Channels Splitting Strategy for Optical Local Area Networks of Passive Star Topology

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***Abstract*—**In this paper, we present a network configuration for a WDM LANs of passive star topology that assume that the set of data WDM channels is split into two separate sets of channels, with different access rights over them. Especially, a synchronous transmission WDMA access algorithm is adopted in order to increase the probability of successful transmission over the data channels and consequently to reduce the probability of data packets transmission cancellation in order to avoid the data channels collisions. Thus, a control pre-transmission access scheme is followed over a separate control channel. An analytical Markovian model is studied and the average throughput is mathematically derived. The performance is studied for several numbers of data channels and various values of control phase duration.

***Keywords*—**Access algorithm, channels division, collisions avoidance, wavelength division multiplexing.

I. Introduction

Control mechanisms are commonly used for the data communication coordination in Wavelength Division Multiplexing (WDM) [1] networks. In literature, several studies for WDM Local Area Networks (LANs) have been proposed that occupy a separate control channel, explicitly assigned to all stations for the control information exchange prior to the actual data packets transmission, like in [2]. Also, many other studies have involved more than one control channels to the control pre-transmission coordination strategy. Such studies, like [3]-[5], adopt the Multi-Channel Control Architecture (MCA) in order to reduce the probability of a control packet collision over the single control channel.

In such studies, the nature of the transmission WDMA Access (WDMA) protocol followed may vary. Thus in the synchronous transmission WDMA protocols, a common clock is assumed for all stations, while all stations are synchronized for transmission and reception [2]-[5]. On the other hand, in the asynchronous transmission, WDMA protocols the packets arrival to the common shared medium occurs in an asynchronous way, independently one from the others [2], [6].

In this study, we consider a WDM LAN of passive star topology that occupies a single control WDM channel explicitly assigned to all stations for the control information exchange. A pre-transmission coordination access scheme is followed according which the stations which attempt data packets transmission, have first to inform the others about their attempt by transmitting a control packet over the control channel. The control information processing at each station provides the appropriate knowledge in order to run in a decentralized way an efficient WDMA algorithm that totally avoids the collisions over the WDM data channels. Especially, the data channels are assumed to be split into two separate data channels sets. This assumption is exploited in order to provide additional transmission capabilities and consequently to enhance the performance.

In order to evaluate the proposed WDMA protocol performance, we developed an analytical Markovian model that gives the formula for the average throughput estimation. Extensive numerical results are studied for diverse values of the performance parameters.

This study is carried out and presented in this paper as follows: Section II describes the network model and assumptions. In Section III, the Markovian analysis is provided. The performance evaluation is given in Section IV, while Section V outlines some conclusions.

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II. Network Model and Assumptions

We assume a passive star network that interconnects a finite number M of stations. The total fiber bandwidth is divided into (N+1) parallel WDM channels, each operating in a different wavelength $\{\lambda_0, \lambda_1, \lambda_2,... \lambda_N\}$, where N is an even integer. The channel λ_0 is called control channel and it transmits the control packets, while the remaining channels $\{\lambda_1, \lambda_2,... \lambda_N\}$ are called data channels and they transmit the data packets.

The set of data channels $\{\lambda_1, \lambda_2,... \lambda_N\}$ is divided into two sets with equal number of data channels, as Fig. 1 depicts. The first set S_1 consists of the channels $\{\lambda_1, \lambda_2, ... \lambda_{N/2}\}$, while the second S_2 consists of the channels $\{\lambda_{1+N/2}, \lambda_{2+N/2},... \lambda_N\}$. In this way, the data channel λ_x, where: $x \in \{1, 2,... N/2\}$ from the set S_1 has an one to one correspondence to the data channel λ_y, where: $y \in \{N/2+1, N/2+2, ... N\}$ from the set S_2. Each station network interface is equipped with a tunable transmitter that can be tuned to all channels $\{\lambda_0, \lambda_1, \lambda_2,... \lambda_N\}$.

The control packet transmission time is defined as time unit reference and is called control mini-slot. Thus, the data packet transmission time normalized in time units is L and is called data slot (L>1). The control packet consists of the source address, the destination address and the data channel λ_x that belongs to the set S_1 and has been chosen for the corresponding data packet transmission.

Both control and data channels use the same time reference, referred to as a cycle. We define as a cycle as the time interval that includes W ($W \geq 1$) time units for the control packets transmissions plus the normalized data packet transmission time L, as Fig. 2 shows. Thus, the cycle time duration C is:

$$C = W + L \qquad \text{time units} \qquad (1)$$

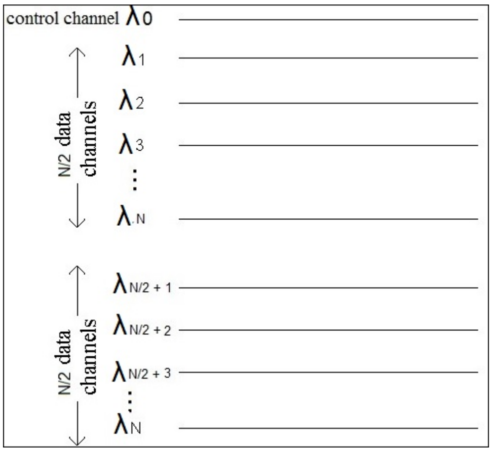

Fig. 1 Network architecture

We assume a synchronous transmission access scheme. The time interval during which all the stations activities take place is called a cycle. Each cycle consists of the control and the data phase, as Fig. 2 shows. The control phase consists of W time units, while the control packets transmissions occur. The data phase lasts for L time units, while the data packets transmissions take place. At the beginning of a cycle data phase, each station is able to transmit at a given wavelength λ_T. In our study, we assume that the tunable transmitter has negligible tuning time and very large tuning range.

At the beginning of a cycle, each station monitors the control channel λ_0 to be informed about the control packets transmissions from all stations during the control phase. Also, if it has to send a data packet to another station, it tunes its tunable transmitter to the control channel λ_0. Then, the station chooses randomly one of the data channels from the set S_1 for the data packet transmission, let us say data channel $\lambda_i \in \{\lambda_1, \lambda_2,... \lambda_{N/2}\}$. Also, it chooses randomly one of the control mini-slots for the control packet transmission, let us say the control mini-slot $j \in \{1, 2,... W\}$. Then, it informs the other stations about the λ_i selection, by transmitting a control packet during the j-th control mini-slot with its tunable transmitter.

The control packets from all stations compete according to the Slotted Aloha scheme. The station continuously monitors the control channel during the control phase. Thus, the station is aware of the data channel claims for transmission of all stations, grace to the broadcast nature of the control channel. We can say that the successfully transmitted control packets are uniformly distributed to the $N/2$ data channels with equal and constant probability $2/N$. So, if one or more other stations have selected the same j-th control mini-slot for transmission, the corresponding control packets have collided during the j-th control mini-slot and are all aborted. On the contrary, if the control packet has been successfully transmitted over the j-th control mini-slot, the station has to check the data channel field of the other successfully transmitted control packets. Thus, if exactly one more station has chosen the same data channel λ_i for transmission and its control packet transmission was successful, then the i-th data channel from the set S_1 is assigned to the first station for transmission, while the i-th data channel from the set S_2 is assigned to the second station for transmission, i.e. the λ_i and $\lambda_{i+N/2}$ data channels, respectively. Also, if more than one of the stations have chosen the same data channel λ_i for transmission and their control packets transmissions were successful, then an arbitration rule for the data channels assignment may be considered, such as priority. In this case, only two of these stations gain access to the data channels λ_i and λ_i+N/2 for transmission during the cycle data phase, while the other data packets' transmissions are cancelled. The stations who gain the access over the data channels start the transmission immediately.

Packets are generated independently at each station following a geometric distribution, i.e. a packet is generated at each cycle with birth probability p. A backlogged station retransmits the unsuccessfully transmitted packet following a geometric distribution with probability r.

We assume that each station is equipped with a transmitter buffer with capacity of one data packet. If the buffer is empty, the station is said to be free, otherwise it is backlogged. If a station is backlogged and generates a new packet, the packet is lost. Free stations, which unsuccessfully transmit on the control channel or in case of loss at the channel assignment competition during a cycle, are getting backlogged in the next cycle. A backlogged station is getting free at the next cycle, if it manages to retransmit without collision over a control channel and its data packet retransmission is not aborted due to the channel assignment competition.

III. Analysis

A discrete time Markov chain is used for the system description. The system state X_t, t=1, 2,… and X_t =0,1,…M is the number of backlogged stations at the beginning of each cycle. Let:

$S_W(k)$=The number of successful control packet (re)transmissions over the W control mini-slots, conditional that k free and/or backlogged stations attempt transmission during a cycle, and $0 \le S_W(k) \le \min(W,k)$.

The probability of s successes over the W control mini-slots from k (re)transmissions during a cycle is given by [5]:

$$\Pr[S_W(k) = s] = \frac{(-1)^s W! k!}{W^k s!} \sum_{j=s}^{\min(W,k)} \frac{(-1)^j (W-j)^{k-j}}{(j-s)!(W-j)!(k-j)!} \qquad (2)$$

and $0 \le s \le \min(W,k)$. Also, let: $A(s)$=The number of successfully transmitted data packets over the N data channels, conditional that s successful (re)transmissions occurred over the W control mini-slots during a cycle, $1 \le A(s) \le S_W(k)$ for every $S_W(k) > 0$.

The probability $\Pr[A(s) = y]$ of y successfully transmitted data packets over the N data channels from s successfully (re)transmitted packets over the W control mini-slots during a cycle is given by [7]:

$$\Pr[A(s)=y]=\sum_{all\ \text{sets}}\frac{\left(\frac{N}{2}\right)!s!}{\left(\frac{N}{2}\right)^{s}\prod_{i=0}^{s}k_i!\prod_{z=1}^{s}(z!)^{k_z}} \tag{3}$$

where: the sets of integers $\{k_0,k_1,k_2,...,k_{N/2}\}$, $i,k_i\in\{0,1,2,...,N/2\}$ satisfy the equations:

$$\sum_{i=0}^{s}k_i=\frac{N}{2} \tag{4}$$

$$\sum_{i=0}^{s}ik_i=s \tag{5}$$

$$k_1+\sum_{i=2,k_i\neq 0}^{s}2i=y \tag{6}$$

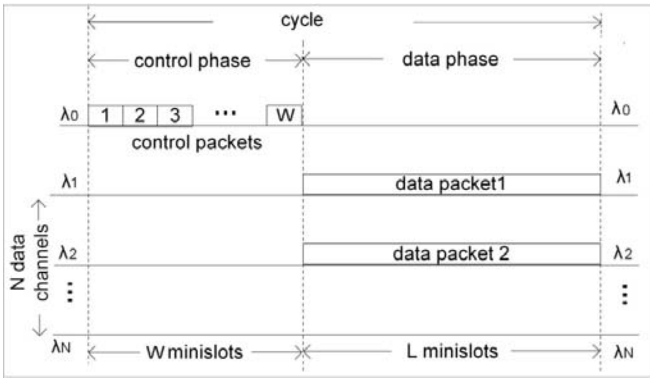

Fig. 2 Cycle definition

We define the function $F(k,y)$ as the sum of all possible products of the probability of y successfully transmitted data packets over the N data channels from k transmissions over the W control mini-slots, during a cycle. It is given by:

$$F(k,y)=\sum_{s=0\&s\geq y}^{\min(W,k)}\Pr[S_W(k)=s]\Pr[A(s)=y] \tag{7}$$

The Markov chain X_t, t=1,2.. is homogeneous, aperiodic and irreducible. The one step transition probabilities are given by:

$$P_{ij}=(X_{t+1}=j\,|\,X_t=i) \tag{8}$$

where:

Case A: $j<i\text{-}W$, then:

$$P_{ij}=0 \tag{9}$$

Case B: $j=i-W$, then:

$$P_{ij}=\binom{i}{W}r^{W}(1-r)^{j}(1-p)^{(M-j-W)}F(W,W) \tag{10}$$

Case C: $j=i-1$, then:

$$P_{ij}=\sum_{k=0}^{M}\sum_{y=0}^{\min(W,k)}\left[\binom{i}{k-y+1}r^{(k-y+1)}(1-r)^{(i-1-k+y)}\times \binom{M-i}{y-1}p^{(y-1)}(1-p)^{(M-(i-1)-y)}F(k,y)\right] \tag{11}$$

Case D: if $j=i$, then:

$$P_{ij}=\sum_{k=0}^{M}\sum_{y=0}^{\min(W,k)}\left[\binom{i}{k-y}r^{(k-y)}(1-r)^{(i-k+y)}\times \binom{M-i}{y}p^{y}(1-p)^{(M-i-y)}F(k,y)\right] \tag{12}$$

Case E: if $j=i+1$, then:

$$P_{ij}=\sum_{k=0}^{M}\sum_{y=0}^{\min(W,k)}\left[\binom{i}{k-y-1}r^{(k-y-1)}(1-r)^{(i+1-k+y)}\times \binom{M-i}{y+1}p^{(y+1)}(1-p)^{(M-i-1-y)}F(k,y)\right] \tag{13}$$

Case F: if $j>i+1$ or $i-W<j<i-1$, then:

$$P_{ij}=\sum_{k=0}^{M}\sum_{y=0}^{\min(W,k)}\left[\binom{i}{k-j+i-y}r^{(k-j+i-y)}(1-r)^{(j-k+y)}\times \binom{M-i}{j-i+y}p^{(j-i+y)}(1-p)^{(M-j-y)}F(k,y)\right] \tag{14}$$

A. Performance Measures

Since the Markov chain X_t, t=1,2… is ergodic, the steady state probabilities can be obtained by solving the system of the linear equations:

$$\pi=\pi P \tag{15}$$

$$\sum_{i=0}^{M}\pi_i=1 \tag{16}$$

where **P** is the transition matrix with elements the probabilities and $\boldsymbol{\pi}$ is a row vector with elements the steady state probabilities.

The conditional throughput $Thr(i)$ is the expected value of the successfully transmitted data packets over the N data channels during a cycle, given that the number of the backlogged stations at the beginning of the cycle is i, i.e.:

$$Thr(i) = \sum_{k=0}^{M} k \left(\sum_{y=0}^{\min(W,k)} yF(k,y) \right) \times \sum_{j=\max(0,k-1)}^{\min(k,M-i)} \binom{i}{k-j} r^{(k-j)} (1-r)^{(i-k-j)} \binom{M-i}{j} p^{j} (1-p)^{(M-i-j)} \tag{17}$$

Thus, the steady state average throughput Thr is given by:

$$Thr = \frac{L}{C} E[S(i)] = \frac{L}{C} \sum_{i=0}^{M} S(i) \pi_i \tag{18}$$

The steady state average number B of busy stations is:

$$B = E[i] = \sum_{i=0}^{M} i \pi_i \tag{19}$$

The average delay D is defined as the average number of cycles that a packet has to wait until its successful transmission. Delay is calculated by Little's formula. It is:

$$D = (W+L) + (W+L) \frac{B}{Thr} \tag{20}$$

IV. Performance Evaluation

In this section, the proposed WDMA protocol performance is extensively studied. For the numerical results, we assume that L=50 time units and r=0.3.

Fig. 3 presents the average throughput Thr versus the birth probability p, for W=6 control mini-slots and N=12, 20 data channels. As it is presented, for the same birth probability p, the throughput Thr curves increase, as the number N of data channels increases. This behavior can be explained by the fact that as the number N increases, the data packets -that are originated by stations whose control packets transmission was successful- are distributed into more data channels, trying to get access to the data multi-channel system. This fact gives rise to the probability of a data packet successful transmission over the two considered sets of data channels. As a result, the throughput Thr increases as the number N is getting higher. For example, for p=0.9 control packets/cycle, the throughput Thr is: 1.71 data packets/cycle for N=12 and 1.97 data packets/cycle for N=20.

In order to deeply study the proposed network configuration and WDMA protocol performance, we assume the probability that a station faces of cancelling a data packet transmission in order to avoid the channels collisions, although its control packet was successfully transmitted. This is a case when a station is informed that there is more than one other station -whose control packets were also successfully transmitted- and that they have selected the same data channel from the set S_1 for transmission. According to the arbitration rule followed, only two stations will finally transmit over the selected data channel from the set S_1 and the corresponding one from the set S_2, while the others will cancel their transmission to avoid the collisions.

The average probability of transmission cancellation to avoid channels collisions versus the birth probability p, for W=6 control mini-slots and N=12, 20 data channels is presented in Fig. 4. As it is noticeable, this probability is a decreasing function of N, for a given value of birth probability p. For example, for p=0.9 control packets/cycle, the transmission cancellation probability is: 0.01% for N=12 and 0.005% for N=20. The explanation comes from the discussion on Fig. 3. Thus, as the number N of data channels is getting higher, the probability of successful data packets transmission rises, providing lower values of transmission cancellation probability.

The variatio,n of the transmission cancellation probability as the number W of control mini-slots varies is studied in Fig. 5. Especially, Fig. 5 illustrates the average transmission cancellation probability versus the birth probability p, for N=20 data channels and W=6, 8, 10 control mini-slots. It is obvious that number W denotes the duration of the cycle control phase. Thus, as Fig. 5 depicts, as the duration of the cycle control phase increases for a given value of birth probability p, the transmission cancellation probability reaches higher values. This behavior is explained by the fact that as the number W is getting higher, the probability of a successful control packet transmission is also higher. As a result, the number of offered data packets over the data channels increases, and consequently, the probability of selecting the same data channel for transmission also increase. This is the reason why, the average transmission cancellation probability decreases, while the number W increase. For example, for p=0.9 control packets/cycle, the transmission cancellation probability is: 0.011% for N=10, 0.009% for N=8 and 0.005% for W=6.

V. Conclusion

In this study, we propose a network architecture in conjunction with an efficient access algorithm for synchronous transmission WDM LANs. The proposed network architecture occupies a separate control channel over which the appropriate control information is exchanged for the data packets transmission coordination. The adopted WDMA protocol assumes the separation of the data channels set into two different sets, with different access rights over them. As a result, we achieve eliminating the probability of cancelling the data packets transmission in order to avoid the collisions over the data channels.

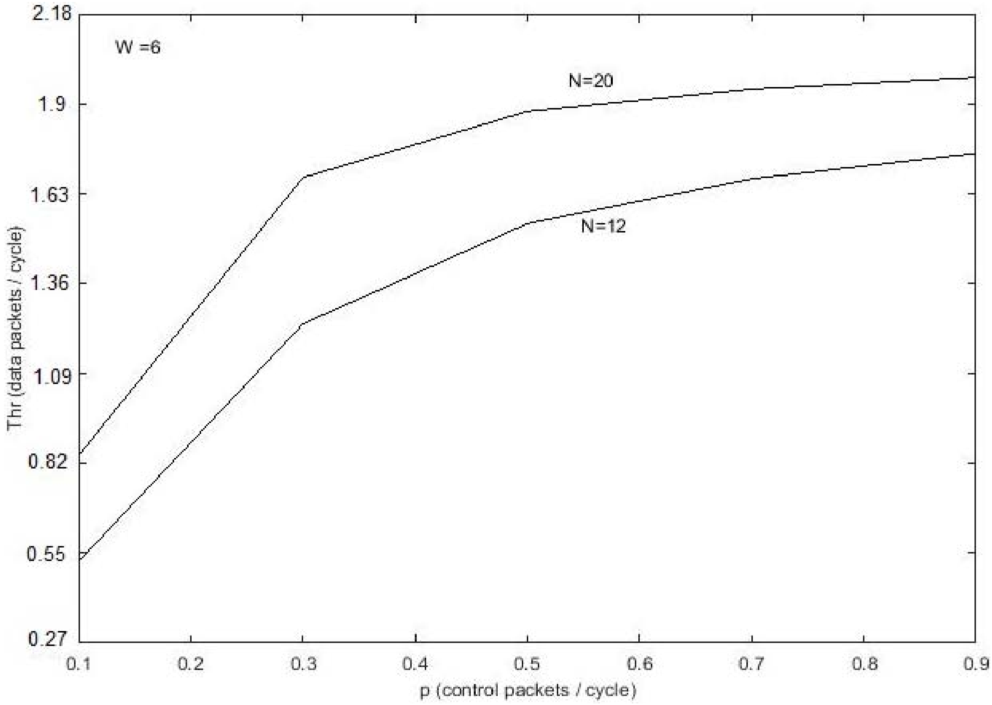

Fig. 3 Average throughput *Thr* vs. the birth probability *p*, for *W*=6, *N*=12, 20

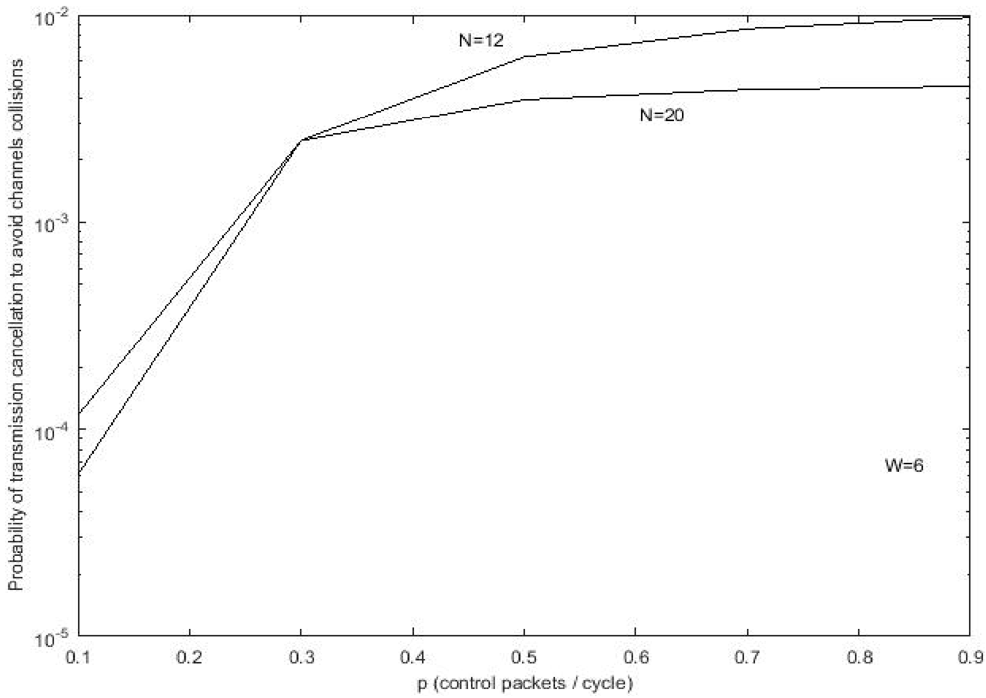

Fig. 4 Average probability of transmission cancellation to avoid channels collisions vs. the birth probability *p*, for *W*=6, *N*=12, 20

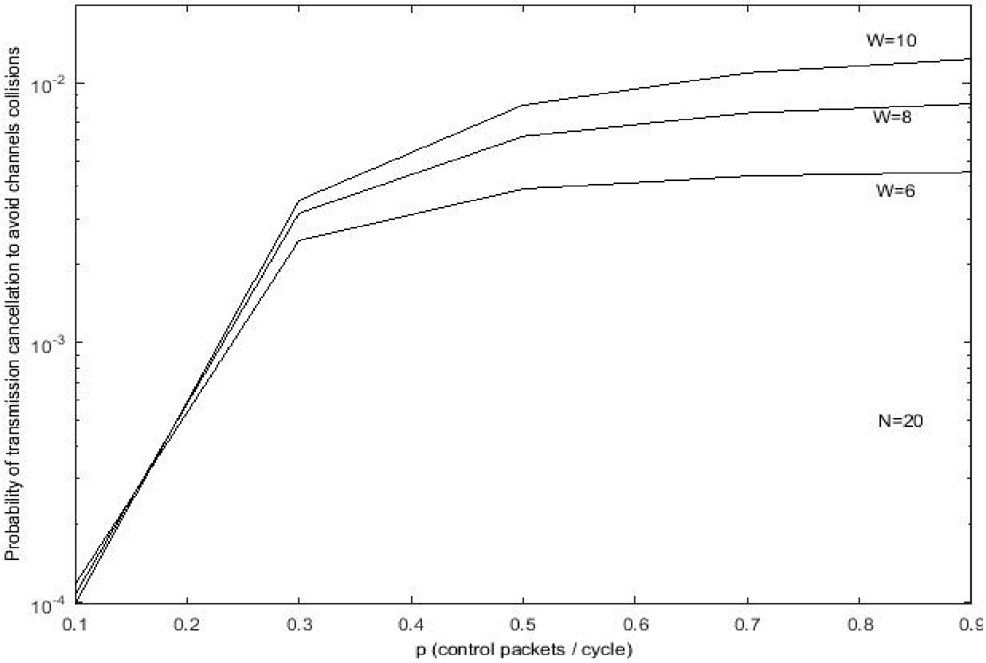

Fig. 5 Average probability of transmission cancellation to avoid channels collisions vs. the birth probability p, for N=20, W=6, 8, 10

REFERENCES

[1] Jun Zheng, and H. T. Mouftah, *Optical WDM Networks: Concepts and Design Principles*. J. Willey & Sons Inc. Publication - IEEE Press, 2004, pp. 3-6.

[2] I. E. Pountourakis, "Performance Evaluation with Receiver Collisions Analysis in Very High-Speed Optical Fiber Local Area Networks Using Passive Star Topology", *IEEE Journal of Lightwave Technology*, Vol. 16, No. 12, pp. 2303-2310, Dec. 1998.

[3] P.A.Baziana: "An Approximate Protocol Analysis with Performance Optimization for WDM Networks", *Optical Fiber Technology*, Vol. 20, Issue 4, pp. 414-421, 2014.

[4] I.E. Pountourakis, P.A. Baziana, G. Panagiotopoulos, "Propagation Delay and Receiver Collision Analysis in WDMA Protocols", *in Proc. 5th International Symposium on Communication Systems Networks and Digital Signal Processing (CSNDSP 2006)*, Patra, Greece, 2006, pp. 120-124.

[5] P.A. Baziana: "Performance Analysis and Transmission Strategies Comparison for Synchronous WDM Passive Star LANs", *Springer Photonic Network Communications Journal*, Vol. 31, Issue 3, pp 457–465, June 2016.

[6] M. I. Habbab, M. Kavehrad, and C. E. W. Sundberg, "Protocols for very high-speed optical fiber local area networks using a passive star topology", *J. Lightwave Technol*., vol. LT-5, pp. 1782–1794, Dec. 1987.

[7] P.A. Baziana: "Criteria for Rejection Probability Reduction in WDM LANs", *in Proc. of 17th International Conference on Transparent Optical Networks 2015 (ICTON 2015)*, pp. 1-4, July 5-9, 2015, Budapest, Hungary.