Non-Linear Vibration and Stability Analysis of an Axially Moving Beam with Rotating-Prismatic Joint

M. Najafi, F. Rahimi Dehgolan

***Abstract*—**In this paper, the dynamic modeling of a single-link flexible beam with a tip mass is given by using Hamilton's principle. The link has been rotational and translational motion and it was assumed that the beam is moving with a harmonic velocity about a constant mean velocity. Non-linearity has been introduced by including the non-linear strain to the analysis. Dynamic model is obtained by Euler-Bernoulli beam assumption and modal expansion method. Also, the effects of rotary inertia, axial force, and associated boundary conditions of the dynamic model were analyzed. Since the complex boundary value problem cannot be solved analytically, the multiple scale method is utilized to obtain an approximate solution. Finally, the effects of several conditions on the differences among the behavior of the non-linear term, mean velocity on natural frequencies and the system stability are discussed.

***Keywords*—**Non-linear vibration, stability, axially moving beam, bifurcation, multiple scales method.

I. Introduction

AXIALLY moving beam with rotating prismatic-joint models may be used for many engineering devices; e.g., robots applications, telescopic members of loading vehicles, space craft antenna, magnetic tape drivers, printers, flexible transmission lines, band saws, weaving mechanisms and furnace conveyor belts all are classified as axially moving beams with rotating prismatic-joint.

Moving beams can be modeled as linear or non-linear. Velocity can be constant or harmonically variable. Time-dependent transport velocity means a mean velocity plus small periodic fluctuations. In fact, many real mechanisms can be represented as axially moving beams with time-dependent velocity.

There are many researches which have been carried out on axially moving systems in literatures. Wang and Wei [1] analyzed a flexible single link with a prismatic joint. Translational and rotational motion effects were analyzed on the vibratory motion. The lateral vibration of model was solved with Galerkin method. By solving some typical problems, the numerical results were obtained. Kane et al. [2] studied cantilever beam which is moved with rotational and translational motion. They studied the effects of centrifugal stiffening and investigated vibrations of beam with Coriolis forces. Also, they tried to challenge certain multi body computer programs used to simulate these problems. In a similar work, Gaultier and Cleghorn [3] investigated the vibration of a translational and rotational beam which is model of elastic link manipulators by using finite element method. Pan et al. [4] analyzed the vibration of flexible manipulators with prismatic joint. The prismatic joint was modeled as a telescopic manipulator composed of two elastic links. Equations of motion and boundary conditions were obtained by Lagrange's equation of motion. Pan et al. [5] consider the dynamic model of an axially moving beam and in order to validate the dynamic model used an experimental outline. Yuh and Young [6] investigated the dynamics of a rotational and translational beam. An approximation method was extended by using assumed modes method. With using by a series of experimental work, the validity of the approximate model was evaluated. By using the computer simulation, the dynamic response of elastic beam with various motions was investigated. Tadikonda and Baruh [7] considered the vibration and control of an elastic beam with an end mass at the end. The model analyzed the effect of elastic behavior and translational motions. The elastic arm was assumed to move in a rigid prismatic joint.

Al-Bedoor and Khulief [8] investigated the dynamics of an elastic arm reciprocating in a rotating prismatic joint. In order to calculate for the prismatic joint and the effect of an end mass, time varying boundary conditions were used. Kalyoncu and Botsalı [9] analyzed an elastic robot arm moving in a rotating prismatic joint. The equations of motion are given in ordinary differential equations. They investigated effect of rotary inertia, change of length, and natural frequencies of the elastic robot arm. Finally, they have shown tip deflections in graphical form and discussed physical trends of the given numerical results. Yuksel and Gurgoze [10] solved the flexural vibrations of an axially moving robotic arm sliding in prismatic joint while the joint was undergoing vertical translation and rotary motion. Yang and Sadler [11] investigated a modal database procedure for analyzing the dynamics of a tracing manipulator. Robot arm always carries a mass at the end. But this matter was not follow in Yang and Sadler's investigation. Al-Bedoor and Khulief [12] considered a dynamic model for a robot arm sliding through a prismatic joint where the prismatic joint was move planar motion. Also, a finite element model with a fixed number of elements was developed, where the element length was constant. Kalyoncu and Botsalı [13] analyzed the effect of axial vibration on the bending vibrations of an elastic sliding link in a rotating prismatic joint. Chalhoub and Chen [14] presented a general method to derive the equations of motion of flexible open kinematic chains. They analyzed the serial characteristic of the kinematic chain by integrating the 4 × 4 Denavit–Hartenberg

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transformation matrix with a 4 × 4 structural flexibility matrix. The approach was specified based on a rotating coordinate system which presented the formulation applicable to both translational and rotational motion. Gurgoze and Yuksel [15] considered the vibrations of an axially moving flexible beam moving through a rotating prismatic joint, restricted to move planar motion. Bauchau [16] considered modeling of prismatic joints in flexible multi-body systems. For most of rigid bodies, the classical formulation of prismatic joints is used. A sliding joint was considered, that involves kinematic constraints at the point of contact between the sliding bodies. Kalyoncu and Botsalı [17] considered lateral and torsional vibrations of elastic manipulators with prismatic joint. The arm was assumed to carry an end mass. The specified perspective of this investigation was consideration of time varying end mass at the end of sliding beams in a rotating prismatic joint.

Ankaralı et al. [18] analyzed a single flexible robot arm with ended mass which was moved by a flexible shaft. Hamilton's principle was applied in giving the dynamic model. Basher [19] considered the dynamic modeling of a flexible beam with rotational and translation movements. They studied the effects of higher-order dynamic response of the flexible beam. By investigated an infinite number of modes, an analytical model of the beam was considered. For considering of equation of motion Euler–Bernoulli beam equation and modal expansion method was used. Farid and Salimi [20] investigated an inverse dynamic method to specify the needed torque and force for an in plane arm sliding through manipulator with rotational prismatic joints with an ended mass. All of the large rotation's non-linear terms were included. Khadem and Pirmohammadi [21] analyzed a mathematical model of a three-dimensional flexible 3-degree of freedom manipulator, with both rotational and translational joint. This model was used for studying the longitudinal, transversal, and torsional vibration specifications of the robot arm. The equations of motion show longitudinal, transversal, and torsional vibration specifications solved in no discretization parametric form. Stoenescu and Marghitu [22] focused on the effect of prismatic joint inertia on frictional dynamics of planar chains. The mathematical model of a planar chain with a prismatic joint and a revolute joint was obtained using Lagrange's equations. They also analyzed effect of the slider inertia on the position of the application point. Akbaba and Yuksel [23] analyzed an elastic beam moving in a prismatic. The beam fluctuation was assumed that became in two planes and torsional elastic displacements, and elastic beam having a mass point at the end. With this assumption, the equations of motion were obtained by using Hamilton's Principle. Dehgolan et al. [24] studied linear frequencies and stability of a flexible rotor-disk-blades system. They used Euler-Bernoulli beam theory and utilized it to model the blade and shaft. They analyzed the effects of various system parameters on the natural frequencies and clarified the decay rates (stability condition).

Recently, variable speed non-linear beams with Euler-Bernoulli theory have been considered. In the present investigation, a harmonically varying speed non-linear beam with mean velocity variation effects is considered. Applying multiple scales method, stability and bifurcation of non-trivial and trivial steady state response are analyzed. Numerical examples helped us to show the effect of non-linear term and mean velocity on natural frequencies, critical speeds, bifurcation points and stability of trivial and non-trivial solutions. Finally, frequency-response curves are drawn.

II. Equations of Motion

A beam with axial stiffness of EA and the flexural rigidity of EI is shown in Fig. 1. Additionally, this beam is assumed as an Euler–Bernoulli beam. The mass and flexible properties are assumed to be distributed uniformly along the flexible arm. The prismatic joint is assumed to be rigid. The flexible arm slides in the prismatic joint. The sliding motion of the flexible manipulator is assumed to be frictionless. The initial length of the beam is denoted as $l_0$and a harmonically varying transport speed, v. As shown in Fig. 1, w(x,t) describes transverse displacements of the beam. For this model, shear deflection is not considered, Euler-Bernoulli theory with rotary inertia effect is applied, and out of plane motion is neglected. Deformation due to pretension is neglected.

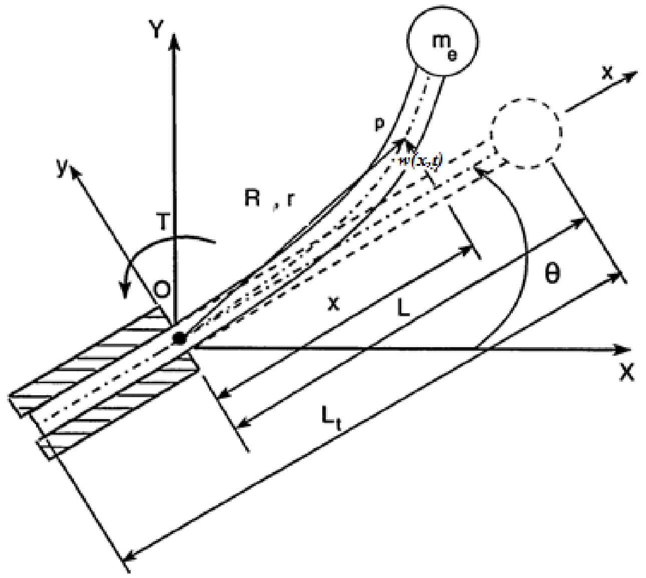

Fig. 1 Axially moving beam with rotating prismatic-joint

The kinetic energy is given by:

$$T = \frac{1}{2}\int_0^l \rho \left[(\dot{x} \frac{\partial w}{\partial x} + \dot{w} + x\,\dot{\theta})^2 + (\dot{x} - w\,\dot{\theta})^2 \right] dx \tag{1}$$

where ρ is the constant mass per unit length. Non-linear strain is used in order to calculate potential energy. Then, the non-linear strain and potential energy are shown as:

$$\varepsilon_{xx} = y\,\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}(\frac{\partial w}{\partial x})^2 \tag{2}$$

$$V = \frac{1}{2}\int_0^l EI\,(\frac{\partial^2 w}{\partial x^2})^2 dx + \frac{1}{8}\int_0^l EA\,(\frac{\partial w}{\partial x})^4 dx + \frac{\rho\dot{\theta}^2}{4}\int_0^l (L^2 - x^2)(\frac{\partial w}{\partial x})^2 dx$$
$$-\frac{\ddot{L}}{2}\int_0^l \rho(L-x)(\frac{\partial w}{\partial x})^2 dx + \frac{1}{2}m_e(L\dot{\theta}^2 - \ddot{L})\int_0^l (\frac{\partial w}{\partial x})^2 dx \tag{3}$$

The governing partial-differential equation and the associated boundary conditions are derived from the Hamilton's principle and the geometrical relations as

$$\delta\int_{t1}^{t2}(T-U)dt = 0 \tag{4}$$

Introducing dimensionless quantities

$$y = \frac{w}{\varepsilon^{0.5}L} \quad z = \frac{x}{L} \quad T = t\sqrt{\frac{EI}{\rho AL^4}} \quad v = LT\,\bar{v} \tag{5}$$

$$\alpha_1 = \frac{m_e}{\rho AL} \quad \alpha_2 = \frac{AL^2}{I} \quad \dot{\theta} = \frac{\partial\theta}{\partial T} = \bar{\omega} \quad \omega = T\,\bar{\omega} \tag{6}$$

Using (4), after simplification, the coupled non-linear equations would be

$$\left(\frac{\partial v}{\partial t} - \omega^2\right)y + (2v)\frac{\partial y}{\partial t} + \left[z\left(\omega^2 - \frac{\partial v}{\partial t}\right)\right]\frac{\partial y}{\partial z} + \frac{\partial^2 y}{\partial t^2} +$$
$$+\left[v^2(1-z)^2 + (1-z)\frac{\partial v}{\partial t} - \frac{\omega^2}{2}(1-z^2) + \alpha_1\left(\omega^2 - \frac{\partial v}{\partial t}\right)\right]\frac{\partial^2 y}{\partial z^2}$$
$$+2v(1-z)\frac{\partial^2 y}{\partial z\,\partial t} + \frac{\partial^4 y}{\partial z^4} - \frac{3}{2}\alpha_2\varepsilon\left(\frac{\partial^2 y}{\partial z^2}\right)\left(\frac{\partial y}{\partial z}\right)^2 + z\frac{\partial\omega}{\partial t} + 2v\,\omega = 0 \tag{7}$$

$$y = 0 \quad \& \quad \frac{\partial y}{\partial z} = 0 \tag{8}$$

$$\begin{cases} \dfrac{\partial^2 y}{\partial z^2} = 0 \\ \left(\alpha_1\dfrac{\partial v}{\partial t} - v^2 - \alpha_1\omega^2\right)y + (2\alpha_1 v - v)\dfrac{\partial y}{\partial t} - \\ \left(\alpha_1(1+z)\dfrac{\partial v}{\partial t} + v^2(1-z) - \alpha_1\omega^2\right)\dfrac{\partial y}{\partial z} - \\ -2\alpha_1 vz\dfrac{\partial^2 y}{\partial t\,\partial z} + \alpha_1\dfrac{\partial^2 y}{\partial t^2} + \alpha_1 v^2 z^2\dfrac{\partial^2 y}{\partial z^2} + \\ \dfrac{\varepsilon}{2}\alpha_2\left(\dfrac{\partial y}{\partial z}\right)^3 - \left(\dfrac{\partial^3 y}{\partial z^3}\right) = v\,\omega - 2\alpha_1 v\,\omega - \alpha_1\left(\dfrac{\partial\omega}{\partial z}\right) \end{cases} \tag{9}$$

In reality, the longitudinal disturbances propagate significantly faster than the transverse one. To use the multiple scales method, the non-linear term must be weak. Then, using transformation $w = \sqrt{\varepsilon}u$ and its substitution into (7), one obtains

$$\left(\frac{\partial v}{\partial t} - \omega^2\right)u + (2v)\frac{\partial u}{\partial t} + \left[z\left(\omega^2 - \frac{\partial v}{\partial t}\right)\right]\frac{\partial u}{\partial z} + \frac{\partial^2 u}{\partial t^2} +$$
$$2v(1-z)\frac{\partial^2 u}{\partial z\,\partial t} + \frac{\partial^4 u}{\partial z^4} - \frac{3}{2}\alpha_2\varepsilon\left(\frac{\partial^2 u}{\partial z^2}\right)\left(\frac{\partial u}{\partial z}\right)^2 +$$
$$\begin{bmatrix} v^2(1-z)^2 + (1-z)\dfrac{\partial v}{\partial t} - \dfrac{\omega^2}{2}(1-z^2) + \\ \alpha_1\left(\omega^2 - \dfrac{\partial v}{\partial t}\right) \end{bmatrix}\frac{\partial^2 u}{\partial z^2} =$$
$$-z\frac{\partial\omega}{\partial t} - 2v\,\omega + \left(\frac{\partial v}{\partial t} - \omega^2\right)(\mu z^2 - \frac{\mu}{3}z^3) +$$
$$\left[z\left(\omega^2 - \frac{\partial v}{\partial t}\right)\right](2\mu z - \mu z^2) +$$
$$\begin{bmatrix} v^2(1-z)^2 + (1-z)\dfrac{\partial v}{\partial t} - \dfrac{\omega^2}{2}(1-z^2) + \\ \alpha_1\left(\omega^2 - \dfrac{\partial v}{\partial t}\right) \end{bmatrix}(2\mu - 2\mu z) -$$
$$\frac{3}{2}\alpha_2\varepsilon\left(4\mu^2 z - 6\mu^2 z^2 + 2\mu^2 z^3\right) \tag{10}$$

$$\mu = \frac{(1-2\alpha_1)v_0\omega_0}{\frac{2}{3}\left(-v_0^2 - \alpha_1\omega_0^2\right) + \alpha_1\omega_0^2 - 2} \tag{11}$$

$$at \quad z = 0 \quad \Rightarrow u = 0 \quad \& \quad \frac{\partial u}{\partial z} = 0$$
$$at \quad z = 1 \Rightarrow$$
$$\begin{cases} \dfrac{\partial^2 u}{\partial z^2} = 0 \\ \left(-v_0^2 - \alpha_1\omega_0^2\right)u + (2\alpha_1 - 1)v_0\dfrac{\partial u}{\partial t_0} + \left(\alpha_1\omega_0^2\right)\dfrac{\partial u}{\partial z} - \\ \alpha_1\dfrac{\partial^2 u}{\partial t_0^2} + (2\alpha_1 v_0)\dfrac{\partial^2 u}{\partial z\,\partial t_0} + \alpha_1 v_0^2\dfrac{\partial^2 u}{\partial z^2} - \dfrac{\partial^3 u}{\partial z^3} = 0 \end{cases} \tag{12}$$

in which ε is a very small parameter (namely, $\varepsilon \ll 1$). As mentioned above, the beam is moving with a harmonically varying velocity about a constant mean velocity, i.e.

$$v = v_0 + \varepsilon v_1\sin\Omega_1 t \qquad \omega = \omega_0 + \varepsilon\omega_1\sin\Omega_2 t \tag{13}$$

In which $\Omega_{1\&2}$ is the frequency of varying speed, v_0 is the mean velocity and εv_1 is the amplitude. In order to find an approximated solution in a finite dimensional function space, the Galerkin method is used in this study. The solution of (7) is approximated by a series of comparison functions that satisfy both the essential and natural boundary conditions. The trial function for the approximated solution may be expressed as

$$u\left(z,t\right)=\sum_{n=1}^{N}\left[\phi_n\left(z\right)q_n(t)\right] \tag{14}$$

where N is the total number of comparison functions, $q_n(t)$ are unknown functions of time to be determined, and w(x,t) are the eigenfunctions for the bending vibration of the stationary cantilever beam:

$$\phi(x)=c_1\left[\begin{array}{l}\left(\sin\lambda_n z-\sinh\lambda_n z\right)-\\ \dfrac{\sin\lambda_n z+\sinh\lambda_n z}{\cos\lambda_n z+\cosh\lambda_n z}\times\left(\cos\lambda_n z-\cosh\lambda_n z\right)\end{array}\right] \tag{15}$$

The weighting function or the virtual function corresponding to (14) is given by:

$$\bar{u}\left(x,t\right)=\sum_{n=1}^{\infty}\left[\phi_n\left(z\right)\bar{q}_n(t)\right] \tag{16}$$

Discretized equations of motion are determined by using (14) and (15). Consider an equation obtained by substituting (14) into (10), multiplying the resultant equation by (16) and then integrating it over the domain $0\le x\le 1$: If this equation is collected with respect to $\bar{q}(t)$, their coefficients provide the discretized equations since $\bar{q}(t)$ are arbitrary. The discretized equations of axially moving beam with rotating prismatic-joint may then be expressed as:

$$\begin{aligned}&(\frac{\partial v}{\partial t}-\omega^2)A_n q_n+2v\ A_n\dot{q}_n+(\omega^2-\frac{\partial v}{\partial t})\sum_{m=1}^{N}B_{nm}q_n+A_n\ddot{q}_n+v^2\sum_{m=1}^{N}C_{nm}q_n+\\&\frac{\partial v}{\partial t}\sum_{m=1}^{N}D_{nm}q_n-(\frac{\omega^2}{2})\sum_{m=1}^{N}E_{nm}q_n+\alpha_1(\omega^2-\frac{\partial v}{\partial t})\sum_{m=1}^{N}F_{nm}q_n+2v\sum_{m=1}^{N}G_{nm}\dot{q}_n+\\&\sum_{m=1}^{N}H_{nm}q_n-\frac{3}{2}\alpha_2\varepsilon\sum_{m=1}^{N}I_{nm}q_n^3++J_n\frac{\partial\omega}{\partial t}+2K_n\omega v=\\&-\mu\left[\begin{array}{l}\dfrac{\partial v}{\partial t}-\omega^2)(\dfrac{2M_n}{3}-L_n)+\\ 2v^2P_n+\dfrac{\partial v}{\partial t}O_n-\dfrac{\omega^2}{2}Q_n+\\ \left.\alpha_1(\omega^2-\dfrac{\partial v}{\partial t})N_n\right]\end{array}\right]+\frac{3}{2}\alpha_2\varepsilon\ \mu^2\left(4J_n-6L_n+2M_n\right)\end{aligned} \tag{17}$$

where the superposed dot represents the differentiation with respect to time; are given by

$$\begin{aligned}&A_n=\int_0^1\phi_n^2dz\qquad B_{nm}=\int_0^1 z\,\phi_n\phi_m' dz\qquad C_{nm}=\int_0^1(1-z)^2\phi_n\phi_m''dz\\&D_{nm}=\int_0^1(1-z)\phi_n\phi_m''dz\ E_{nm}=\int_0^1(1-z^2)\phi_n\phi_m''dz\qquad F_{nm}=\int_0^1\phi_n\phi_m''dz\\&G_{nm}=\int_0^1(1-z)\phi_n\phi_m'dz\qquad H_{nm}=\int_0^1\phi_n\phi_m^{(4)}dz\ I_{nm}=\int_0^1\phi_n\phi_m''(\phi_m')^2dz\\&J_n=\int_0^1 z\,\phi_n dz\qquad K_n=\int_0^1\phi_n dz\qquad L_n=\int_0^1 z^2\phi_n dz\qquad M_n=\int_0^1 z^3\phi_n dz\\&N_n=\int_0^1(1-z)\phi_n dz\qquad O_n=\int_0^1(1-z)^2\phi_n dz\qquad P_n=\int_0^1(1-z)^3\phi_n dz\\&Q_n=\int_0^1(1-z)(1-z^2)\phi_n dz\end{aligned} \tag{18}$$

Note that the dimensionless natural frequency of the stationary cantilever beam, ω_n, is equal to the square of the root of λ_n.

III. Multiple Scales Method, Stability and Bifurcations

The straightforward expansion techniques fail to correctly represent a proper solution for problems which have secular terms. This deficiency is overcome by assuming the solution to be a function of multiple independent time-variables, or scales [25]. Then, one assumes the expansion of the form [26], [27]

$$u(x,t;\varepsilon)=u_0(x,T_0,T_1)+\varepsilon u_1(x,T_0,T_1)+... \tag{19}$$

in which $T_0=t$ and $T_1=\varepsilon t$. Substitution of (15) and (16) into (14) shows that

$$\begin{aligned}
&O\left(\varepsilon^0\right)\Rightarrow\\
&(-\omega_0^2)A_n q_{n0}+2v_0 A_n \dot{q}_{n0}+(\omega_0^2)\sum_{m=1}^{N}B_{nm}q_{n0}+A_n\ddot{q}_{n0}+v_0^2\sum_{m=1}^{N}C_{nm}q_{n0}-\\
&-(\frac{\omega_0^2}{2})\sum_{m=1}^{N}E_{nm}q_{n0}+\alpha_1(\omega_0^2)\sum_{m=1}^{N}F_{nm}q_{n0}+2v_0\sum_{m=1}^{N}G_{nm}\dot{q}_{n0}+\sum_{m=1}^{N}H_{nm}q_{n0}=\\
&=-2K_n\omega_0 v_0-\mu\left[(-\omega_0^2)(\frac{2M_n}{3}-L_n)+2v_0^2P_n-\frac{\omega_0^2}{2}Q_n+\alpha_1(\omega_0^2)N_n\right]
\end{aligned} \tag{20}$$

$$\begin{aligned}
&O\left(\varepsilon^1\right)\Rightarrow\\
&(-\omega_0^2)A_n q_{n1}+2v_0 A_n\dot{q}_{n1}+(\omega_0^2)\sum_{m=1}^{N}B_{nm}q_{n1}+A_n\ddot{q}_{n1}+\\
&v_0^2\sum_{m=1}^{N}C_{nm}q_{n1}--(\frac{\omega_0^2}{2})\sum_{m=1}^{N}E_{nm}q_{n1}+\alpha_1(\omega_0^2)\sum_{m=1}^{N}F_{nm}q_{n1}+\\
&2v_0\sum_{m=1}^{N}G_{nm}\dot{q}_{n1}+\sum_{m=1}^{N}H_{nm}q_{n1}==-2K_n\left(v_0\omega_1\sin\Omega_2 t+v_1\omega_0\sin\Omega_1 t\right)-\\
&\mu\left[(v_1\Omega_1\cos\Omega_1 t-2\omega_0\omega_1\sin\Omega_2 t)(\frac{2M_n}{3}-L_n)+\right.\\
&\left.+4(v_0v_1\sin\Omega_1 t)P_n+(\omega_0\omega_1\sin\Omega_2 t)(2\alpha_1 N_n-Q_n)\right]-\\
&\left(v_1\Omega_1\cos\Omega_1 t-2\omega_0\omega_1\sin\Omega_2 t\right)A_n q_{n0}-\\
&-\left(2v_0A_n\right)\frac{\partial q_{n0}}{\partial t_1}-\left(2A_n v_1\sin\Omega_1 t\right)\dot{q}_{n0}+\\
&\left(v_1\Omega_1\cos\Omega_1 t-2\omega_0\omega_1\sin\Omega_2 t\right)\sum_{m=1}^{N}B_{nm}q_{n0}-2A_n\frac{\partial^2 q_{n0}}{\partial t_0\partial t_1}-\\
&-\left(2v_0v_1\sin\Omega_1 t\right)\sum_{m=1}^{N}C_{nm}q_{n0}-\left(v_1\Omega_1\cos\Omega_1 t\right)\sum_{m=1}^{N}D_{nm}q_{n0}-\\
&\left(\omega_0\omega_1\sin\Omega_2 t\right)\sum_{m=1}^{N}E_{nm}q_{n0}++\left(v_1\Omega_1\cos\Omega_1 t-2\omega_0\omega_1\sin\Omega_2 t\right)\times\\
&\sum_{m=1}^{N}F_{nm}q_{n0}-\left(2v_0\right)\sum_{m=1}^{N}G_{nm}\frac{\partial q_{n0}}{\partial t_1}-\left(2v_1\sin\Omega_1 t\right)\sum_{m=1}^{N}G_{nm}q_{n0}-\\
&-\sum_{m=1}^{N}H_{nm}q_{n0}+\frac{3}{2}\alpha_2 q_{n0}^3-\left(\omega_1\Omega_2\cos\Omega_2 t\right)J_n+\\
&\frac{3}{2}\alpha_2\mu^2\left(4J_n-6L_n+2M_n\right)-\frac{3}{2}\alpha_2\sum_{m=1}^{N}I_{nm}q_{n0}^3
\end{aligned} \tag{21}$$

One supposes that the solution of (20) is

$$q_0^n\left(t_0,t_1\right)=\theta_n\left(t_1\right)e^{i\omega_n t_0}+\overline{\theta}_n\left(t_1\right)e^{-i\omega_n t_0} \tag{22}$$

in which ω_n is the natural frequency and $\theta_n(t_1)$ is the amplitude. Substitution of (22) into (21) shows that

$$\begin{aligned}
&O\ \left(\varepsilon^1\right)\Rightarrow\\
&(-\omega_0^2)A_n q_{n1}+2v_0 A_n \dot{q}_{n1}+(\omega_0^2)\sum_{m=1}^{N}B_{nm}q_{n1}+A_n\dot{q}_{n1}+v_0^2\sum_{m=1}^{N}C_{nm}q_{n1}-\\
&-(\frac{\omega_0^2}{2})\sum_{m=1}^{N}E_{nm}q_{n1}+\alpha_1(\omega_0^2)\sum_{m=1}^{N}F_{nm}q_{n1}+2v_0\sum_{m=1}^{N}G_{nm}\dot{q}_{n1}+\sum_{m=1}^{N}H_{nm}q_{n1}=\\
&=-2K_n\left(v_0\omega_1\sin\Omega_2 t+v_1\omega_0\sin\Omega_1 t\right)-\\
&\mu\left[(\frac{2M_n}{3}-L_n)(v_1\Omega_1\cos\Omega_1 t-2\omega_0\omega_1\sin\Omega_2 t)\right]+\\
&+4(v_0 v_1\sin\Omega_1 t)P_n+(\omega_0\omega_1\sin\Omega_2 t)\left[2\alpha_1 N_n-Q_n\right]-\\
&\left[\left(v_1\Omega_1\cos\Omega_1 t-2\omega_0\omega_1\sin\Omega_2 t\right)\times\times A_n(\theta_n\left(t_1\right)e^{i\omega_n t_0}+q_{n0}^p)\right]-\\
&\left(2v_0 A_n\right)\left(\theta_n'\left(t_1\right)e^{i\omega_n t_0}\right)-\left(2A_n v_1\sin\Omega_1 t\right)\left(\theta_n\left(t_1\right)i\,\omega_n e^{i\omega_n t_0}\right)+\\
&+\left(v_1\Omega_1\cos\Omega_1 t-2\omega_0\omega_1\sin\Omega_2 t\right)\sum_{m=1}^{N}B_{nm}(\theta_n\left(t_1\right)e^{i\omega_n t_0}+q_{n0}^p)-\\
&2A_n\left(\theta_n'\left(t_1\right)i\,\omega_n e^{i\omega_n t_0}\right)-\\
&\left[\left(2v_0 v_1\sin\Omega_1 t\right)\sum_{m=1}^{N}C_{nm}+\left(v_1\Omega_1\cos\Omega_1 t\right)\sum_{m=1}^{N}D_{nm}+\left(\omega_0\omega_1\sin\Omega_2 t\right)\sum_{m=1}^{N}E_{nm}\right]\times\\
&(\theta_n\left(t_1\right)e^{i\omega_n t_0}+q_{n0}^p)+\\
&\left[\left(v_1\Omega_1\cos\Omega_1 t-2\omega_0\omega_1\sin\Omega_2 t\right)\sum_{m=1}^{N}F_{nm}-\left(2v_1\sin\Omega_1 t\right)\sum_{m=1}^{N}G_{nm}\right]\times\\
&(\theta_n\left(t_1\right)e^{i\omega_n t_0}+q_{n0}^p)-\\
&-\left(2v_0\right)\sum_{m=1}^{N}G_{nm}\left(\theta_n'\left(t_1\right)e^{i\omega_n t_0}\right)-\sum_{m=1}^{N}H_{nm}(\theta_n\left(t_1\right)e^{i\omega_n t_0}+q_{n0}^p)+\\
&\frac{3}{2}\alpha_2(\theta_n\left(t_1\right)e^{i\omega_n t_0}+q_{n0}^p)^3--\left(\omega_1\Omega_2\cos\Omega_2 t\right)J_n+\\
&\frac{3}{2}\alpha_2\mu^2\left(4J_n-6L_n+2M_n\right)+cc+h.o.t
\end{aligned} \tag{23}$$

in which when Ω_1 is close to $2\omega_n$, principal parametric resonances will occur. Let us consider

$$\Omega_1=\omega_n+\varepsilon\sigma \tag{24}$$

where σ is the detuning parameter. The solvability condition can be obtained using (21) as:

$$\frac{d\theta_n}{dt_1}+\zeta_{n1}\theta_n^2\overline{\theta}+\left(\zeta_{n2}\right)\theta_n+\left(\eta_{n3}+\eta_{n4}\right)e^{i\sigma T_1}=0 \tag{25}$$

in which

$$\begin{aligned}
\zeta_{n1}&=\frac{\left(\frac{9}{2}\alpha_2\sum_{m=1}^{N}I_{nm}\right)}{-2\left[A_n\left(v_0+i\,\omega_n\right)+\left(v_0\right)\sum_{m=1}^{N}G_{nm}\right]}\\
\zeta_{n2}&=\frac{\left(\frac{9}{2}\alpha_2 q_{n0}^{2p}\sum_{m=1}^{N}I_{nm}-\sum_{m=1}^{N}H_{nm}\theta_n\right)}{-2\left[A_n\left(v_0+i\,\omega_n\right)+\left(v_0\right)\sum_{m=1}^{N}G_{nm}\right]}\\
\eta_{n3}&=\frac{\left[-\frac{\mu}{2}(\frac{2M_n}{3}-L_n)\right]}{-2\left[A_n\left(v_0+i\,\omega_n\right)+\left(v_0\right)\sum_{m=1}^{N}G_{nm}\right]}\\
\eta_{n4}&=\frac{\left[\left(i\left(K_n v_1\omega_0-2P_n v_0 v_1\right)\right)\right]}{-2\left[A_n\left(v_0+i\,\omega_n\right)+\left(v_0\right)\sum_{m=1}^{N}G_{nm}\right]}
\end{aligned} \tag{26}$$

Let

$$\theta_n(T_1) = \frac{1}{2} a_n(T_1)\, \mathrm{e}^{i\beta_n(T_1)} \tag{27}$$

Using (23) and (25), one has

$$\frac{\partial a_n}{\partial T_1} = -\frac{1}{4}\mathrm{Re}(\zeta_{n1}) a_n^3 - \mathrm{Re}(\zeta_{n2}) a_n - \left[\mathrm{Re}(\eta_{n3}+\eta_{n4})\cos(\gamma_n) - \mathrm{Im}(\eta_{n3}+\eta_{n4})\sin(\gamma_n)\right] \tag{28}$$

$$\frac{\partial \gamma_n}{\partial T_1} = \sigma + \frac{1}{4}\mathrm{Im}(\zeta_{n1}) a_n^2 + \mathrm{Im}(\zeta_{n2}) + \frac{1}{a_n}\left[\mathrm{Im}(\eta_{n3}+\eta_{n4})\cos(\gamma_n) + \mathrm{Re}(\eta_{n3}+\eta_{n4})\sin(\gamma_n)\right] \tag{29}$$

in which

$$\gamma_n = \sigma T_1 - \beta_n$$
$$\frac{\partial \beta_n}{\partial T_1} = \sigma - \frac{\partial \gamma_n}{\partial T_1} \tag{30}$$

As one considers the stationary response, the value of a_n' and γ_n' will be equal to zero. Elimination of η_n between (28) and (29) leads to

$$\sigma = -\frac{1}{4}\mathrm{Im}(\zeta_{n1}) a_n^2 - \mathrm{Im}(\zeta_{n2}) \pm \sqrt{\frac{1}{a_n^2}\left[[\mathrm{Re}(\eta_{n3}+\eta_{n4})]^2 + [\mathrm{Im}(\eta_{n3}+\eta_{n4})]^2\right] - \left[\frac{1}{4}\mathrm{Re}(\zeta_{n1}) a_n^2 + \mathrm{Re}(\zeta_{n2})\right]^2} \tag{31}$$

Using (28) and (29) and constructing the Jacobian matrix, one has

$$\left| J - \lambda I \right| = 0 \tag{32}$$

$$\begin{bmatrix} \left(-\frac{3}{4}\mathrm{Re}(\zeta_{n1}) a_n^2 - \mathrm{Re}(\zeta_{n2}) - \lambda\right) & \left(\begin{array}{l}\frac{1}{2}\mathrm{Im}(\zeta_{n1}) a_n + \mathrm{Im}(\zeta_{n2}) + \\ \frac{1}{a_n}\left[\sigma + \frac{1}{4}\mathrm{Im}(\zeta_{n1}) a_n^2 + \mathrm{Im}(\zeta_{n2})\right]\end{array}\right) \\ \left(-a_n\left[\sigma + \frac{1}{4}\mathrm{Im}(\zeta_{n1}) a_n^2 + \mathrm{Im}(\zeta_{n2})\right]\right) & \left(-\frac{1}{4}\mathrm{Re}(\zeta_{n1}) a_n^2 - \mathrm{Re}(\zeta_{n2}) - \lambda\right) \end{bmatrix} \tag{33}$$

From (31) and (33), one has

$$\lambda^2 + \left\{a_n^2\,\mathrm{Re}(\zeta_{n1}) + 2\,\mathrm{Re}(\zeta_{n2})\right\}\lambda + \\ + \left\{\sigma + \frac{1}{4}\mathrm{Im}(\zeta_{n1}) a_n^2 + \mathrm{Im}(\zeta_{n2})\right\} \times \\ \left\{\frac{3}{4}\mathrm{Im}(\zeta_{n1}) a_n^3 + \frac{1}{4}\mathrm{Im}(\zeta_{n1}) a_n^2 + \mathrm{Im}(\zeta_{n2}) a_n + \mathrm{Im}(\zeta_{n2}) + \sigma\right\} = 0 \tag{34}$$

By using the Routh-Hurwitz criterion, the stability condition can be obtained as below

$$\varsigma_n = \sqrt{\frac{1}{a_n^2}\left[[\mathrm{Re}(\eta_{n3}+\eta_{n4})]^2 + [\mathrm{Im}(\eta_{n3}+\eta_{n4})]^2\right] - \left[\frac{1}{4}\mathrm{Re}(\zeta_{n1}) a_n^2 + \mathrm{Re}(\zeta_{n2})\right]^2}$$
$$\left\{a_n^2\,\mathrm{Re}(\zeta_{n1}) + 2\,\mathrm{Re}(\zeta_{n2})\right\} > 0 \tag{35}$$

(For σ_1)

From (31) and (34) and using the Routh-Hurwitz criterion, the stability condition can be obtained as below

$$\left\{\frac{3}{4}\mathrm{Im}(\zeta_{n1}) a_n^3 \{\varsigma_n\} + \mathrm{Im}(\zeta_{n2}) a_n \{\varsigma_n\} + \varsigma_n^2\right\} > 0 \tag{36}$$

(For σ_2)

IV. Simulation

In this section, the objectives are to study natural frequencies and critical speed variations according to mean velocity. Also, the effects of non-linear term, mean velocity on stability of trivial and non-trivial solutions are investigated. In the other words, one would like to assess how the natural frequencies, critical speeds, stability and bifurcation points will change when system parameters change.

Figs. 2 and 3 show that increasing the mean velocity or would lead to a reduction in the first two natural frequencies of system.

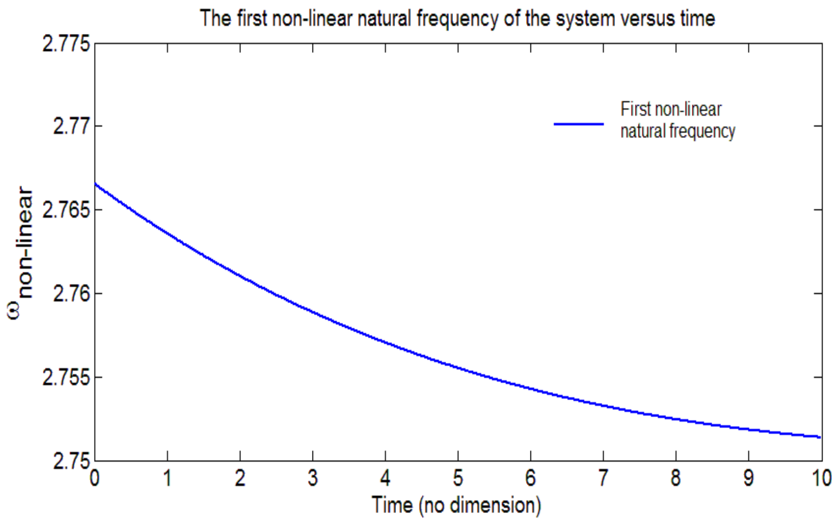

Fig. 2 First natural frequency versus the mean velocity and rotary inertia for the first two modes

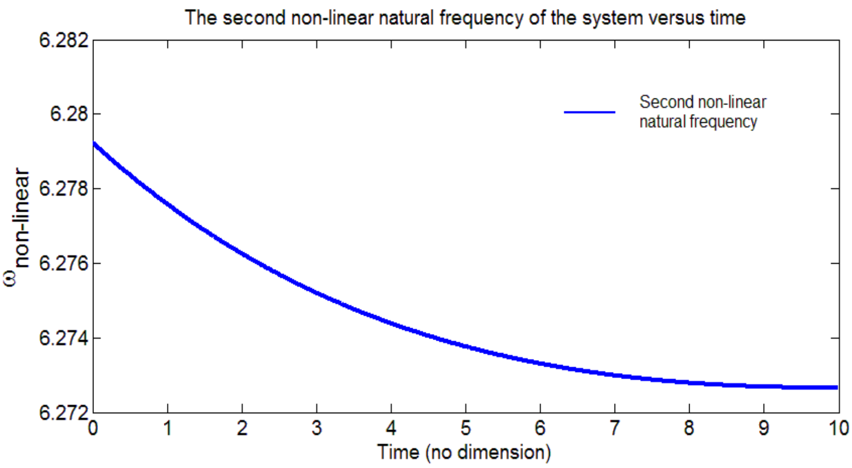

Fig. 3 Second natural frequency versus the mean velocity and rotary inertia for the first two modes

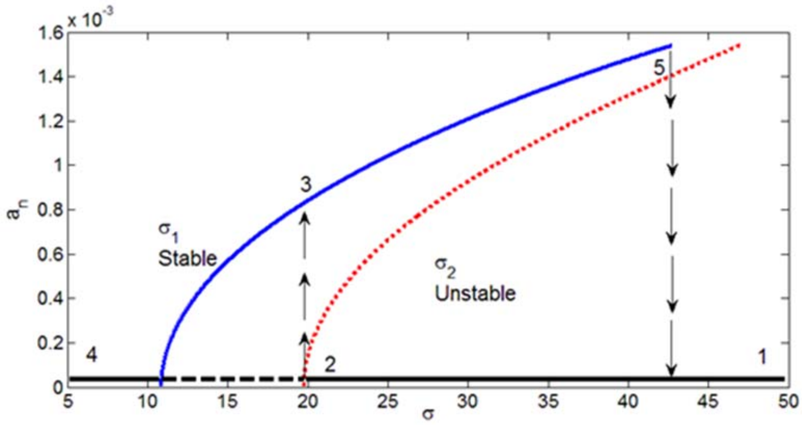

Fig. 4 Stability and bifurcation points' variation for the first mode (dashed line: unstable and solid line: stable)

V. Stability

This section is investigated stability under variation of the mean velocity (v_0), mean angular velocity (ω_0) and non-linear term (α_2). Fig. 4 shows that when $\sigma_1 < \sigma < \sigma_2$, the trivial solution is unstable and bifurcation points arise at $\sigma = \sigma_1$ and $\sigma = \sigma_2$. The larger mean velocities are likely to make "σ_1" stable. As for Euler-Bernoulli beam, the curve of detuning parameter "σ_2" is always unstable. Increasing "v_0"

leads to a larger instability area for trivial solution. It means that the bifurcation point will appear sooner.

In Figs. 5-7, when $\sigma < \sigma_1$, only stable trivial solution exists. When $\sigma = \sigma_1$, the trivial solution will be unstable and a stable nontrivial solution occurs. When $\sigma = \sigma_2$, the trivial solution starts to be stable again, and an unstable nontrivial solution occurs. In Figs. 5-7, at $\sigma < \sigma_1$, a stable trivial solution exists. When $\sigma = \sigma_1$, the trivial solution starts to be unstable, and an unstable nontrivial solution occurs. At $\sigma = \sigma_2$, the trivial solution starts to be stable again, and an unstable nontrivial solution bifurcates. As for the Euler-Bernoulli beam, the curve for detuning parameter "σ_2" is always unstable. Increasing "ζ" leads to a smaller instability interval for trivial solution.

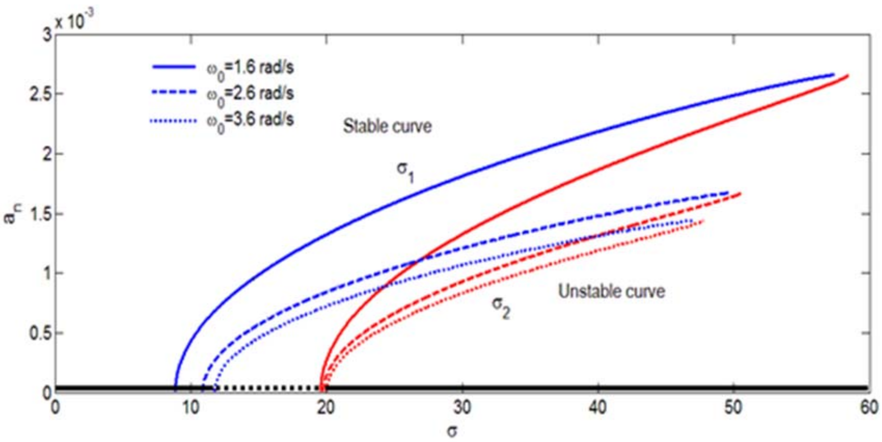

Fig. 5 Stability and bifurcation point variation under the mean translational velocity variation for the first mode

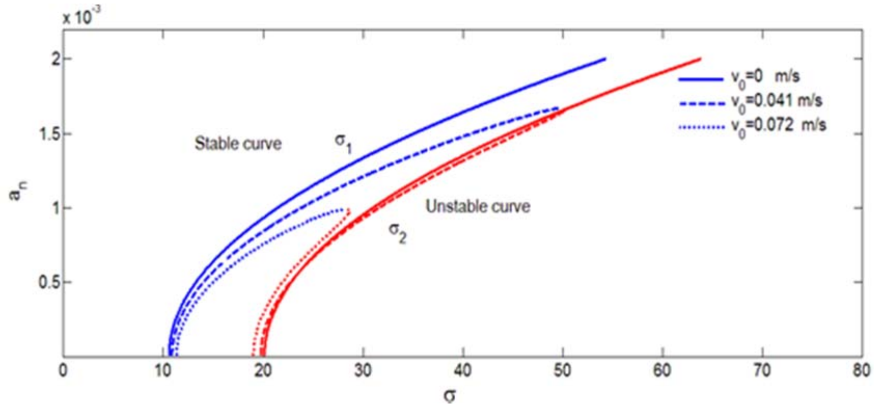

Fig. 6 Stability and bifurcation point variation under the mean rotational velocity variation for the first mode

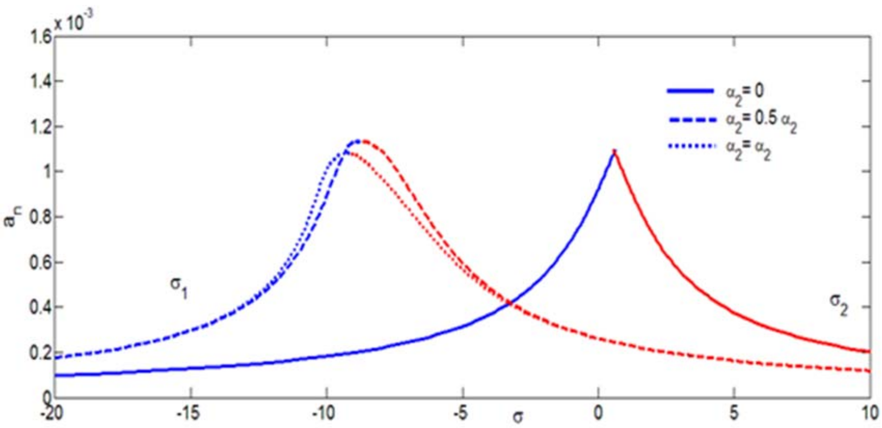

Fig. 7 Stability and bifurcation point variation under the non-linear term variation for the first mode

VI. Conclusions

In this paper, free non-linear vibration of axially moving beam with rotating prismatic joint was investigated. The beam is moving under constant a mean velocity with small periodic fluctuations. The equations were obtained to PDE equations. Then, with using multiple scales method, equations were transferred to ODE. The objectives were to study effects of non-linear term, mean translational and rotational velocity on stability of trivial and non-trivial solutions. In this paper, it was shown that increasing the mean translational velocity causes to a reduction in first natural frequencies of system. Also, the principal parametric resonance would arise when speed fluctuation frequency is near to the natural frequency. Also, increasing mean rotational velocity made a frequency increased. Stability and bifurcation of non-trivial and trivial steady state responses were analyzed by using Routh-Hurwitz criterion. The effects of non-linear term and mean velocity on bifurcation points and stability of trivial and non-trivial solutions also were investigated, and the frequency-response curves were depicted.

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