

Abstract — Optimal load factors (dead, live and seismic) used for the design of buildings may be different, depending on the seismic ground motion characteristics to which they are subjected, which are closely related to the type of soil conditions where the structures are located. The influence of the type of soil on those load factors, is analyzed in the present study. A methodology that is useful for establishing optimal load factors that minimize the cost over the life cycle of the structure is employed; and as a restriction, it is established that the probability of structural failure must be less than or equal to a prescribed value. The life-cycle cost model used here includes different types of costs. The optimization methodology is applied to two groups of reinforced concrete buildings. One set (consisting on 4-, 7-, and 10-story buildings) is located on firm ground (with a dominant period $T_s \approx 0.5$ s) and the other (consisting on 6-, 12-, and 16-story buildings) on soft soil ($T_s \approx 1.5$ s) of Mexico City. Each group of buildings is designed using different combinations of load factors. The statistics of the maximum inter-story drifts (associated with the structural capacity) are found by means of incremental dynamic analyses. The buildings located on firm zone are analyzed under the action of 10 strong seismic records, and those on soft zone, under 13 strong ground motions. All the motions correspond to seismic subduction events with magnitudes $M \geq 6.9$. Then, the structural damage and the expected total costs, corresponding to each group of buildings, are estimated. It is concluded that the optimal load factors combination is different for the design of buildings located on firm ground than that for buildings located on soft soil.

Keywords — Life-cycle cost, optimal load factors, reinforced concrete buildings, total costs, type of soil.

I. INTRODUCTION

A design process focuses on getting the best cost/reliability ratio. Some reliability-based design methods are described in [1]. The expected costs during the useful life of the structures play an important role when establishing parameters for the design. The determination of costs in an issue that has been extensively studied in recent decades by various authors [2]-[9]; however, these methodologies, in general, are limited to particular cases.

Recently, general methodologies have been proposed to establish optimal load factors combination that guarantees the minimum total cost expected in the useful life of the structures [10].

II. METHODOLOGY

The methodology used in the present study is as follows:

1. Different reinforced concrete buildings are designed using different load combinations in accordance with a given code. In this study, we used the Mexico City Building Code (MCBC-04).

2. The maximum inter-story drift (MID) of each structure (associated with the structural capacity), is obtained by means of Incremental Dynamic Analysis (IDA).

3. For each load combination corresponding to each building, the annual rate of exceedance of a certain MID is evaluated as [11], [12]:

$$\nu_D(d) = \int_{d(D > d | S_a)} p(D > d | S_a) d(S_a) \ (1)$$

where, $\nu_D(d)$: maximum drift demand hazard curve, $d$: value of the MID, $D$: structural demand, represented by the MID; $S_a$: pseudo-acceleration associated with the fundamental period of the building; $p(D > d | S_a)$: fragility curve, which represents the conditional probability that $D$ exceeds the value $d$, given an intensity $S_a$; and $\nu(S_a)$: seismic hazard curve for the site of interest, which represents the average annual number that a seismic ground motion occurs with an intensity equal or greater than $S_a$.

4. Then, the average annual rate of structural failure for each combination is calculated using (2) [13], [14]:

$$\nu_f = \int \left[ \frac{d\nu_p(d)}{d(d)} \right] p(C \leq d) d(d) \ (2)$$

where $\nu_f$: average annual rate of structural failure. It represents the average number of times per year that the demand $d$ exceeds the capacity $C$, and $p(C \leq d)$: probability that the structural capacity $C$ (near collapse limit-state) is smaller than $d$.

The buildings whose designs have an average annual rate of structural failure greater than that implicit in the MCBC-04 are discarded; so, the following condition is established:

$$\nu_f (\gamma) \leq (\nu_{0,MCBC-04}) \ (3)$$

5. Based on the simulated seismic demands and on the capacity of the structure, the damage index of the structures is calculated. With this, it is possible to...
calculate the total costs, which must be transported to present value with the following expression:

\[ VP = \frac{VF}{(1+i)^n} \]  
(4)

where \( VP \): value at time 0 (i.e., at present); \( VF \): value in time \( n \) (i.e., at future); \( i \): discount rate; and \( n \): number of years to be considered.

6. For each load combination, the estimated total cost \( C_T \) associated with the life-cycle of the structure is estimated (buildings are assumed to have a life-cycle of 50 years) as:

\[ C_T(\gamma) = C_I(\gamma) + C_d(\gamma) \]  
(5)

where, \( C_T \): total cost associated with the life-cycle of the structure, \( C_I \): initial cost, \( C_d \): cost associated with structural damage, \( \gamma \): specific load combination

7. The optimal combination of the factors corresponding to dead, live, and earthquake loads is obtained, based on the minimum total cost. That is, the total cost is minimized as:

\[ \min[C_T(\gamma)] \]  
(6)

III. STRUCTURAL CAPACITY AND SEISMIC STRUCTURAL DEMANDS

A. Structural Capacity

Structural capacity curves are obtained from incremental dynamic analyses, IDA [15]. In this study, the parameter to measure the damage is the MID that is plotted versus the seismic intensity at which the seismic record is scaled, and this graphic representation is known as curve IDA.

To determine the structural behavior in the seismic environment, it is necessary to obtain a collection of capacity curves using different seismic records of the same family.

From these curves, the statistics of the yielding limit-state and of the collapse limit-state are defined.

B. Seismic Structural Demands

To estimate the total cost during the life-cycle of the structures it is necessary to obtain the seismic structural demands. In this study, the structural demands are given in terms of the MID. Different values of this parameter (chosen to measure structural damage) are obtained from the structural demand hazard curves, by means of Monte Carlo simulations. For the numerical simulation of the seismic structural demands, the inverse transformation method is used [16].

It is assumed that the occurrence times of the seismic events follow a Poisson process. Thus, the waiting times between events are modeled with an exponential probability distribution.

IV. COSTS ANALYSIS

The total cost includes the initial cost and the cost associated with structural damage that occurs during the life-cycle of the structure.

A. Initial Cost

The initial cost \( C_I \) includes direct cost, indirect cost and the utility paid to the builder. Equation (7) is useful for estimating the initial cost through the costs of materials (concrete and steel) [17].

\[ C_I = 1.93C_M \]  
(7)

where \( C_M \) is the cost of materials.

B. Cost Associated with Structural Damage

The cost of damages \( C_d \) can be considered as the sum of the following costs: repair or reconstruction, \( C_{PR} \); loss of content, \( C_{PC} \); indirect losses, \( C_{PL} \); loss of life, \( C_{PL} \); and injuries, \( C_{PR} \). Then, the cost associated with damages is expressed as [2]:

\[ C_d = C_{PR} + C_{PC} + C_{PL} + C_{PR} + C_{PL} \]  
(8)

Structural damage costs are estimated from a measure of physical damage, represented by the damage index, \( DI \), which takes values between 0 and 1. Thus, for the case of total damage \( DI \) is equal to 1, while it is 0 when there is no damage. \( DI \) is given by [18]:

\[ DI = \frac{\delta_d - \delta_y}{\delta_u - \delta_y} \]  
(9)

where \( \delta_d \): MID demand in the structure; \( \delta_y \): MID associated with the service limit-state (structure without damage); and \( \delta_u \): MID associated with the collapse limit-state.

1. Repair or Reconstruction Cost

In many cases, the damages are very severe, and by security it is necessary to demolish the structure. Reference [19] establishes that, from an ID>0.7 in reinforced concrete structures, the repair can no longer be carried out, so it is necessary to demolish. The cost for repair or reconstruction is given by [10]:

\[ C_{PR} = \begin{cases} C_I(DI^2); 0 < DI < 0.7 \\ 1.2(C_I); DI \geq 0.7 \end{cases} \]  
(10)

2. Loss of Contents

The maximum cost for loss of contents (ID≥1) is a fraction of the initial cost of the building, adopting a fraction of 50% [20]. For the case of ID <1, a cost variation is considered as a function of the ID in a linear way. Thus, the cost of contents loss is defined as [2]:

\[ C_{PC} = \begin{cases} 0.5(C_I)(DI); 0 < ID < 1 \\ 0.5(C_I); DI \geq 1 \end{cases} \]  
(11)
3. Indirect Loss
In this study, the structures analyzed are office buildings, so
that indirect losses are associated with not generating money
due to the rent during the time the structure is repaired or
rebuilt. The cost for indirect losses is estimated using [2]:

\[ C_{PI} = \begin{cases} R(P_R)(A)(D^2); 0 < D < 1 \\ R(P_R)(A); D \geq 1 \end{cases} \] (12)

where \( P_R \) is the maximum period (in months) of
reconstruction, \( A \) is the building area (in square meters) and \( R \)
is the cost per square meter of rent per month.

4. Cost of Fatality
To estimate the cost of loss of life, it is necessary to
evaluate the average number of people killed within a
construction area, during intense seismic events. To do this, a
nonlinear regression to estimate the number of deaths \( (N_d) \) as a
function of the collapsed area was employed. The regression is
based on the total area of collapsed buildings in Mexico City
during the 1985 earthquake [21] and the number of deaths
[22]. Here, \( N_d \) is defined as:

\[ N_d = 45.48 + 5.531744(A^2) \] (13)

where \( A \) is the area of collapsed building in 1000m².

The maximum cost for life loss (ID≥1) is equal to the
number of deaths multiplied by the expected value of their
income during their working life \((IVL)\). Therefore, the cost for
life loss is calculated by using (14) [2]. For ID <1, a cost
variation is assumed as a function of the \( ID \) raised to the
fourth power.

\[ C_{PL} = \begin{cases} N_d(IVL)(D^4); 0 < D < 1 \\ N_d(IVL); D \geq 1 \end{cases} \] (14)

5. Cost of Injuries
The cost of injuries refers to the costs involved during the
hospital stays of people injured in an earthquake.

According with [2], the average number of people injured
per unit of collapsed area of buildings is equal to 0.0168/m².
This amount is the result of dividing the number of injuries
reported in the 1985 earthquake that affected Mexico City [22]
by the total area of collapsed buildings [21].

The maximum cost for injuries (ID≥1) is calculated by
using (15) [2], where the number of people with disability is
equal to 10% of the total number of injured, while the
remaining 90% have injuries without disability. For ID <1 a
cost variation is assumed as a function of the ID raised to the
second power.

\[ C_{PI} = \begin{cases} 0.1CLI + 0.9CLS \times 0.0168(A)(D^2); 0 < D < 1 \\ 0.1CLI + 0.9CLS \times 0.0168(A); D \geq 1 \end{cases} \] (15)

where CLI is the cost for injuries with disability, and CLS is
the cost for injuries without disability.

V. CHARACTERISTICS OF THE BUILDINGS ANALYZED
In the present study three reinforced concrete buildings with
different number of stories and different separation of bays for
each type of soil (firm ground and soft soil) are analyzed. The
structuring is based on orthogonal rigid frames, with three
bays in both directions at constant separations and with a stor y
height constant of 4 m for all stories. Therefore, the buildings
have a symmetrical distribution both in floor and in elevation.
Table I shows the geometric characteristics of the buildings
analyzed.

| TABLE I
<p>| GEOMETRIC CHARACTERISTICS OF REINFORCED CONCRETE BUILDINGS |</p>
<table>
<thead>
<tr>
<th>Soft soil</th>
<th>Firm ground</th>
</tr>
</thead>
<tbody>
<tr>
<td>Num. of stories</td>
<td>Separation of bays (m)</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>16</td>
<td>8</td>
</tr>
</tbody>
</table>

A. Analysis and Structural Design Specifications
The buildings are designed in accordance with Mexico City Building Code (MCBC-2004). Table II shows the different
combinations used for the buildings design.

VI. RESULTS
The methodology mentioned in Section II was applied to
the two groups of reinforced concrete buildings designed with
the load factors combinations listed in Table II. Results
concerning to the structural systems located on the two soil
types are shown below.

A. Soft Soil
Table III shows a summary of costs normalized with respect
to the initial cost of combination 1, \( C_{IC}(1) \), expected on average
during the life-cycle of the three buildings (6, 12, and 16
stories) located on soft soil, corresponding to each load
combination.

From Table III it can be seen that combination 4
corresponding to \( FC_{M}=1.1, FC_{V}=1.1 \) and \( FC_{S}=1.4 \), is associated
with the minimum total cost.

| TABLE II
| LOAD FACTORS COMBINATIONS |
|----------|----|----|----|
| Combination | \( FC_{M} \) | \( FC_{V} \) | \( FC_{S} \) |
| 1 | 1.1 | 1.1 | 1.1 |
| 2 | 1.1 | 1.1 | 1.2 |
| 3 | 1.1 | 1.1 | 1.3 |
| 4 | 1.1 | 1.1 | 1.4 |
| 5 | 1.1 | 1.1 | 1.5 |
| 6 | 1.1 | 1.1 | 1.6 |
| 7 | 1.1 | 1.1 | 1.7 |
| 8 | 1.1 | 1.1 | 1.9 |
| 9 | 1.1 | 1.1 | 3 |

\( FC_{M} \): dead load factor; \( FC_{V} \): live load factor; \( FC_{S} \): seismic load factor

Fig. 1 shows the behavior of each of the standardized costs
with respect to the initial cost of combination 1, \( C_I(1) \), which can be expected during the life-cycle of the three buildings (6, 12, and 16 stories) corresponding to combination 1 to 9.

### TABLE III
SUMMARY OF STANDARDIZED COSTS (6, 12 AND 16 STORIES)

<table>
<thead>
<tr>
<th>Combination</th>
<th>( C_I )</th>
<th>( C_d/ C_I(1) )</th>
<th>( C_T/ C_I(1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>1.153</td>
<td>2.153</td>
</tr>
<tr>
<td>2</td>
<td>1.038</td>
<td>0.958</td>
<td>1.997</td>
</tr>
<tr>
<td>3</td>
<td>1.090</td>
<td>0.880</td>
<td>1.970</td>
</tr>
<tr>
<td>4</td>
<td>1.130</td>
<td>0.826</td>
<td>1.957</td>
</tr>
<tr>
<td>5</td>
<td>1.177</td>
<td>0.793</td>
<td>1.970</td>
</tr>
<tr>
<td>6</td>
<td>1.294</td>
<td>0.766</td>
<td>2.060</td>
</tr>
<tr>
<td>7</td>
<td>1.341</td>
<td>0.726</td>
<td>2.067</td>
</tr>
<tr>
<td>8</td>
<td>1.507</td>
<td>0.616</td>
<td>2.123</td>
</tr>
<tr>
<td>9</td>
<td>2.783</td>
<td>0.176</td>
<td>2.959</td>
</tr>
</tbody>
</table>

\( C_I \): initial cost; \( C_d \): structural damage cost; \( C_T \): total cost

#### B. Firm Ground

Table IV shows a summary of costs normalized with respect to the initial cost of combination 1, \( C_I(1) \), expected on average during the life-cycle of the three buildings (4, 7, and 10 stories) located on firm ground, corresponding to each load combination.

From Table IV, it can be seen that combination 1 associated with an \( F_{CM}=1.1 \), \( F_{CV}=1.1 \) and \( F_{CS}=1.1 \), generates the minimum total cost.

### TABLE IV
SUMMARY OF STANDARDIZED COSTS (4, 7 AND 10 STORIES)

<table>
<thead>
<tr>
<th>Combination</th>
<th>( C_I )</th>
<th>( C_d/ C_I(1) )</th>
<th>( C_T/ C_I(1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.144</td>
<td>1.144</td>
</tr>
<tr>
<td>2</td>
<td>1.008</td>
<td>0.143</td>
<td>1.150</td>
</tr>
<tr>
<td>3</td>
<td>1.015</td>
<td>0.141</td>
<td>1.156</td>
</tr>
<tr>
<td>4</td>
<td>1.024</td>
<td>0.130</td>
<td>1.154</td>
</tr>
<tr>
<td>5</td>
<td>1.031</td>
<td>0.128</td>
<td>1.159</td>
</tr>
<tr>
<td>6</td>
<td>1.043</td>
<td>0.124</td>
<td>1.167</td>
</tr>
<tr>
<td>7</td>
<td>1.053</td>
<td>0.113</td>
<td>1.166</td>
</tr>
<tr>
<td>8</td>
<td>1.080</td>
<td>0.112</td>
<td>1.192</td>
</tr>
<tr>
<td>9</td>
<td>1.298</td>
<td>0.081</td>
<td>1.379</td>
</tr>
</tbody>
</table>

\( C_I \): initial cost; \( C_d \): structural damage cost; \( C_T \): total cost

Fig. 2 shows the behavior of each of the standardized costs with respect to the initial cost of combination 1, \( C_I(1) \), which can be expected during the life-cycle of the three buildings (4, 7, and 10 stories), corresponding to combinations 1 to 9.

![Fig. 2 Standardized cost with respect to the initial cost of combination 1, \( C_I(1) \) of the three buildings (4, 7, and 10 stories) on firm ground](image)

VII. CONCLUSION

Based on the results obtained, it is concluded that, for structures located on firm ground, the optimal combination of design load factors, that gives place to the minimum total structural cost is:

\[ F_{CM}=1.1, \quad F_{CV}=1.1 \quad \text{and} \quad F_{CS}=1.1 \]

On the other hand, the costs associated with earthquake damage during the life-cycle of structures located on soft soils are important and have a significant influence on the load factors selection. For structures located on soft soil of Mexico City, the optimal combination of design load factors that gives place to the minimum total structural cost is:

\[ F_{CM}=1.1, \quad F_{CV}=1.1 \quad \text{and} \quad F_{CS}=1.4 \]

Therefore, it was shown that the optimal combination of load factors is different for the design of buildings located on firm ground and for buildings located on soft soil.

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REFERENCES


