given by

$$s_1, s_2 = -\zeta \omega_n \pm j\omega_d \sqrt{1-\zeta^2}$$  \hspace{1cm} (14)

where $\omega_d$ is the damped frequency of oscillation given by

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$  \hspace{1cm} (15)

For positive damping, roots of the characteristic equation must have magnitude real part if synchronizing power coefficient $P_s$ is positive. The response is bounded and the system is stable.

REFERENCES


[16] Engineering Recommendation G59/1, Recommendations for the connection of Embedded Generating plant to the Regional Electricity Companies.


