Effects of Thermal Radiation on Mixed Convection in a MHD Nanofluid Flow over a Stretching Sheet Using a Spectral Relaxation Method

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Abstract—The effects of thermal radiation, Soret and Dufour parameters on mixed convection and nanofluid flow over a stretching sheet in the presence of a magnetic field are investigated. The flow is subject to temperature dependent viscosity and a chemical reaction parameter. It is assumed that the nanoparticle volume fraction at the wall may be actively controlled. The physical problem is modelled using systems of nonlinear differential equations which have been solved numerically using a spectral relaxation method. In addition to the discussion on heat and mass transfer processes, the velocity, nanoparticles volume fraction profiles as well as the skin friction coefficient are determined for different important physical parameters. A comparison of current findings with previously published results for some special cases of the problem shows an excellent agreement.

Keywords—Non-isothermal wedge, thermal radiation, nanofluid, magnetic field, Soret and Dufour effects.

I. INTRODUCTION

MAGNETOHYDRODYNAMICS (MHD) flow and heat and mass transfer over a stretching surface has applications in polymer technology, glass-fiber production and in metallurgical industries. Many metallurgical processes involve cooling continuous strips or filaments by drawing them through a quiescent fluid. The properties of the final product depend on the rate of cooling which can be controlled by drawing such strips in an electrically conducting fluid subject to a magnetic field to achieve the desired characteristics of the final product. Indeed, many engineering processes, such as transport via conveyor belts, possess the characteristics of a moving continuous surface (see Subhas and Veena [1], Prasad et al. [2]). Blasius [3] was the first to report the boundary layer flow over a flat plate in a uniform free stream. Howarth [4] provided numerical solutions to the Blasius problem while Crane [5] expanded the study to stretched surfaces. Other features such as porosity, magnetic field effects and viscoelasticity or permeable surfaces, were studied by, among others, Gupta and Gupta [6]. Some recent contributions have come from Aiyesimi et al. [7]. Liao [8] and Xu et al. [9] obtained series solutions of the boundary layer equations.

In the past few years, convective heat and mass transfer in nanofluids has become a topic of major contemporary interest. Nanofluids have a significant role in enhancing the heat transfer properties of fluids, for instance, nanofluids have been shown to have higher thermal conductivity rates compared to common fluids such as water which makes these fluids ideal as advanced heat transfer fluids. Masuda et al. [10] for instance have suggested the use of nanofluids in cooling advanced nuclear systems. The most important properties of nanofluids are enhanced effective fluid thermal conductivity and heat transfer coefficient (see [11]-[17]). Studies on steady and unsteady nanofluid flow due to a stretching sheet were reported by Gbadeyan et al. [18], Subhakar and Gangadhar [19], Mahapatra et al. [20], Shakhaoath et al. [21] and Singh et al. [22]. Rohni et al. [23] presented a numerical solution for the unsteady flow over a continuously shrinking surface with wall mass suction using the nanofluid model proposed by Buongiorno [24]. In addition, a numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids was carried out by Oztop and Abu-Nada [25]. Gharagozloo et al. [26] investigated aggregation and thermal conductivity in nanofluids while Philip et al. [27] proposed a nanofluid with tunable thermal properties. Recently, Haroun et al. [28] studied unsteady MHD mixed convection in a nanofluid due to a stretching/shrinking surface with suction/injection using the spectral relaxation method.

The aim of the present study is to analyze thermal radiation, Soret and Dufour effects on steady mixed convection in boundary layer flow of a nanofluid over a non-isothermal with a chemical reaction and viscous dissipation using spectral relaxation method (SRM) (see Motsa [29]). A comparative study for a special case is presented which shows excellent agreement with Yih [30].

II. MATHEMATICAL FORMULATIONS

Consider steady two-dimensional incompressible nanofluid flow over a non-isothermal stretching wedge (see Fig. 1). The temperature and nanoparticle concentration at the stretching surface are \( T_w \) and \( C_w \) respectively, and in the ambient nanofluid these are \( T_\infty \) and \( C_\infty \), respectively. The radiation heat flux in the \( x \)-direction is negligible compared to the flux in the \( y \)-direction. The \( x \) and \( y \) directions are in the plane of, and perpendicular to the sheet, respectively. The continuity, momentum, heat and mass transfer equations of a steady, an incompressible nanofluid boundary layer flow are (see Tiwari and Das [31])

\[ \text{Determination} \]
\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1) \]

\[ u_0 \frac{\partial u}{\partial x} + v_0 \frac{\partial u}{\partial y} = \nu_{nf} \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} - u \left( \frac{\sigma B^2}{\rho_{nf}} \right) + g\beta_T (T - T_\infty) + g\beta_C (C - C_\infty), \quad (2) \]

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} - \frac{1}{(pcp)_{nf}} \frac{\partial q}{\partial y} + \frac{\rho_f D_m K_T}{C_s (pcp)_{nf}} \frac{\partial^2 C}{\partial y^2} + \frac{\nu_{nf}}{(pcp)_{nf}} \left( \frac{\partial u}{\partial y} \right)^2, \quad (3) \]

\[ \alpha_{nf} \frac{\partial C}{\partial x} + \nu_{nf} \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} - R(C - C_\infty), \quad (4) \]

where \( u \) and \( v \) are the fluid velocity components in the \( x \) and \( y \) directions, respectively, \( \nu_{nf}, \rho, \rho_{nf}, \sigma, B_0, \mu_{nf}, g \) are the nanofluid kinematic viscosity, the pressure, nanofluid density, electrical conductivity, the uniform magnetic field in the \( y \)-direction, the effective dynamic viscosity of the nanofluid and gravitational acceleration, \( \beta_T, \beta_C, T, C, \alpha_{nf}, (pcp)_{nf} \) are the volumetric thermal expansion coefficient, volumetric solutal expansion coefficient, temperature of fluid in the boundary layer, fluid solutal concentration, the thermal diffusivity of the nanofluid, the nanofluid heat capacitance respectively, \( \rho_f, D_m, K_T, C_s, (cp)_{nf}, T_m \). \( R \) is the density of the base fluid, the mass diffusivity of the concentration, thermal diffusion ratio, concentration susceptibility, specific heat of fluid at constant pressure, mean fluid temperature and the thermal reaction parameter respectively. Here \( q_c \) is the radiation heat flux given by

\[ q_c = \frac{4\sigma^* \partial T^4}{3K^* \partial y}, \quad (5) \]

where \( \sigma^* \) is the Stefan-Boltzmann constant and \( K^* \) is the Rosseland mean absorption coefficient. Expanding \( T^4 \) about \( T_\infty \) we obtain, \( T^4 \approx 4T^4\left(T - T_\infty\right) \) (see Singh et al. [22]). For a uniform- stream, the momentum equation (2) becomes

\[ u_\infty \frac{dU_\infty}{dx} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\sigma B^2}{\rho_{nf}} U_\infty \quad (6) \]

where \( U_\infty = ax^n \) is the velocity of the potential flow, \( n = \beta/(2 - \beta) \) and \( \beta \) is the Hartree pressure gradient parameter with \( \beta = \Omega/\pi \) for a total angle of the wedge, \( a \) is a positive real number.

Substituting (6) in (2) and (5) in (3), the transport equations are written as

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \nu_{nf} \frac{\partial^2 u}{\partial y^2} + U_\infty \frac{dU_\infty}{dx} + (U_\infty - u) \left( \frac{\sigma B^2}{\rho_{nf}} \right) + g\beta_T (T - T_\infty) + g\beta_C (C - C_\infty), \quad (7) \]

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{1}{(pcp)_{nf}} \frac{\partial q}{\partial y} + \frac{\rho_f D_m K_T}{C_s (pcp)_{nf}} \frac{\partial^2 C}{\partial y^2} + \frac{\nu_{nf}}{(pcp)_{nf}} \left( \frac{\partial u}{\partial y} \right)^2, \quad (8) \]

subject to the boundary conditions

\[ u = v = 0, \; T = T_w(x) = T_\infty + T_0 x^{2n}, \]

\[ C = C_w(x) = C_\infty + C_0 x^{2n} \; \text{at} \; y = 0, \]

\[ u = U_\infty = ax^n , \; T = T_\infty, \; C = C_\infty \; \text{as} \; y \to \infty, \; 0 \leq n \leq 1, \quad (9) \]

and initial conditions

\[ u = 0, \; v = 0, \; T = T_w, \; C = C_w, \; \forall x, y, \quad (10) \]

where \( T_0 \) and \( C_0 \), are positive real numbers. When \( n = 0 \), we have the boundary - layer flow over a stationary flat plate. When \( n = 1 \), we obtain flow near the stagnation point of an infinite wall.

The effective viscosity of the nanofluid is (see Brinkman [32])

\[ \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \quad (11) \]

where \( \phi \) and \( \mu_f \) are the solid volume fraction of nanoparticles and the dynamic viscosity of the base fluid. In (1)-(4);

\[ \rho_{nf} = (1 - \phi)(\rho_p)_f + \phi(\rho_p)_s \]

\[ (pcp)_{nf} = (1 - \phi)(pcp)_f + \phi(pcp)_s \]

\[ \nu_{nf} = \frac{\mu_{nf}}{\rho_{nf}} \]

\[ k_{nf} = \frac{\bar{k}_s + k_f}{\bar{k}_s + k_f} \]

\[ k_f = \frac{\bar{k}_s + k_f}{\bar{k}_s + k_f + \phi(k_f - k_s)} \]

where \( k_{nf}, k_f, k_s, \rho_{nf}, (pcp)_f, (pcp)_s \) are the thermal conductivity of the nanofluid, the thermal conductivity of the fluid, the thermal conductivity of the solid fractions, the density of the solid fractions, the heat capacity of base fluid and the effective heat capacity of nanoparticles, respectively, (see Abu-Nada [33]). \( \alpha_f = k_f/(pcp)_f \) and \( \nu_f = \mu_f/\rho_f \) are the thermal diffusivity and kinetic viscosity of the base fluid, respectively.

The continuity equation (1) is satisfied by introducing a stream function \( \psi(x, y) \) such that

\[ u = \frac{\partial \psi}{\partial y}, \; v = -\frac{\partial \psi}{\partial x}, \quad (13) \]

Introducing the following non-dimensional variables, (see Yih [30])

\[ \eta = \sqrt{\frac{U_\infty x y}{\nu_f}}, \; \xi = \frac{\sigma B^2}{\rho_f U_\infty}, \; \psi = \sqrt{U_\infty \nu_f x f(\xi, \eta)} \]

\[ \theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty}, \; \Phi(\xi, \eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad (14) \]

where \( \eta \) and \( \xi \) are dimensionless variables. By using (14) the governing equations (4), (7) and (8) along with the boundary
conditions (9) are reduced to the following boundary value problem

\[
f''' + \phi_1 \left( \frac{n+1}{2} f'' + n (1 - f'^2) + \xi (1 - f') \right) + Gr_t \theta + Gr_c \phi = \phi_1 (1 - n) \xi \left[ f' \frac{\partial f'}{\partial \xi} - \nu f'' \frac{\partial f}{\partial \xi} \right],
\]

subject to the boundary conditions

\[
f(\xi, 0) = f'(\xi, 0) = 0, \theta(\xi, 0) = 1, \Phi(\xi, 0) = 1,
\]

when \( \eta \to 0, \xi \geq 0, \)

\[
f'(\xi, \infty) = 1, \theta(\xi, \infty) = 0, \Phi(\xi, \infty) = 0,
\]

when \( \eta \to \infty, \xi \geq 0, \)

where primes denote differentiation with respect to \( \eta. \) The non-dimensional parameters appearing in (15)-(17) namely, \( Gr_t, Gr_c, Pr, Nr, Ec, Sc, D_f, \gamma \) and \( Sr \) denote the local temperature Grashof number, local concentration Grashof number, Prandtl number, thermal radiation parameter, Eckert number, the Schmidt number, Dufour number, scaled chemical reaction parameter and the Soret number.

The Eckert number represents the kinetic energy of the flow relative to the boundary layer enthalpy difference. The Dufour effect describes the energy flux created when a system is under a concentration gradient. These parameters are defined mathematically as

\[
Gr_t = \frac{g \beta_T (T_w - T_{\infty}) x}{U_{\infty}^2}, Gr_c = \frac{g \beta_C (C_w - C_{\infty}) x}{U_{\infty}^2},
\]

\[
Pr = \frac{\nu_f}{\alpha_f}, Nr = \frac{16 \sigma^* T_{\infty}^3}{3K^*(\rho c_p) \nu_f}, Ec = \frac{U_{\infty}^2}{(ep)f(T_w - T_{\infty})},
\]

\[
Sc = \frac{\nu_f}{D_m}, D_f = D_m K_T (C_w - C_{\infty}) C_f \nu_f (T_w - T_{\infty}),
\]

\[
\nu_f = \frac{\mu_f}{\rho_f}, \gamma = \frac{R_x}{U_{\infty}}, Sr = \frac{D_m K_T (T_w - T_{\infty})}{\nu_f T_m (C_w - C_{\infty})}.
\]

The nanoparticle volume fraction \( \phi_1 \) and \( \phi_2 \) are defined as

\[
\phi_1 = (1 - \phi)^{2.5} \left[ 1 - \phi + \phi \left( \frac{\rho_s}{\rho_f} \right) \right],
\]

\[
\phi_2 = \left[ 1 - \phi + \phi \left( \frac{\rho c}{\rho_c f} \right) \right].
\]
The heat transfer rate at the surface is given by
\[ q_w = -k_{nf} \left( \frac{\partial T}{\partial y} \right)_{y=0} = -k_{nf} \left( \frac{T_w - T_{\infty}}{x} \right) \sqrt{\frac{U_{xc}}{\nu_f}} \theta'(0, \xi). \] (25)

The Nusselt number is defined as
\[ Nu_x = \frac{xq_w}{k_f (T_w - T_{\infty})}. \] (26)

Using (25) in (26), the wall heat transfer rate is obtained as
\[ Nu_x \left( \frac{k_f}{k_{nf}} \right) = -\theta'(0, \xi). \] (27)

The mass flux at the wall surface is given by
\[ q_m = -D \left( \frac{\partial C}{\partial y} \right)_{y=0} = -D \left( \frac{C_w - C_{\infty}}{x} \right) \Phi'(0, \xi), \] (28)

and the Sherwood number is defined as
\[ Sh_x = \frac{xq_m}{D (C_w - C_{\infty})}. \] (29)

Using (2.28) in (2.29) the wall mass transfer rate is obtained as
\[ Sh_x \left( \frac{C_w}{C_{\infty}} \right) = -\Phi'(0, \xi). \] (30)

IV. SOME PARTICULAR CASES OF INTEREST

In some limiting cases, (15)-(17) reduce to ordinary differential equations.

Case(1): For steady-state flow when \( \phi = 0 \) (regular fluid) with \( \xi = 0 \), \( Gr_t = 0 \) and \( Gr_c = 0 \), (15) is approximately as Yih [30]
\[ f''' + \frac{1}{2} f'' f' + n(1 - f^2) = 0. \] (31)

The solution of (31) is presented in Table II for the skin friction for various values of \( n \).

Case(2) For a regular fluid when \( \xi = 0 \), and \( n, Nr, Ec, D_f, Gr_t \) and \( Gr_c \) are set to zero, (15) and (16) reduce to
\[ f''' + \frac{1}{2} f'' f' = 0. \] (32)
\[ \frac{1}{Pr} \theta''' + \frac{1}{2} f\theta' = 0. \] (33)

Equation (32) is decoupled from the energy equation (33).

Case(3): For this case when \( \phi = 0 \); the physical parameters \( n = 0, Nr = 0, Ec = 0, D_f = 0, Gr_t = 0 \) and \( Gr_c = 0 \), (15)-(16) reduce to (see Yih [30]):

<table>
<thead>
<tr>
<th>( n )</th>
<th>Present result (SRM)</th>
</tr>
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<tbody>
<tr>
<td>0.05</td>
<td>0.213484</td>
</tr>
<tr>
<td>0.0</td>
<td>0.332057</td>
</tr>
<tr>
<td>0.5</td>
<td>0.332057</td>
</tr>
<tr>
<td>1/3</td>
<td>0.757448</td>
</tr>
</tbody>
</table>

TABLE III

COMPARISON OF THE VALUES OF \( f'''(0, 0) \) FOR VARIOUS VALUES OF \( Pr \), when \( \phi = 0 \) (REGULAR FLUID), \( Nr = Ec = D_f = n = 0 \) AND \( \xi = 0 \) (STEADY STATE)

<table>
<thead>
<tr>
<th>( Pr )</th>
<th>( \theta'(0, 0) )</th>
<th>( \theta'(0, 0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.140034</td>
<td>0.140034</td>
</tr>
<tr>
<td>1.0</td>
<td>0.332057</td>
<td>0.332057</td>
</tr>
<tr>
<td>10</td>
<td>0.728141</td>
<td>0.728141</td>
</tr>
<tr>
<td>100</td>
<td>1.571831</td>
<td>1.571831</td>
</tr>
<tr>
<td>1000</td>
<td>3.387083</td>
<td>3.396962</td>
</tr>
<tr>
<td>10000</td>
<td>7.297402</td>
<td>7.351156</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( Pr )</th>
<th>( \theta'(0, 0) )</th>
<th>( \theta'(0, 0) )</th>
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</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.357022</td>
<td>0.356986</td>
</tr>
<tr>
<td>1.0</td>
<td>0.382558</td>
<td>0.382558</td>
</tr>
<tr>
<td>1.5</td>
<td>0.398264</td>
<td>0.398239</td>
</tr>
<tr>
<td>2.0</td>
<td>0.409147</td>
<td>0.409147</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( Pr )</th>
<th>( \theta'(0, 0) )</th>
<th>( \theta'(0, 0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.332057</td>
<td>0.332057</td>
</tr>
<tr>
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<tr>
<td>2.0</td>
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<td>0.465962</td>
</tr>
</tbody>
</table>

TABLE IV

COMPARISON OF THE VALUES OF \( \theta'(0,\xi) \) FOR VARIOUS VALUES OF \( Pr \) WITH \( \phi = 0 \) (REGULAR FLUID), \( Nr = Ec = D_f = n = 0 \) and \( \xi = 0 \) (STEADY STATE)
The linearized form of (15) - (17) is

\[ f''_{r+1} + a_{1,r} f''_{r+1} + a_{2,r} f'_{r+1} - \phi_1 \xi (1-n) f'_{r+1} \frac{\partial f'_{r+1}}{\partial \xi} = R_{1,r}, \]

\[ \theta'_{r+1} + b_{1,r} \theta'_{r+1} + b_{2,r} \theta_{r+1} = \frac{k_f}{k_n} Pr \phi_2 \xi (1-n) f'_{r+1} \frac{\partial \theta_{r+1}}{\partial \xi} = R_{2,r}, \]

\[ \phi''_{r+1} + c_{1,r} \phi''_{r+1} + c_{2,r} \phi_{r+1} - Sc \xi (1-n) f'_{r+1} \frac{\partial \phi_{r+1}}{\partial \xi} = R_{3,r}, \]

where

\[ a_{1,r} = \phi_1 \left[ \frac{(n+1)}{2} f_r + (1-n) \xi \frac{\partial f'}{\partial \xi} \right], \quad a_2 = -\phi_1 \xi, \]

\[ R_{1,r} = -\phi_1 \xi (1 - f''_r) + \xi + Gr \theta_r + Gr \phi_1, \]

\[ b_{1,r} = \frac{k_f}{k_n} Pr \phi_2 \left[ \frac{(n+1)}{2} f_r + \xi (1-n) \frac{\partial f'}{\partial \xi} \right], \]

\[ b_{2,r} = \frac{k_f}{k_n} Pr \phi_2 f'_r, \]

\[ R_{2,r} = \frac{k_f}{k_n} Pr D_f \theta_r = \frac{k_f}{k_n} Pr \left( 1 - \phi \right)^2 (f''_r)^2, \]

\[ c_{1,r} = \frac{(n+1)}{2} f_{r+1} Sc + Sc \xi (1-n) \frac{\partial f'}{\partial \xi}, \]

\[ c_{2,r} = -Sc \xi (2nf'_{r+1} + \gamma), \]

\[ R_{3,r} = -Sc \xi \phi_{r+1}. \]

It must be noted that (36)-(38) are linear and being decoupled, can be solved sequentially to obtain approximate solutions \( f(\eta, \xi), \theta(\eta, \xi) \) and \( \phi(\eta, \xi) \). In this study, the Chebyshev spectral collocation method was used to discretize in \( \eta \) and finite differences with central differencing for is used to discretize in \( \xi \). Starting from initial guesses for \( f, \theta \) and \( \phi \), (36)-(38) were solved iteratively until the approximate solutions converged to within a certain prescribed tolerance level. The accuracy of the results was validated against results from literature for some special cases of the governing equations.

\[ \text{V. RESULTS AND DISCUSSION} \]

In this study, we considered the flow of two nanofluids, namely Cu-water and Ag-water nanofluids. The conservation equations are solved numerically using the spectral relaxation method for \( 0 \leq \xi \leq 1 \).

The thermophysical properties of the nanofluids and the base fluid water used in the numerical simulations are given in Table I. To determine the accuracy of our numerical results, we have compared our results for the local skin friction coefficient and the local Nusselt number with previous results by Yih [30]. The results are shown in Tables II-IV. The comparison with previously published results shows excellent agreement which is useful in establishing the validity of our approach. The Nusselt number for various values of the Prandtl number \( Pr \) and \( \xi \) are given in Tables III and IV.

Figs. 2-8 show the effect of nanoparticle volume fraction on the velocity profiles, temperature profiles, concentration profiles, skin friction coefficient, heat transfer coefficient and the mass transfer coefficient respectively. Increased nanoparticle volume fraction leads to an increase in the fluid velocity, temperature profiles and the mass transfer coefficient while the concentration profiles, skin friction coefficient and heat transfer coefficients are reduced. Figs. 6 and 7 show that the skin friction and the heat transfer coefficient decrease with the increasing value of \( \xi \), whereas in Figs. 8 the mass...
Fig. 6 Effect of change in the nanoparticle volume fraction values $\phi$ on Skin friction coefficient for $Gr_c = 0.01$, $n = 0$, $Gr_c = 0.01$, $Nr = 10$, $Pr = 7$, $Ec = 10$, $D_f = 0.01$, $\gamma = 3$, $Sc = 1$ and $Sr = 1$

Fig. 7 Effect of change in the nanoparticle volume fraction values $\phi$ heat transfer coefficient for $Gr_c = 0.01$, $n = 0$, $Gr_c = 0.01$, $Nr = 10$, $Pr = 7$, $Ec = 10$, $D_f = 0.01$, $\gamma = 3$, $Sc = 1$ and $Sr = 1$

Fig. 8 Effect of change in the nanoparticle volume fraction values $\phi$ on mass transfer coefficient for $Gr_c = 0.01$, $n = 0$, $Gr_c = 0.01$, $Nr = 10$, $Pr = 7$, $Ec = 10$, $D_f = 0.01$, $\gamma = 3$, $Sc = 1$ and $Sr = 1$

Fig. 9 Effect of thermal radiation $Nr$ on temperature profiles for $n = 0$, $\phi = 0.3$, $Gr_t = 0.01$, $Gr_c = 0.01$, $Pr = 7$, $Ec = 10$, $D_f = 0.01$, $\gamma = 3$, $Sc = 1$, $Sr = 1$ and $\xi = 0.5$

Fig. 10 Effect of thermal radiation $Nr$ on concentration profiles for $n = 0$, $\phi = 0.3$, $Gr_t = 0.01$, $Gr_c = 0.01$, $Pr = 7$, $Ec = 10$, $D_f = 0.01$, $\gamma = 3$, $Sc = 1$, $Sr = 1$ and $\xi = 0.5$

Fig. 11 Effect of thermal radiation $Nr$ on Skin friction coefficient for $n = 0$, $\phi = 0.3$, $Gr_t = 0.01$, $Gr_c = 0.01$, $Pr = 7$, $Ec = 10$, $D_f = 0.01$, $\gamma = 3$, $Sc = 1$ and $Sr = 1$
transfer coefficient increases with the increasing value of. This is due to the fact that increased nanoparticle volume...
fraction enhances the thermal conductivity causing higher flow rates at the surface. Comparing the effect of the nanoparticle volume fraction on Cu-water and Ag-water nanofluids, (Figs. 2-4 and 8) it is noted that the effect is greater in the case of a Ag-water nanofluid than in a Cu-water nanofluid. The reverse is true for solutal concentration profiles, skin friction coefficient and the heat transfer coefficient where the effect is greater for Cu-water nanofluids (see Figs. 5-7). These finding are similar to the results reported by Kameswaran et al. [37]. Figs. 9-13 show the influence of the thermal radiation parameter on the temperature profiles, concentration profiles, skin friction coefficient, heat transfer coefficient and mass transfer coefficient. Fig. 9 indicates that as $\eta$ increase, the temperature profile increases correspondingly up to a certain range and after that it shows the opposite trend. The results show a greater sensitivity for a Ag-water nanofluid regarding the temperature profiles, and mass transfer coefficient while the Cu-water nanofluid is more sensitive for concentration profile, the skin friction coefficient and the heat transfer coefficient. These results are qualitatively similar to those obtained by Kameswaran and Sibanda [38].

Figs. 14-17 show the effects of the Dufour number for different values of the nanoparticle volume fraction ($\phi = 0.3, 0.5, 0.7$) on the temperature profiles, concentration profiles, heat transfer coefficient and mass transfer coefficient, respectively. The temperature profiles and mass transfer coefficient increase with increases in the Dufour number while the concentration profiles and heat transfer coefficient are reduced with increasing value of Dufour number. As with the previous set of graphs, the Ag-water nanofluid shows a greater sensitivity to increases in the Dufour number compared to the Cu-water nanofluid. The heat transfer coefficient is greater for a Cu-water nanofluid.

Figs. 18 and 19 show the effect of the chemical reaction parameter on the concentration profiles and the mass transfer coefficient. The local solute concentration is reduced by the increasing reaction rate. The concentration profiles decrease with an increase in the chemical reaction parameter whereas the mass transfer coefficient increases with an increase in the chemical reaction parameter. The mass transfer coefficient increases as $\xi$ increases.

Figs. 20 and 21 show the impact of the Soret number on the concentration profiles and the mass transfer coefficient. Where the concentration profiles grow less while the mass transfer coefficient increases with an increase in the Soret number. Again Figs. 20 and 21 show that as Soret number increases, the boundary layer thickness for the solute concentration reduces. The mass transfer coefficient is increasing when the Soret number is positive.

VI. CONCLUSION

The steady state mixed convection in MHD nanofluid boundary layer flow and heat, mass transfer from a non-isothermal wedge has been studied. We have considered the flow of Cu-water and Ag-water nanofluids and assumed that the nanoparticle volume fraction can be actively controlled at the boundary surface. The equations were solved using the spectral relaxation method and to benchmark our solutions, these were compared with some limiting cases in the literature. The comparison showed a good agreement with the published results. From the numerical simulations, the following conclusions may drawn:

1) The velocity profiles increase with increases in the nanoparticle volume fraction.
2) The temperature profiles increase with increasing nanoparticle volume fraction, thermal radiation parameter and the Dufour number.
3) The concentration profiles decrease with increasing nanoparticle volume fraction, chemical reaction parameter, Soret and Dufour numbers while the concentration profiles increase with increases in the thermal radiation parameter.
4) The skin friction coefficient decreases with an increase in the nanoparticle volume fraction while the opposite trend is observed for increasing thermal radiation parameter values.
Fig. 20 Effect of the Soret number on the concentration profiles for $n = 0, \phi = 0.3, Gr_1 = 0.01, Gr_2 = 0.01, Pr = 7, \gamma = 0.1, Ec = 10, D_f = 0.01, Sc = 1$ and $\eta = 3$.

Fig. 21 Effect of the Soret number on the local Sherwood number for $n = 0, \phi = 0.3, Gr_1 = 0.01, Gr_2 = 0.01, Pr = 7, Nr = 10, Ec = 10, D_f = 0.01, Sc = 1$ and $\gamma = 3$.

5) The heat transfer coefficient decreases with an increase in the nanoparticle volume fraction and the Dufour number while the heat transfer coefficient increase with an increase in the values of thermal radiation parameter.

6) The mass transfer coefficient increases with an increase in the nanoparticle volume fraction, chemical reaction parameter, Soret and Dufour number while the opposite trend is observed for increasing values of thermal radiation parameter.

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