

# Forecasting Materials Demand from Multi-Source Ordering

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**Abstract**—The downstream manufactures will order their materials from different upstream suppliers to maintain a certain level of the demand. This paper proposes a bivariate model to portray this phenomenon of material demand. We use empirical data to estimate the parameters of model and evaluate the RMSD of model calibration. The results show that the model has better fitness.

**Keywords**—Farlie-Gumbel-Morgenstern family of bivariate distributions, multi-source ordering, materials demand quantity, recency, ordering time.

## I. INTRODUCTION

TO predict the materials demand in the manufacturing process is an important issue and is also discussed in many previous researches [1]-[4]. In order to avoiding the materials shortage or price monopoly [3], the downstream manufactures will purchase materials not only from one upstream supplier [5], [7]. The multi-source ordering can help downstream manufactures to make sure the stable inventory [2], [3], [6], [8]. Thus, this paper focuses on the multi-source topic to provide a new model which is different from previous researches and compare this new model with previous one.

Huang [9] proposes the materials demand model in which the total demand quantities consist of the quantity that the downstream orders in the past (is called “ordering quantity of past”) and the time interval between the last purchase and the end of observation time (is called “recency of ordering time”) [10], [11]. In the research conducted by [9], the “ordering quantity of past” is considered as log normal distribution and “recency of ordering time” follows a renew process with exponential distribution. In this model [9], the “ordering quantity of past” is only from one source of upstream supplier. But in generally speaking, downstream manufactures want to keep the adequate supply of materials through ordering materials from different suppliers [7], [8].

To portray this phenomenon, Huang [12] extend her model by computing the characteristic function to demonstrate multi-source ordering from various upstream suppliers. In this extended model [12], total materials demand quantities are still composed of “ordering quantity of past” and “recency of ordering time”. But the “ordering quantity of past” follows a characteristic function.

The goal of this research is to propose different distributions of the “ordering quantity of past” (different from characteristic function) to compare the validation with Huang’s [12] model.

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We also assume if the “recency of ordering time” is following exponential distribution, then the materials demand model will totally different from the previous one.

This paper is organized as follows: first we will introduce the concept of materials demand model. The probability distribution of “ordering quantity of past” and “recency of ordering time” will be demonstrated. Two types of assumption of probability distribution with “recency of ordering time” will also be compute. Secondly, cumulative distribution function (cdf) and the probability density function (pdf) of full materials demand quantity model will be derived. Thirdly, the empirical data will be used to estimate the parameters. We use the results of estimation to simulate data. Then the comparison of empirical data and simulation data will be calculated and the model validation is shown. Finally, the conclusions are made.

## II. THE MODEL

### A. The Materials Demand Model

Based on Huang’s [9], [12] model, we also consider the materials demand model which is to predict the total demand quantity (denoted as DQ) is composed by the ordering quantity of past (denoted as OP) and the recency of ordering time (denoted as T).

$$DQ = OP \cdot RT \quad (1)$$

### B. The Ordering Quantity

According Farlie-Gumbel-Morgenstern family of bivariate distributions [13]-[15], there are two independent univariate distributions and combine these two into a correlated bivariate distribution. We consider the OP is from two different sources which are denoted as upstream supplier  $S_Z$  and  $S_X$ . Then the order quantities X and Y are the random variables”. The pdf of x is

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right) \quad (2)$$

and its cdf is:

$$F_x(x) = \frac{1}{2} \left[ 1 + \operatorname{erf}\left(\frac{(x-\mu_x)}{\sqrt{2\sigma_x^2}}\right) \right] \quad (3)$$

The pdf of y is:

$$f_y(y) = \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left(-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right) \quad (4)$$

and its cdf is:

$$F_y(y) = \text{erf}\left(\frac{(y-\mu_y)}{\sqrt{2\sigma_y^2}}\right) \quad (5)$$

Based on Farlie-Gumbel-Morgenstern family of bivariate distributions [13]-[15], the joint density of x and y is:

$$\begin{aligned} f_{xy}(x, y|\beta) &= f_x(x) f_y(y) \{1 + \beta [1 - 2F_x(x)] [1 - 2F_y(y)]\} \\ &= \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right) \cdot \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left(-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right) \cdot \\ &\quad \left\{1 + \beta \cdot \text{erf}\left(\frac{(x-\mu_x)}{\sqrt{2\sigma_x^2}}\right) \cdot \text{erf}\left(\frac{(y-\mu_y)}{\sqrt{2\sigma_y^2}}\right)\right\} \\ &= \frac{1}{2\pi\sigma_x\sigma_y} \exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2} - \frac{(y-\mu_y)^2}{2\sigma_y^2}\right) \cdot \\ &\quad \left\{1 + \beta \cdot \text{erf}\left(\frac{(x-\mu_x)}{\sqrt{2\sigma_x^2}}\right) \cdot \text{erf}\left(\frac{(y-\mu_y)}{\sqrt{2\sigma_y^2}}\right)\right\} \end{aligned} \quad (6)$$

### C. The Recency of Ordering Time

We consider two types of recency of ordering model. One is considering the recency of ordering time (denoted as  $RT_i$ ) as renew process [10] and the purchase interval time is a random variable which follows exponential distribution. The pdf and cdf in the recency of ordering time (denoted as  $rt_i$ ) are computing as:

$$f_{RT_i}(rt_i|\theta) = \frac{1}{\theta^2} \exp\left(-\frac{rt_i}{\theta}\right) \quad (7)$$

and

$$F_{RT_i}(RT_i|\theta) = \frac{1 - \exp\left(-\frac{RT_i}{\theta}\right)}{\theta} \quad (8)$$

Another one is considering the recency of ordering time (denoted as  $RT_2$ ) as an exponential distribution. Its pdf and cdf are shown as:

$$f_{T_2}(t_2|\gamma_2) = \frac{1}{\gamma_2} \exp\left(-\frac{t_2}{\gamma_2}\right) \quad (9)$$

and

$$F_{T_2}(t_2|\gamma_2) = 1 - \exp\left(-\frac{t_2}{\gamma_2}\right) \quad (10)$$

### D. The Full Model

According to (1), the materials demand model is composed by OP multiplying RT and we consider two types of the RT to demonstrate model 1 and model 2. Thus, we can compute two kinds of full model.

### E. Model 1

In order to calculate the cdf of full model, we denote random variable K as demand quantity. Then, its cdf is:

$$\begin{aligned} F_{M_1}(k) &= P(T_1 \cdot A < k) \\ &= \int_0^{\infty} P(T_1 \cdot A | T_1 = t_1, A = x, y) f_{T_1}(t_1|\gamma_1) dt_1 \\ &= \int_0^{\infty} P\left(A < \frac{k}{T_1}\right) f_{T_1}(t_1|\gamma_1) dt_1 \\ &= \int_0^{\frac{k}{\gamma_1}} \int_0^{\frac{k}{\gamma_1}} \left\{ \frac{1}{2\pi\sigma_x\sigma_y} \exp(C) \{G\} \right\} dx dy f_{T_1}(t_1|\gamma_1) dt_1 \\ &= \frac{1}{2\pi\sigma_x\sigma_y\gamma_1^2} \int_0^{\frac{k}{\gamma_1}} \int_0^{\frac{k}{\gamma_1}} \exp\left(C - \frac{t_1}{\gamma_1}\right) \{G\} dx dy dt_1 \end{aligned}$$

where  $C = -\frac{(x-\mu_x)^2}{2\sigma_x^2} - \frac{(y-\mu_y)^2}{2\sigma_y^2}$ ,  $G = 1 + \beta \cdot \text{erf}\left(\frac{(x-\mu_x)}{\sqrt{2\sigma_x^2}}\right) \cdot \text{erf}\left(\frac{(y-\mu_y)}{\sqrt{2\sigma_y^2}}\right)$

Based on (11), we can compute the pdf of the proposed model. It shows in (12).

$$\begin{aligned} f_{M_1}(k) &= \frac{d}{dk} F_{M_1}(k) \\ &= \frac{1}{2\pi\sigma_x\sigma_y\gamma_1^2} \frac{d}{dk} \int_0^{\frac{k}{\gamma_1}} \int_0^{\frac{k}{\gamma_1}} \exp\left(C - \frac{t_1}{\gamma_1}\right) \{G\} \end{aligned} \quad (11)$$

### F. Model 2

According to (1), the materials demand model is composed by OP multiplying RT and we consider two types of the RT to demonstrate model 1 and model 2. Thus, we can compute two kinds of full model.

$$\begin{aligned} F_{M_2}(s) &= P(T_2 \cdot A < s) \\ &= \int_0^{\infty} P(T_2 \cdot A | T_2 = t_2, A = x, y) f_{T_2}(t_2|\gamma_2) dt_2 \\ &= \int_0^{\infty} P\left(A < \frac{s}{T_2}\right) f_{T_2}(t_2|\gamma_2) dt_2 \\ &= \int_0^{\frac{s}{\gamma_2}} \int_0^{\frac{s}{\gamma_2}} \left\{ \frac{1}{2\pi\sigma_x\sigma_y} \exp(C) \{G\} \right\} dx dy f_{T_2}(t_2|\gamma_2) dt_2 \\ &= \frac{1}{2\pi\sigma_x\sigma_y\gamma_2} \int_0^{\frac{s}{\gamma_2}} \int_0^{\frac{s}{\gamma_2}} \exp\left(C - \frac{t_2}{\gamma_2}\right) \{G\} dx dy dt_2 \end{aligned} \quad (12)$$

According to (13), we can compute pdf as:

$$\begin{aligned}
 & f_{M_2}(s) \\
 &= \frac{d}{ds} F_{M_2}(s) \\
 &= \frac{1}{2\pi\sigma_x\sigma_y\gamma_2} \frac{d}{ds} \int_0^{\frac{s}{\gamma_2}} \int_0^{\frac{s-t_2}{\gamma_2}} \exp\left(C - \frac{t_2}{\gamma_2}\right) \{G\} dx dy dt_2
 \end{aligned} \tag{13}$$

### III. MATH

We use 5328 samples which are from a downstream manufacture of solar energy generation. These data include the information about the total demand quantity in each month, ordering quantity from two different upstream suppliers (company H and Q) and the recency of ordering time of this downstream manufacture.

We compare the proposed model (model 1 and model 2) with Huang's model. Thus, we use these empirical data respectively to estimate the parameters of the proposed model 1, model 2 and Huang's model [12].

The analysis process follows these steps:

- Step1: The parameters are estimated by these empirical data.
- Step2: We use the results of parameter estimation of each model (model 1, model 2 and Huang's model [12]) to poll the simulation data.
- Step3: In this step, we use root-mean-square deviation (RMSD) to calculate the difference between empirical data and simulation data.

#### A. The Parameters Estimation

We use MLE (maximum likelihood estimate) to estimate the parameters.

- Model 1: Let  $k_j$  denote the materials demand quantities by upstream  $j$ , that is:

$$\begin{aligned}
 & L_{M_1}(\mu_x, \sigma_x^2, \mu_y, \sigma_y^2, \gamma_1, \beta) \\
 &= \prod_{j=1}^n f_{M_1}(k_j) \\
 &= \frac{1}{2\pi\sigma_x\sigma_y\gamma_1^2} \frac{d}{dk} \int_0^{\frac{k}{\gamma_1}} \int_0^{\frac{k-t_1}{\gamma_1}} \exp\left(C - \frac{t_1}{\gamma_1}\right) \{G\} dx dy dt_1
 \end{aligned} \tag{14}$$

- Model 2. Let  $s_k$  denote the materials demand quantities by upstream  $p$ , that is:

$$\begin{aligned}
 & L_{M_2}(\mu_x, \sigma_x^2, \mu_y, \sigma_y^2, \gamma_2, \beta) \\
 &= \prod_{p=1}^m f_{M_2}(s_p) \\
 &= \frac{1}{2\pi\sigma_x\sigma_y\gamma_2} \frac{d}{ds} \int_0^{\frac{s}{\gamma_2}} \int_0^{\frac{s-t_2}{\gamma_2}} \exp\left(C - \frac{t_2}{\gamma_2}\right) \cdot \{G\} dx dy dt_2
 \end{aligned} \tag{15}$$

## IV. RESULTS AND DISCUSSION

The estimation results are shown in Table I.

TABLE I  
 THE RESULTS OF PARAMETERS ESTIMATION

Parameters	Huang's Model [12]	Parameters	New Model A	New Model B
$\mu_u$	589	$\mu_x$	577	560
$\sigma_u^2$	7.89	$\sigma_x^2$	3.22	2.23
$\mu_w$	648	$\mu_z$	624	645
$\sigma_w^2$	10.2	$\sigma_z^2$	8.25	7.35
$\Theta$	3.35	$\theta$	3.55	---
$P$	4.27	$\lambda$	---	3.72
		$\beta$	-1.31	-0.55

TABLE II  
 THE RESULTS OF RMSD

	RMSD*
Huang's model [12]	0.9892
New model 1	0.8753
New model 2	0.7024

We can find that the parameter "  $\beta$  " which in the Farlie-Gumbel-Morgenstern family of bivariate distributions can reflect the relationship between ordering quantities of upstream supplier H and Q. They are negative relations. If downstream manufacture orders more material quantities from supplier H, then the less ordering will happen in supplier Q.

We compute the root-mean-square deviation (RMSD) to make comparison between empirical data and simulation data in Huang's model [12], the proposed model 1 and model 2. The results show that new model 2 has best fitness than new model 1 and Huang's model [12]. It means the new model 2 have predictive power to forecast the total materials demand. The results of prediction are most close to the real data.

## V. CONCLUSION

This paper considers multi-source from different upstream suppliers and demonstrates more complicate phoneme when the recency of ordering time are both in renew process or exponential distribution. It shows the combination of exponential distribution (as recency of ordering time) and Farlie-Gumbel-Morgenstern family of bivariate distributions (as ordering quantity of past) are best than that of renew process and Farlie-Gumbel-Morgenstern family of bivariate distributions or renew process and characteristic function. The proposed model provides the information to detect the relationship between upstream suppliers. This can reflect not only different source ordering but also the competitive situation on the supplier side. Thus, these two proposed models are more fitness with real data. In the future, the researchers can use different data to test the relations between upstream supplier and extend these models in different industries.

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#### REFERENCES

- [1] J. L. Zeballosa and C. A. Méndeza, A. P. Barbosa-Povoab, A. Q. Novais, 'Multi-period design and planning of closed-loop supply chains with uncertain supply and demand', *Computers & Chemical Engineering*, vol 66, no.4, pp. 151–164, 2014.
- [2] M. Zorgdrager and R. Curran, W. J. C. Verhagen, B. H. L. Boesten, and C. N. Water, "A predictive method for the estimation of material demand for aircraft non-routine maintenance", *20th ISPE International Conference on Concurrent Engineering: Proceedings*, pp. 509-515, 2013.
- [3] H. Stadler, "Supply chain management and advanced planning-basics, overview and challenges", *European Journal of Operational Research*, vol. 163, no.3, pp. 575-588, 2005.
- [4] H. Lu, H. Wang, Y. Xie and H. Li, "Construction material safety-stock determination under nonstationary stochastic demand and random supply yield", *IEEE Transactions on Engineering Management*, vol. 63, no.2, pp.201-212,
- [5] A. Gupta, C. D. Maranas and C. M. McDonald, "Mid-term supply chain planning under demand uncertainty: customer demand satisfaction and inventory management", *Computers & Chemical Engineering* vol. 24, no.12, pp. 2613-2621, 2000.
- [6] T. C. Poona, K. L. Choya, F. T. S. Chana and H.C.W., Lau, "A real-time production operations decision support system for solving stochastic production material demand problems", *Expert Systems with Applications*, vol. 38, no.5, pp. 4829-4838, 2011.
- [7] E. A. Martínez Ceseña and P. Mancarella, "Practical recursive algorithms and flexible open-source applications for planning of smart distribution networks with demand response", *Sustainable Energy, Grids and Networks*, In Press, 2016.
- [8] S. R. Cardoso, A. Paula, F.D. Barbosa-Póvoa and S. Relvas, "Design and planning of supply chains with integration of reverse logistics activities under demand uncertainty", *European Journal of Operational Research*, vol. 226, no.3, pp. 436-451, 2013.
- [9] H. H. Huang, "A materials demand model with ordering quantity of past and recency of ordering time", *Key Engineering Materials*, In Press, 2016.
- [10] H. H. Huang, "A detection model of customer alive in information management application", *Advanced Materials Research*, vol.684, pp.505-508, 2013.
- [11] H. H. Huang, "Data mining application of marketing: a bivariate model of customer purchase monetary and interpurchase time", *Information and Knowledge Management*, vol.45, pp.154-157, 2012.
- [12] H. H. Huang, "Combining Recency of Ordering Time and Different Sources from Upstream to Predict the Materials Demand of Downstream Manufactures", *Advances in Engineering Research*, In Press, 2016.
- [13] N. L., Johnson and Kotz, S., "on some generalized Farlie-Gumbel-Morgenster distributions", *Communications in Statistics*, vol.4, no.4, pp.415-27, 1975.
- [14] N. L., Johnson and Kotz, S., "on some generalized Farlie-Gumbel-Morgenster Distributions-II: Regression, correlation and further generalizations", *Communications in Statistics*, vol.6, vol.6, pp. 485-96, 1977.
- [15] D. J. G., Farlie, "The Performance of Some Correlation Coefficients for a General Bivariate Distribution", *Biometrika*, vol.47, pp.307-23, 1960.