Type–2 Fuzzy Programming for Optimizing the Heat Rate of an Industrial Gas Turbine via Absorption Chiller Technology

T. Ganesan, M. S. Aris, I. Elamvazuthi, Momen Kamal Tageldeen

Abstract—Terms set in power purchase agreements (PPA) challenge power utility companies in balancing between the returns (from maximizing power production) and securing long term supply contracts at capped production. The production limitation set in the PPA has driven efforts to maximize profits through efficient and economic power production. In this paper, a combined industrial-scale gas turbine (GT) - absorption chiller (AC) system is considered to cool the GT air intake for reducing the plant’s heat rate (HR). This GT-AC system is optimized while considering power output limitations imposed by the PPA. In addition, the proposed formulation accounts for uncertainties in the ambient temperature using Type-2 fuzzy programming. Using the enhanced chaotic differential evolution (CEDE), the Pareto frontier was constructed and the optimization results are analyzed in detail.

Keywords—Absorption chillers, turbine inlet air cooling, power purchase agreement, multiobjective optimization, type-2 fuzzy programming, chaotic differential evolution.

I. INTRODUCTION

The overall efficiency of the generation system is a key element in power production. Optimizing the system increases its efficiency resulting in higher power production. Most current power producers are bound contractually with the consumer by a PPA [1], [2]. Some of these contracts require that the amount of power supplied remains fixed throughout the period of purchase. Although engineers and plant personnel manage to optimize the plant efficiency significantly, the surplus power produced could not be sold to the consumer due to the terms in the PPA. PPA imposes an upper limit on the power that the supplier can produce for the client (according to the terms in the PPA). This happens while reducing the plant’s heat rate (HR). This system is considered to cool the GT air intake for reducing the plant’s heat rate (HR). This GT-AC system is optimized while considering power output limitations imposed by the PPA. In addition, the proposed formulation accounts for uncertainties in the ambient temperature using Type-2 fuzzy programming. Using the enhanced chaotic differential evolution (CEDE), the Pareto frontier was constructed and the optimization results are analyzed in detail.

There are several methods used to optimize GT operations. For instance, in [3], the thrust and specific fuel consumption was optimized using the General Algebraic Modeling System (GAMS) computer program. In [4], a GT engine was optimized using an evolutionary approach called StudGA. The optimization was aimed to improve the GT thrust while minimizing the blade temperature and maintaining the fuel consumption. Similarly, in [5], a swarm intelligence strategy was employed to improve the performance of the turbine by optimizing the fuel flow controller.

As industrial systems become more complex, engineers and decision makers often find themselves in situations involving multiple objectives or criteria [6]-[8]. Multicriteria scenarios have been encountered in power generation and aerospace, especially when dealing with GTs. For instance, in [9], a Multiobjective Genetic Algorithm (MOGA) was employed for optimizing the geometry of aerofoil GT discs. In that work, the authors considered fatigue life prediction and total geometrical mass as objective functions. In [10], the modeling and optimization of a micro turbine cycle was done, where the authors considered the design parameters as: compression ratio, compressor isentropic efficiency, combustion chamber inlet temperature, and turbine inlet temperature. The authors considered power exergy efficiency, total cost, and carbon dioxide emission of the plant as the three objectives to be optimized. In this work the Differential Evolution (DE) technique is employed in conjunction to the weighted sum approach [11]. To obtain a solution closer to the global optima, the random generator of the conventional DE technique is enhanced with a chaotic component - producing the Chaotic-Driven DE Technique (CDDE). The Hypervolume Indicator (HVI) was then employed for the purpose of measuring the solution quality [12].

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This paper presents the novel modeling of a real-world combined GT and AC multicriteria optimization problem for an open cycle plant. The ambient air conditions directly influence the turbine inlet air temperature and subsequently the plant’s HR. To account for uncertainties in the ambient air temperature, the combined GT-AC system was modeled using a Type-2 fuzzy programming approach. This multicriteria fuzzy optimization problem was solved using the improved CDDE strategy [13]. Optimization results are analyzed and discussed in this paper.

II. AMBIENT TEMPERATURE MODELING USING TYPE-2 FUZZY LOGIC

Type-2 fuzzy sets are generalizations of the conventional or type-1 fuzzy sets [14]. The primary feature of the Type-1 fuzzy set is its membership function, \( \eta(x) \in [0,1] \) and \( x \in X \). Type-2 fuzzy logic (FL) employs a membership function of a second order, \( \mu_{\eta}(y, \eta_{\eta}(x)) \in [0,1] \) such that \( y \in Y \).

Therefore, \( \mu(x, \eta_{\eta}(x)) \) is a membership function that requires three-dimensional inputs. The type-2 fuzzy set is defined as:

\[
\tilde{F} = \left\{ (y, \eta_{\eta}(x)), \mu_F(y, \eta_{\eta}(x)) : \forall x \in X, \forall y \in Y, \eta_{\eta}(x) \in [0,1] \right\}
\]

The type-2 membership function has two membership grades: primary and secondary memberships. Thus, a crisp set (or function) undergoes fuzzification twice such that the first fuzzification transforms it to a type-1 fuzzy set (via the primary membership function). The secondary membership function the type-1 fuzzy set is transformed to a type-2 fuzzy set. In simpler terms; the type-2 fuzzy set results from the fuzzification of a type-1 fuzzy set. This operation aims to improve its efficacy and accuracy in capturing uncertainties. The region covered by the type-1 fuzzy sets in type-2 FL systems is represented by the footprint of uncertainty (FOU). This region of uncertainty is contained by the uppermost and lowermost type-1 membership functions \( \eta_{\eta}^U(x) \) and \( \eta_{\eta}^L(x) \) respectively. A type-2 FL system usually consists of four subcomponents: fuzzifier, inference engine, type reducer and defuzzifier. The fuzzifier directly transforms the crisp set into a type-2 fuzzy set. The inference engine functions to combine rules to map the type-2 fuzzy set from crisp inputs. Therefore, each rule is interpreted as a type-2 fuzzy implication in the inference engine. In this work, all the consequent and antecedent sets are generalized type-2 fuzzy sets. The rule, \( R_i \) from a type-2 FL system could be represented as follows:

\[
R_i: \text{IF } x_1 \text{ is } M_1 \text{ AND } \ldots \text{ AND } x_j \text{ is } M_j \text{ THEN } y_1 \text{ is } N_1, \ldots, y_k \text{ is } N_k \text{ such that } i \in [1, Z]
\]

where \( j \) is the number of fuzzy inputs, \( k \) is the number of fuzzy outputs and \( i \) is the number of rules. The type reducer functions to transform (or reduce) the type-2 fuzzy set to a type-1 fuzzy set. Various type-reduction approaches have been developed in the past. For instance: centroid type reduction [15] vertical slice-centroid type reduction [16], alpha cuts/planes [17] and the random sampling technique [18]. Defuzzification on the other hand reduces the type-1 fuzzy set to a crisp output similar to operations in conventional type-1 FL systems. There are various defuzzification techniques which are employed selectively to suit specific data representations and applications [19].

Energy systems that rely on their surroundings are often difficult to design if the surroundings contain irregularities or uncertainties. In GT-AC systems, the ambient temperature is a weather-dependent variable. Using meteorological data, the type-2 FL is employed to model and incorporate ambient temperature considerations into the problem formulation. The meteorological information for Sepang – Kuala Lumpur International Airport AB (KLIA) at Malaysia was retrieved from the weather database [20]. The daily average ambient temperature (K) was obtained for the month of January in 2016. The primary membership function, \( \eta_F(x) \) was employed to model the weekly data, while the secondary membership function, \( \mu_F(y, \eta_F(x)) \) was used to model the overall monthly data. The S-curve function is employed as the primary and secondary membership functions. Therefore type-2 fuzzification is performed on the ambient temperature using the S-curve membership function. This is carried out by determining the average, maximum, and minimum values of insolation and ambient temperature from the meteorological data. The S-curve membership function is as:

\[
\mu_{\eta_F}(a, b) = \begin{cases} 
1 & \text{if } b_i \leq b_i^a \\
\frac{B}{1+Ce^{\frac{a(b_i-b_i^a)}{(b_i^b-b_i^a)}}} & \text{if } b_i^a \leq b_i \leq b_i^b \\
0 & \text{if } b_i \geq b_i^b
\end{cases}
\]

where \( B \) and \( C \) are parameters which are tuned heuristically such that the membership fits the meteorological data effectively. Using Zadeh’s extension principle, all crisp variables (ambient temperature (\( Z_a \)) and insolation (\( Z_s \))) and their respective constraints are transformed via type-2 fuzzification. Assuming a credibility level \( \varepsilon \), \((0 < \varepsilon < \frac{b_i-b_i^a}{b_i^b-b_i^a})\) chosen by the Decision Maker (DM), as the DM takes a risk and ignores all the membership degrees smaller than the \( \varepsilon \) levels [21]. The FOU is the union of all the primary memberships [22]. In this case, the union of all the primary S-curve memberships, \( \eta_{\eta_F}(x) \) for each day depicts the FOU:

Let \( \eta_{\eta_F}(x) \in \left( J_i^{\varepsilon} \subseteq [0,1] \right) \) such that \( i = [L, U] \), THEN
\[ FOU = \bigcup_{\alpha \in \mathcal{X}} J_{\alpha} \quad (4) \]

where \( J_{\alpha} \) is the fuzzy set, \( L \) is the lower bound and \( U \) is the upper bound. A graphical depiction of the FOU generated by the primary S-curve memberships in this work is given in Fig. 2. There are many readily available techniques for type-reduction and defuzzification. In this work, the alpha-plane approach [17] was employed for type-reduction, while the conventional alpha-cut approach [23] was used for the defuzzification. An alpha cut can be defined on a fuzzy set, \( \overline{F} \) via its decomposed form as:

\[ \overline{F} = \bigcup_{\alpha \in [0,1]} \alpha \cdot F_{\alpha} \quad (5) \]

where \( F_{\alpha} \) is an \( \alpha \) – level set. Similarly, an alpha-cut on a type-2 fuzzy set could be performed via the decomposition theorem. Since this operation is performed on a type-2 fuzzy set, it is defined as an alpha-plane instead of an alpha cut:

\[ \overline{F} = \bigcup_{\alpha \in [0,1]} \alpha \cdot \overline{F}_{\alpha} \quad (6) \]

where \( \overline{F}_{\alpha} \) is a type-2 \( \alpha \) – level set. It should be noted that by using Zadeh’s extension principle, the alpha-planes could be utilized to execute type-2 fuzzy operations using interval type-2 fuzzy sets. This is analogous to implementations in type-1 fuzzy sets since the extension principle could be evoked to extend functions that interrelate crisp, type-1 fuzzy as well as type-2 fuzzy sets.

III. GT-AC DYNAMIC MODEL

The model developed in this work is of a combined GT and AC system – where the AC is used to cool the compressor inlet air of the GT. The AC system uses a lithium bromide (LiBr) – water solution as a refrigerant and working fluid. This cooling is aimed to enhance the performance of the GT in terms of: overall thermal efficiency (\( \eta_{th} \)) as well as lowering the HR.

The GT simulation in this work was modeled based on a V94.2 Siemens GT with a rated speed of 3000 rpm and a rated capacity of 131.5 MW. The GT in the plant as well as in the simulation is operated at a base load 123 MW. The heat exchangers design, pumps, and compressor formulations were neglected in this model since they have very minimal effect on the overall optimization. A transient refrigeration load was considered in this work. The main components (evaporator, condenser, generator and absorber) thermal dynamics and balances in the AC are given as:

\[ \frac{dQ_e}{dt} + \frac{dQ_c}{dt} + \frac{dQ_g}{dt} + \frac{dQ_a}{dt} \quad (kcal/hr) \quad (7) \]

where \( Q_e, Q_c, Q_g, Q_a \) are the heat transferred by the evaporator, condenser, generator and absorber (in kcal). The AC performance is gauged using the real and ideal coefficient of performance (COP) along with the relative performance ratio (RPR):

\[ \text{COP} = \frac{\text{refrigeration load}}{\text{external heat input}} = \left( \frac{Q_e}{Q_g} \right) = \left( \frac{dQ_e / dt}{dQ_g / dt} \right) \quad (8) \]

\[ \text{RPR} = \frac{\text{COP}}{\text{COP}_i} \quad (9) \]

where COP, is the ideal COP. The GT performance indicators are specific fuel consumption (SFC) (kg/kWh), HR (kJ/kWh) and thermal efficiency, nth:

\[ SFC = 3600 \frac{m_{fuel}}{W_{net}} \quad (10) \]

\[ HR = SFC \cdot LHV \quad (11) \]

\[ n_{th} = \frac{3600}{SFC} \cdot LHV \quad (12) \]

where \( m_{fuel} \) is the mass flowrate of the fuel fed into the GT (in kg/s) and LHV is the lower heating value of natural gas (in kJ/kg). This optimization problem is formulated where the constraints were based on the chiller design specifications. Due to the dynamic nature of the model, the objective functions are time-averaged. Since the ambient temperature is fuzzified, the objective functions (HR’, COP’ and \( n_{th} \)) become fuzzy as well:

Max \( \rightarrow \) Overall Thermal Efficiency, \( \bar{n}_{th} \)

Min \( \rightarrow \) Heat Rate, \( \overline{HR} \)

Max \( \rightarrow \) Coefficient of Performance, \( \overline{COP} \)

subject to the following constraints:

\[ 32 \leq t_a \leq 38.5 \leq t_c \leq 18.480 \leq t_g \leq 1000 \leq M_a, M_g \leq 200 \leq Q_e \leq 2040 \quad (13) \]

where \( t_a, t_c, t_g \) and \( t_e \) are the component temperatures (°C) of the absorber, evaporator, generator, and condenser. QE is the refrigeration load (kcal/h), and \( E_L \) is the heat exchanger effectivity in the AC. \( M_a \) and \( M_g \) are the mass storage capacity of the sumps (kg) in the AC. The PPAs impose an additional constraint to the problem formulation. The minimal amount of power required to be supplied at all time is 129 MW. The PPA considered here allows for a maximal supply of 7.8% above minimal supply power which translates to about 10 MW. Therefore, the maximum power supply is limited to as follows:

\[ W_{net} \leq 139 \quad (14) \]
IV. CHAOS-DRIVEN DIFFERENTIAL EVOLUTION (CDDE)

DE is a class of evolutionary meta-heuristic algorithms first introduced in the mid-nineties [24]. The incorporation of perturbative methods into evolutionary techniques is the fundamental theme of DE. DE starts by generating a population of a minimum of four individuals denoted, \( P \). These individuals are real-coded vectors with some specified size, \( N \). The DE algorithm is augmented to enhance its optimization capabilities by the addition of the chaotic component. The chaotic component diversifies the population further enabling it to thoroughly search the objective space. First, the population of vectors, \( x_0 \) was generated. The consequent steps are similar to the regular DE algorithm where one principal parent, \( x_p \) and three auxiliary parents \( x_a \) are randomly selected. Differential mutation is then performed and the mutated vector, \( V_i \) is generated. By recombining \( V_i \) with \( x_p \), the child trial vector, \( x_{child} \) is created. The obtained \( x_{child} \) is used as the input to the chaotic map [25]. The CEDE approach employed in this work follows closely the technique presented in [13]:

\[
N_i(t) = x_{child}^i(t), \quad R_i(t) = \lambda N_i(t) \quad (15)
\]

\[
N_i(t + 1) = R_i(t) N_i(t) [1 - N_i(t)] \quad (16)
\]

\[
R_i(t + 1) = R_i(t) + \lambda' \quad (17)
\]

\[
\frac{x_{child}^i}{x_p} \quad (18)
\]

where \( N(t) \) and \( R(t) \) are variables in the logistic chaotic map, and are predefined relaxation parameters. Then, the logistic mapping is performed until a specific number of iteration is satisfied. The final value at maximum number of iteration of \( N(t_{max}) \) is incorporated into the child trial vector, \( x_{child} \). Hence, the child trial vector, \( x_{child} \) undergoes another round of mutation by the chaotic map. Survival selection in the next generation is performed via ‘knock-out’ competition. The fitness function for the child trial vector, \( x_{child} \) is evaluated. Thus, another variant of the DE algorithm which is chaotically driven was developed. In this work, this algorithm is called the Chaotic-driven DE (CEDE). The algorithm for the CEDE technique is given in Algorithm 1:

**Algorithm 1: Chaos-Driven Differential Evolution (CDDE)**

1. Set parameters: \( N \) and \( P \).
2. Deterministically initialize population vectors, \( x_0^i \).
3. Iterate chaotic logistic map.
4. IF \( n > N_{max} \) go to next step else go to Step 3.
5. Randomly select one principal parent, \( x_p^i \).
6. Randomly select three auxiliary parents, \( x_a^i \).
7. Perform differential mutation & generate mutated vector, \( V_i \).
8. Recombine \( V_i \) with \( x_p^i \) to generate child trial vector, \( x_{child}^i \).
9. Evaluate fitness of the new \( x_{child}^i \).
10. IF the halting conditions are fulfilled halt and print solutions else proceed to step 2.

V. COMPUTATIONAL RESULTS

The fuzzy multicriteria problem is solved via the weighted-sum approach. The COP and thermal efficiency objectives are in the range of 0 to 2 while the second objective, HR has values in the scale of \( 10^3 \). Thus, the HR objective was normalized using the high HR value of 8000 kJ/kWh:

\[
HR' = \frac{HR}{8000} \quad (19)
\]

The objective functions are optimized to obtain the chiller design parameters and the amount of GT air inlet cooling required, \( \Delta T \) - while accounting for uncertainties in the ambient temperature data in the optimization model. The CEDE technique was employed in this work is incorporated with the chaotic component to boost its optimization performance. The relaxation parameter, \( \lambda \) was varied to increase the degree of chaos in the CEDE. After rigorous testing, it was identified that the most optimal frontier is obtained when the CEDE is tuned with maximal chaotic capability (at \( \lambda = 0.9 \)). The nadir point (0.1, 1, 0.1) was employed as a reference in the HVI.

The algorithms implemented in this work were developed using the C++ programming language on a personal computer with an Intel® Core™ i5 processor running at 3.2 GHz. The entire Pareto frontier was constructed using compromised individual solutions obtained at different scalarization. Due to stochastic nature of the algorithm, the best individual solution was taken after five independent runs. The optimization carried out in this work was on a dynamic system for a 24-hour cycle. Therefore, the objective values obtained here are time-averaged. The fuzzified ambient temperatures with respect to various membership grades are presented in Fig. 1.

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**Fig. 1 Fuzzy Ambient Temperature versus membership grades**

The membership grade \( \mu_x(y, \eta_x(x)) \) is more of a tensor since it takes vector values of \( \eta_x(x) \) as an input. To reduce its dimensions, \( \mu_x(y, \eta_x(x)) \) values are averaged back into a vector form as depicted in Fig. 1. The ranges for the membership grades are as follows:
Using type-2 fuzzy formulation to account for the uncertainties in the ambient temperature, the CEDE technique applied to the combined multiobjective GT-AC system. The optimal values for the overall thermal efficiency, $\eta'_{th}$ is given in Fig. 2.

Fig. 2 Overall Thermal Efficiency versus membership grades

Based on Fig. 2, the variation in the thermal efficiency of the GT at all membership grades is 5.146% (corresponding to 0.01608 differences in thermal efficiency). A maximum thermal efficiency of 0.3286 was obtained at $\mu_{F,T} = 0.04298$ and $\eta_{F,T} = 0.13811$. The minimal thermal efficiency at $\mu_{F,T} = 0.00547$ and $\eta_{F,T} = 0.15725$ was 0.3125. Figs. 3 and 4 depict the normalized HR and the COP respectively relative to the primary and secondary membership values.

The maximal variation in terms of the HR relative to the membership grades is 376.06 kJ/kWh. The maximum normalized HR is 0.72003 (5760.27 kJ/kWh) at $\eta_{F,T} = 0.15725$ and $\mu_{F,T} = 0.00547$. The minimal normalized HR is 0.67302 (5384.22 kJ/kWh) at $\eta_{F,T} = 0.13811$ and $\mu_{F,T} = 0.04298$. The HR fluctuates by about 6.9844%. On the other hand, the COP varies by 2.08991% (0.04). It can be observed that the uncertainties in temperature variation significantly impacts HR and thermal efficiency of the GT while affecting the COP of the AC to a lesser degree. From these findings, it could be said that uncertainty in ambient temperatures highly influences the optimization on the GT operations as compared to the design of the AC. The temperature data considered here are only for over a range of a single month. When projected further to an annual scale, the minor variations in the COP may translate to significant cost savings. Besides, this work focuses on a single unit combined GT-AC system; these variations in COP, thermal efficiency and HR may compound to produce a greater impact when considering multiple GT-AC systems.

In Table I, $\Delta T$ is the temperature difference achieved by the AC for cooling the turbine air inlet.
The overall frontier dominance trend follows the trend observed in the dominance of the individual solution points produced by the techniques employed in this work. It can be seen that in Fig. 5, the solutions are diversely spread throughout the objective space. The computational time taken for the complete construction of the Pareto frontier for each membership grade is approximately 50 – 60 seconds. The CEDE approach in this work performed stable computations throughout the objective space. The computational time taken seen that in Fig. 5, the solutions are diversely spread produced by the techniques employed in this work. It can be observed in the dominance of the individual solution points.

The PPA considered in this work limits the power output of other hybrid algorithms [28]. In addition, future projections on metaheuristics could be tested on this problem – e.g. swarm algorithms; Bacteria Foraging [26] and Cuckoo Search [27] or optimization using particle swarm optimization with non-Gaussian random generators. Intelligent Decision Technologies, 10(2), pp.93-103.

The Type-2 fuzzy framework was implemented to optimize the design and operations of the combined GT - AC system. The ambient temperature data contained two moments of the design and operations of the combined GT - AC system. Due to the GT power output constraint imposed by the PPA, the optimization directly impacts the HR – resulting in lower fuel consumption (SFC). In future works, other metaheuristics could be tested on this problem – e.g. swarm algorithms; Bacteria Foraging [26] and Cuckoo Search [27] or other hybrid algorithms [28]. In addition, future projections on long-term cost-benefit analyses could be carried out to observe how the fuzzy optimization in this work affects the GT-AC system in fiscal terms.

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