Workspace Analysis of 6–6 Cable-Suspended Parallel Robots
Arian Bahrami, Amir Teimourian

Abstract—In this paper, the effect of the moving platform size on the workspace volume of 6–6 cable-suspended parallel robots is investigated in details for different geometric configurations and orientations of the moving platform. The obtained hints can be used as a rule of thumb in designing this type of robot.

Keywords—Cable-suspended parallel robot, system analysis and design, workspace analysis.

I. INTRODUCTION

In 2004, Pusey et al. [8] concluded that for any geometry of the moving platform (MP) the same size as the base platform (BP). In other words, for a fixed size of the BP, as the ratio of MP/BP increases, the workspace volume increases as shown in their Fig. 5.

II. KINEMATIC MODELING

The 6-6 cable-suspended parallel robot has a fixed platform, an MP and six cables each connected to a connection point on the MP Ai and one of the connection points on the fixed platform Bi, allowing six degrees of freedom as shown in Fig. 1. Or is considered as the origin of the fixed frame XYZ which is located at the center of the BP and Or is considered as the origin of the body frame xyz attached to the mass center of the MP. rbase is the radius distance from Or to the connection points on the base platform Bi and rend is the radius distance from Or to the connection points on the moving platform Ai. Figs. 2 (a) and (b) show the γ angle of the BP (γbase) and the MP (γend), respectively. The applied diagram of the model is shown in Fig. 3. This expression can be obtained from Fig. 3:

\[ l_i = p + R M a_i - b_i \quad , i = 1, 2, 3, ..., 6 \]  \hspace{1cm} (1)

where bi is the position vector of the connection points on the base platform Bi in the fixed frame XYZ, \( M a_i \) is the position vector of the connection points on the moving platform Ai in the body frame xyz, \( p \) is the position vector of the point Or, \( l_i \) is the cable length vector from \( B_i \) to \( A_i \) and \( R \) is the rotation matrix of the MP with respect to the BP with three rotation angles \( \psi, \theta, \phi \) about the fixed axes of X, Y and Z, respectively which can be defined as:

Arian Bahrami and Amir Teimourian are with the Mechanical Engineering Department, Eastern Mediterranean University, G. Magosa, TRNC Mersin 10, Turkey (phone: +90 392 630 2598; e-mail: arian.bahrami@emu.edu.tr).
The cable length $q_i$ can be defined as:

$$q_i = \|l_i\| = (l_i^T l_i)^{1/2}, \quad i = 1, 2, 3, ..., 6$$

The Jacobian matrix $J$ relates the twist of the $MP$ $\tau = [\tau^T \omega^T]^T$ to the cable velocity vector $\dot{q} = [\dot{q}_1 \dot{q}_2 \dot{q}_3 \dot{q}_4 \dot{q}_5 \dot{q}_6]^T$, written as:

$$\dot{q} = J\tau$$

where $\omega^T$ is the angular velocity vector of the MP with respect to the fixed frame $XYZ$. The Jacobian matrix $J$ is represented such that its $i$th row is:

$$j_i = \begin{bmatrix} l_i^T \\ b_{a_i} \times l_i^T \\ \end{bmatrix}, \quad i = 1, 2, ..., 6$$

where,

$$b_{a_i} = R^M a_i$$

The relation between the cable tensions and external wrench on the MP can be defined as:

$$J^T s = F_{ext}$$

where $s$ is the vector of cable tensions $s = [s_1 s_2 s_3 s_4 s_5 s_6]^T$, and $F_{ext}$ is the vector of external wrench applied to the MP, $F_{ext} = [0 0 -mg 0 0 0]^T$. For calculating tension in the cables, (7) can also be written as:

$$s = J^T F_{ext}$$

The static equilibrium workspace is defined by the number of points where the mass center of the MP can be placed while all the cables must be in tension [8]. Therefore, at each point within the search volume if all the elements of the vector of the cable tensions obtained from (8) have nonnegative values $s_i \geq 0$, that point belongs to static equilibrium workspace volume.
III. RESULTS

A MATLAB program is created for analyzing the workspace volume of the 6-6 cable suspended robot which tests the points within the search space to define whether they belong to the static equilibrium workspace volume of the robot or not. The program input includes the size and the geometry of both platforms (\(r_{\text{base}}, r_{\text{end}}, \gamma_{\text{base}}, \gamma_{\text{end}}\)), the position and orientation of the MP, the desired search volume, and incremental step size for the search. To keep consistency, the values of the step size along all three axes, the search volume for possible workspace volume, and the geometry of both platforms are taken from Pusey et al. [8] as:

- The search volume is a cube with the dimensions: \(-10 \leq Z \leq 0\) m, \(-8\) m \(\leq Y \leq 8\) m, \(-8\) m \(\leq X \leq 8\) m.
- The step size for the search is 0.4 m for all X, Y, and Z axes.
- The geometry of the BP is the same as the MP: \(\gamma = \gamma_{\text{base}} = \gamma_{\text{end}}\).

By setting these values of the parameters and using (8), one can obtain the number of points as the workspace volume index.

![Graph](image1)

**Fig. 4** The effect of the ratio of BP/MP on the workspace volume for orientation \(\psi = \theta = \phi = 0\), \(r_{\text{end}} = 6\) m

![Graph](image2)

**Fig. 5** The effect of the ratio of MP/BP on the workspace volume for orientation \(\psi = \theta = \phi = 0\), \(r_{\text{end}} = 6\) m

The variation of the average workspace volume with respect to the ratio of BP/MP in case of zero rotation for different geometric configurations \(\gamma\) can be observed that for a fixed size of the MP, as the size of the BP increases, the workspace volume increases for all geometric configurations. Fig. 5 shows the effect of MP/BP on the workspace volume in case of zero rotation for different geometric configurations \(\gamma\).

In attempting a broader range of studies besides the case of zero rotation \(\psi = \theta = \phi = 0\), 637 different orientations are considered. The workspace volume is evaluated for all 637 constant orientation combinations of \(\psi, \theta, \phi\) defined in Table I of [8] as:

- \(-30 \leq \psi \leq 30\), \(\Delta \psi = 10\)
- \(-30 \leq \theta \leq 30\), \(\Delta \theta = 10\)
- \(-60 \leq \phi \leq 60\), \(\Delta \phi = 10\)

Due to the vast amount of information involved, the individual workspace volume shapes at every constant orientation are not presented. To reduce the amount of data pertaining to 637 orientations observed, averages of the workspace volume are used. All the obtained workspace volume values for 637 different roll, pitch and yaw angles are averaged for different geometric configurations, the BP and the MP sizes.

![Graph](image3)

**Fig. 6** The effect of the ratio of MP/BP on the average workspace volume \(r_{\text{end}} = 6\) m

The variation of the average workspace volume is presented in Table I and Figs. 6 and 7. It is noticeable that the trend seen in Figs. 6 and 7 is nearly similar to the case of zero rotation presented in Figs. 4 and 5. The obvious difference is that when the size of the BP is constant, as the size of the MP increases, the average workspace volume increases.
Fig. 7 The effect of the ratio of BP/MP on the average workspace volume ($r_{var}=6 \text{ m}$)

TABLE I

<table>
<thead>
<tr>
<th>MP/MP</th>
<th>$\gamma$ =0</th>
<th>15</th>
<th>30</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1538.6</td>
<td>602.9</td>
<td>161.6</td>
<td>18.0</td>
</tr>
<tr>
<td>0.4</td>
<td>1595.5</td>
<td>633.3</td>
<td>176.0</td>
<td>18.3</td>
</tr>
<tr>
<td>0.6</td>
<td>1646.1</td>
<td>657.8</td>
<td>188.0</td>
<td>21.2</td>
</tr>
<tr>
<td>0.8</td>
<td>1681.3</td>
<td>674.7</td>
<td>196.0</td>
<td>24.4</td>
</tr>
<tr>
<td>1.0</td>
<td>1704.0</td>
<td>682.9</td>
<td>198.9</td>
<td>25.2</td>
</tr>
</tbody>
</table>

TABLE II

<table>
<thead>
<tr>
<th>MP/MP</th>
<th>$\gamma$ =0</th>
<th>15</th>
<th>30</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1535.2</td>
<td>602.9</td>
<td>161.6</td>
<td>18.0</td>
</tr>
<tr>
<td>0.4</td>
<td>1578.6</td>
<td>633.0</td>
<td>176.0</td>
<td>18.2</td>
</tr>
<tr>
<td>0.6</td>
<td>1610.8</td>
<td>655.6</td>
<td>187.9</td>
<td>21.1</td>
</tr>
<tr>
<td>0.8</td>
<td>1615.5</td>
<td>666.9</td>
<td>195.2</td>
<td>24.3</td>
</tr>
<tr>
<td>1.0</td>
<td>1594.3</td>
<td>665.4</td>
<td>196.4</td>
<td>25.0</td>
</tr>
<tr>
<td>1.2</td>
<td>1551.7</td>
<td>650.2</td>
<td>193.4</td>
<td>25.4</td>
</tr>
<tr>
<td>1.4</td>
<td>1478.5</td>
<td>621.6</td>
<td>186.4</td>
<td>23.0</td>
</tr>
<tr>
<td>1.6</td>
<td>1404.7</td>
<td>588.4</td>
<td>173.9</td>
<td>19.5</td>
</tr>
</tbody>
</table>

The average workspace volume difference between the smallest and largest sizes of the MP is about 10%, 13%, 23%, and 40% for the geometric configurations $\gamma = 0$, 15, 30, 45, respectively as presented in Table I. It should be mentioned that the possible collision between a physical BP and MP was not considered in [8] and our initial calculation. It is evident that as the size of the MP increases, the average workspace volume decreases due to an increase in the possibility of collision between two platforms for any small rotation of a larger MP near the BP. In other words, the average workspace volume difference would be less considering the possible collision which will be shown later in Fig. 8 and Table II.

Table II and Fig. 8 show the variation of the average workspace volume with respect to the size of the MP when the possible collision between a physical BP and MP is taken into account in the calculation. The z coordinate of all the MP vertices should be below the BP to avoid any possible collision formulated as:

$$\mathbf{K}(\mathbf{p} + \mathbf{R}^{	ext{T}} \mathbf{a}_i) < 0, \ i = 1, 2, 3, ..., 6$$ (9)

It is noticeable from Fig. 8 that there exists an optimal MP size for any geometric configuration where the average workspace volume reaches its maximum value in this case. The average workspace volume reaches its maximum value at the MP/BP ratios of 0.73, 0.80, 0.93, and 1.20 for the geometric configurations $\gamma = 0$, 15, 30, 45, respectively. As a result, the optimal size of the MP increases with increasing the geometric configuration $\gamma$. Moreover, it can be deduced from Tables I and II that the effect of the possible collision on the average workspace volume is more significant for the geometric configuration $\gamma = 0$ while the effect is less significant for the geometric configuration $\gamma = 45$. These hints can be used as a rule of thumb in designing this type of robot and similar configurations.

IV. CONCLUSION

This paper presents the effect of the MP size on the workspace volume of 6–6 cable-suspended parallel robots considering different geometric configurations and orientations of the MP. The following conclusions were drawn:

- For a fixed size of the BP, as the ratio of MP/BP increases, the workspace volume remains constant for all geometric configurations in case of zero rotation of the MP.
- For a fixed size of the MP, as the ratio of BP/MP increases, the workspace volume increases for all geometric configurations in case of zero rotation of the MP.
- For a fixed size of the BP, as the ratio of MP/BP increases, the average workspace volume increases for all geometric configurations.
- The size of the MP has the least effect on the average workspace volume for the geometric configuration $\gamma = 0$
while it has the greatest effect on the average workspace volume for the geometric configuration $\gamma = 45$.

- There exists an optimal MP size for any geometric configuration where the average workspace volume reaches its maximum value when the effect of the possible collision between the BP and the MP is taken into account.
- The optimal size of the MP increases with increasing the geometric configuration $\gamma$.
- The effect of the possible collision on the average workspace volume is more significant for the geometric configuration $\gamma = 0$ while the effect is less significant for the geometric configuration $\gamma = 45$.

REFERENCES