Numerical Simulations on Feasibility of Stochastic Model Predictive Control for Linear Discrete-Time Systems with Random Dither Quantization

Taiki Baba, Tomoaki Hashimoto

***Abstract*—**The random dither quantization method enables us to achieve much better performance than the simple uniform quantization method for the design of quantized control systems. Motivated by this fact, the stochastic model predictive control method in which a performance index is minimized subject to probabilistic constraints imposed on the state variables of systems has been proposed for linear feedback control systems with random dither quantization. In other words, a method for solving optimal control problems subject to probabilistic state constraints for linear discrete-time control systems with random dither quantization has been already established. To our best knowledge, however, the feasibility of such a kind of optimal control problems has not yet been studied. Our objective in this paper is to investigate the feasibility of stochastic model predictive control problems for linear discrete-time control systems with random dither quantization. To this end, we provide the results of numerical simulations that verify the feasibility of stochastic model predictive control problems for linear discrete-time control systems with random dither quantization.

***Keywords*—**Model predictive control, stochastic systems, probabilistic constraints, random dither quantization.

I. Introduction

THE quantization of control signals occurs in many systems equipped with discrete-level actuators/sensors. The control signals are also quantized in communication networks. Thus, the quantized control of systems is one of the most important research topics in recent years.

Recently, the random dither quantization method that transforms a given continuous-valued signal to a discrete-valued signal by adding artificial random noise to the continuous-valued signal before quantization has been proposed in [1]. It has been shown that the random dither quantization method exhibits much better performance than the simple uniform quantization method for linear discrete-time systems with quantized control inputs. Hence, this paper focuses on feedback control systems with random dither quantizers. Although the effectiveness of random dither quantization method has been verified, the constraints imposed on the state variables of systems have not been taken into consideration in [1] for the design of feedback control systems with random dither quantization.

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T. Baba and T. Hashimoto are with the Department of Mechanical Engineering, Osaka Institute of Technology, Asahi-ku 535-8585, Japan (e-mail: info@thashi.net).

Model predictive control (MPC), also known as receding horizon control [2]-[4], is a well-established control method in which a performance index is minimized subject to constraints imposed on the state variables of systems. An important advantage of MPC is its ability to deal with constraints on the state and control variables of systems [5]-[7]. Although several MPC methods [8]-[11] do not take account of uncertain disturbances, another MPC methods [12]-[15] enable us to fulfill constraints imposed on control systems against uncertain disturbances. It is well known that the methods of MPC against uncertain disturbances can be classified into deterministic and stochastic approaches. The uncertain disturbances that occur in control systems with random dither quantization can be considered as the random quantization errors. Thus, we address the stochastic MPC (SMPC) approach where the expected values of the performance indices and probabilistic constraints are considered by exploiting the statistical information of uncertain disturbances.

Recently, several methods for solving optimal control problems subject to probabilistic constraints imposed on the state variables of systems for linear discrete-time control systems have been proposed in [16]-[18]. In particular, a method for solving optimal control problems subject to probabilistic state constraints for linear discrete-time control systems with random dither quantization has been proposed in [19]. However, the feasibility of optimal control problems subject to probabilistic constraints for linear control systems with random dither quantization has not yet been studied. Therefore, the objective of this paper is to examine the feasibility of stochastic model predictive control problems for linear discrete-time control systems with random dither quantization. For this purpose, we conduct on numerical simulations to verify the feasibility of stochastic model predictive control problems for linear discrete-time control systems with random dither quantization. In other words, the obtained results of numerical simulations enable us to verify the effectiveness of the SMPC method for linear discrete-time control systems with random dither quantization subject to state constraints.

This paper is organized as follows: In Section II, we introduce some notations. In Section III, the system model and random dither quantizer are formulated. In Section IV, some preliminary results are provided. The main results are provided in Section V. Finally, some concluding remarks are given in Section VI.

II. Notation

Throughout this paper, we adopt some notations introduced in this section. Let $\mathbb{R}$ and $\mathbb{N}$ denote the sets of real and natural numbers, respectively. Let $\mathbb{R}_+$ denote the set of non-negative real numbers.

For matrix A, let A' denote the transpose of A. For matrices $F = \{f_{i,j}\}$ and $G = \{g_{i,j}\}$, let the inequalities between F and G, such as $F > G$ and $F \geq G$, indicate that they are component-wise satisfied, i.e., $f_{i,j} > g_{i,j}$ and $f_{i,j} \geq g_{i,j}$ hold true for all i and j, respectively. Similarly, let multiplication $F \circ G$ indicate that it is applied component-wise, i.e., $F \circ G = \{f_{i,j} \times g_{i,j}\}$ for all i and j.

Let $\mathcal{P}(S)$ denote the probability that event S occurs. If $\mathcal{P}(S) = 1$ holds true, S almost surely occurs. For a random variable z, let the expected value and variance of z be denoted by $\mathcal{E}(z)$ and $\mathcal{V}(z)$, respectively. For a random vector $x = [x_1, \cdots, x_n]'$ whose components are random variables, let $\mathcal{E}(x)$ and $\mathcal{V}(x)$ denote $\mathcal{E}(x) = [\mathcal{E}(x_1), \cdots, \mathcal{E}(x_n)]'$ and $\mathcal{V}(x) = [\mathcal{V}(x_1), \cdots, \mathcal{V}(x_n)]'$,respectively.

Let q denote the static nearest-neighbor quantizer toward $-\infty$ with the quantization interval d as shown in Fig. 1 of [19].

III. System Model

In this section, we introduce the system model for linear discrete-time control systems with random dither quantization. Here, we consider the following linear discrete-time system:

$$x(t+1) = Ax(t) + Bv(t), \tag{1}$$
$$v(t) = q\left(u(t) + \eta(t)\right), \tag{2}$$

where $t \in \mathbb{N}$ is the time step, $x(t) : \mathbb{N} \to \mathbb{R}^n$ is the state, $u(t) : \mathbb{N} \to \mathbb{R}^m$ is the control input. Moreover, q is the quantizer defined in Section II and $\eta(t) : \mathbb{N} \to \mathbb{R}^m$ is an independent and identically distributed random variable with the uniform probability distribution on $[-d/2, d/2)$. From (2), we can see that the random dither quantization transforms a given continuous-valued signal to a discrete-valued signal by adding artificial random noise to the continuous-valued signal before quantization.

Throughout this paper, the system coefficients A and B are assumed to be known constant matrices. Also, we suppose that the pair (A, B) is controllable. All components of state $x(t)$ are observable, that is, they are exactly known at present time t. Thus, we suppose that $\mathcal{E}(x(t)) = x(t)$ and $\mathcal{V}(x(t)) = 0$.

It has been shown in [19] that the above system is equivalently transformed into the following system:

$$x(t+1) = Ax(t) + B(u(t) + w(t)), \tag{3}$$
$$w(t) = v(t) - u(t), \tag{4}$$

where w denotes the quantization error. From the definition of the random dither quantizer, we note that the quantization error is also a random variable.

The following properties of the expectation and variance of the quantization error w have been shown in [1].

Lemma 1 ([1]): For the fixed quantization interval d, the expectation and variance of the quantization error w are given by

$$\mathcal{E}(w(t)) = 0, \tag{5}$$
$$\mathcal{V}(w(t)) \leq \frac{d^2}{4}. \tag{6}$$

IV. Preliminaries

The SMPC problem of system (3) has been already formulated in [19]. The control input at each time t is determined so as to minimize the performance index given by

$$J := \phi[x(t+N)] + \sum_{k=t}^{t+N-1} L[x(k), u(k)], \tag{7a}$$

where $N \in \mathbb{N}$ denotes the length of the evaluation interval. Moreover, let ϕ and L be defined by

$$\phi := \mathcal{E}[x(t+N)'Px(t+N)], \tag{7b}$$
$$L := \mathcal{E}[x(k)'Qx(k)] + u(k)'Ru(k), \tag{7c}$$

where let P, Q, and R be weighting coefficients that are positive definite constant matrices. Note that $\phi \in \mathbb{R}_+$ is the terminal cost function and $L \in \mathbb{R}_+$ is the stage cost function over the evaluation interval.

For notational convenience, we introduce the so-called expanded vectors as follows: Let $\mathbf{X} \in \mathbb{R}^{nN}$, $\mathbf{U} \in \mathbb{R}^{mN}$ and $\mathbf{W} \in \mathbb{R}^{\ell N}$ be defined by

$$\mathbf{X}(t) := \begin{bmatrix} x(t+1) \\ \vdots \\ x(t+N) \end{bmatrix},$$

$$\mathbf{U}(t) := \begin{bmatrix} u(t) \\ \vdots \\ u(t+N-1) \end{bmatrix},$$

$$\mathbf{W}(t) := \begin{bmatrix} w(t) \\ \vdots \\ w(t+N-1) \end{bmatrix}.$$

We can see that $\mathbf{X}$, $\mathbf{U}$ and $\mathbf{W}$ consist of the system state, control input and quantization error, respectively, over the evaluation interval. Next, we introduce the following assumption.

Assumption 1: We assume that each element of $x(t)$, $\mathbf{U}(t)$ and $\mathbf{W}(t)$ are independent for each time t.

Under the above assumption, it has been shown in [19] that the minimization problem of J in (7) subject to system equation (3) has been reduced to the following quadratic programming problem with respect to $\mathbf{U}$:

$$\min_{\mathbf{U}(t)} J[x(t), \mathbf{X}(t), \mathbf{U}(t)] = \tag{8}$$
$$\min_{\mathbf{U}(t)} \left\{ \begin{array}{l} \mathbf{U}'(t)\left(\mathbf{B}'\mathbf{QB} + \mathbf{R}\right)\mathbf{U}(t) \\ \qquad +2\left(\mathbf{A}x(t) + \mathbf{B}\mathcal{E}(\mathbf{W}(t))\right)'\mathbf{QBU}(t) \end{array} \right\},$$

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where $\mathbf{A} \in \mathbb{R}^{nN \times n}$, $\mathbf{B} \in \mathbb{R}^{nN \times mN}$, $\mathbf{Q} \in \mathbb{R}^{nN \times nN}$, and $\mathbf{R} \in \mathbb{R}^{mN \times mN}$ are the so-called expanded matrices defined in [19]

$$\mathbf{A} := \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix}.$$

$$\mathbf{B} := \begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{bmatrix},$$

$$\mathbf{Q} := \begin{bmatrix} Q & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & Q & 0 \\ 0 & \cdots & 0 & P \end{bmatrix},$$

$$\mathbf{R} := \begin{bmatrix} R & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & R \end{bmatrix}.$$

Here, we introduce a variable p_i that denotes the probability. Let the probability in vector form be denoted by

$$p(t) = \begin{bmatrix} p_1(t) \\ \vdots \\ p_2(t) \end{bmatrix} : \mathbb{N} \to [0\ 1]^n,$$

which means that each component $p_i(t)$ belongs to $[0\ 1]$ for each time t. Let $\mathbf{p} \in \mathbb{R}^{nN}$ be defined by

$$\mathbf{p}(t) = \begin{bmatrix} p(t+1) \\ \vdots \\ p(t+N) \end{bmatrix}.$$

Here, we impose the following probabilistic constraint on the optimization problem: for $k = t+1, \cdots, t+N$ and $i = 1, \cdots, n$,

$$\mathcal{P}\left(\underline{x}_i(k) < x_i(k) < \overline{x}_i(k)\right) \geq p_i(k), \tag{9}$$

where $\underline{x}_i(k)$, $\overline{x}_i(k) \in \mathbb{R}$, and $p_i(k) \in [0\ 1]$ for $k = t+1, \cdots, t+N$ are given constant sequences and their subscript indicates the ith element of the vector. Condition (9) indicates that state x_i over the prediction horizon must remain within the bound $[\underline{x}_i\ \ \overline{x}_i]$ at least with probability p_i.

Let $\underline{\mathbf{X}} \in \mathbb{R}^{nN}$ and $\overline{\mathbf{X}} \in \mathbb{R}^{nN}$ be defined by:

$$\underline{\mathbf{X}}(t) := \begin{bmatrix} \underline{x}(t+1) \\ \vdots \\ \underline{x}(t+N) \end{bmatrix}, \quad \overline{\mathbf{X}}(t) := \begin{bmatrix} \overline{x}(t+1) \\ \vdots \\ \overline{x}(t+N) \end{bmatrix}.$$

Using the above notation, probabilistic constraint (9) is rewritten in vector form as

$$\mathcal{P}\left(\underline{\mathbf{X}}(t) < \mathbf{X}(t) < \overline{\mathbf{X}}(t)\right) \geq \mathbf{p}(t). \tag{10}$$

In general, to solve the quadratic programming problem with probabilistic constraints is not straightforward. In [17], it has been shown that the probabilistic constraints (10) can be converted into deterministic constraints using the concentration inequalities.

Using Proposition 1 shown in [17], it is straightforward to prove the following lemma.

Lemma 2: Suppose that the following condition holds:

$$\mathbf{U}_{\min}(t) \leq \mathbf{B}\mathbf{U}(t) \leq \mathbf{U}_{\max}(t), \tag{11}$$

where $\mathbf{U}_{\min}$ and $\mathbf{U}_{\max}$ are defined by:

$$\begin{aligned} \mathbf{U}_{\min}(t) &:= \underline{\mathbf{X}}(t) + \boldsymbol{\kappa}(t) \circ \sqrt{(\mathbf{B} \circ \mathbf{B})\mathcal{V}(\mathbf{W}(t))} \\ &\quad - \mathbf{A}x(t) - \mathbf{B}\mathcal{E}(\mathbf{W}(t)), \end{aligned} \tag{12a}$$

$$\begin{aligned} \mathbf{U}_{\max}(t) &:= \overline{\mathbf{X}}(t) - \boldsymbol{\kappa}(t) \circ \sqrt{(\mathbf{B} \circ \mathbf{B})\mathcal{V}(\mathbf{W}(t))} \\ &\quad - \mathbf{A}x(t) - \mathbf{B}\mathcal{E}(\mathbf{W}(t)). \end{aligned} \tag{12b}$$

$$\boldsymbol{\kappa}(t) := \left[\frac{1}{\sqrt{1-\mathbf{p}_1(t)}}, \cdots, \frac{1}{\sqrt{1-\mathbf{p}_{nN}(t)}}\right]'. \tag{12c}$$

Then, the probabilistic condition (10) is fulfilled.

Proof: Substituting $\mathbf{B}$ into $\mathbf{C}$ in Proposition 1 of [17], we can complete the proof. ■

Remark 1: From Lemma 2, the minimization problem of (7) with probabilistic constraint (10) is reduced to the quadratic programming problem (8) with deterministic constraint (11), which can be solved using a conventional algorithm.

V. Main Results

In this section, we examine the feasibility of stochastic model predictive control problem (7) subject to probabilistic constraint (10) by conducting on numerical simulations.

Here, we consider the following system model that is used in [1] as an illustrative example.

$$\dot{x}(t) = \begin{bmatrix} 0 & 4 \\ -3 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{13}$$

The given linear continuous-time control system can be transformed into the following linear discrete-time control system using the zero-order hold method with sampling time $\Delta t = 0.01$.

$$x(\tau+1) = \begin{bmatrix} 0.994 & 0.0404 \\ -0.0303 & 1.0196 \end{bmatrix} x(\tau) + \begin{bmatrix} 0.0002 \\ 0.0101 \end{bmatrix} u(\tau) \tag{14}$$

The state of the closed-loop systems with the state feedback controller $u = -[0.2\ \ 2.9]x(t)$ used in [1] is denoted by x^{rdq}. On the other hand, the state of the closed-loop systems with the stochastic model predictive controller proposed here is denoted by x^{smp}.

In the following, we provide the simulation results to verify the effectiveness of the proposed method. The parameters employed in the numerical simulations are as follows: $N = 5$, $P = Q = 100$, $R = 1$, $d = 2$, and $\underline{x}_i = -1$, $\overline{x}_i = 1$, $p_i = 0.8$ for all i. We perform 100 trials for numerical simulations.

Time responses of both expectations of x_1^{rdq} and x_1^{smp} are shown in Fig. 1. We can see from Fig. 1 that x_1^{rdq} breaks the constraint on the state but x_1^{smp} fulfills it. Time responses of both expectations of x_2^{rdq} and x_2^{smp} are shown in Fig. 2. Fig. 2 reveal that x_2^{rdq} breaks the constraint on the state but x_2^{smp}

fulfills it. Time responses of both variances of x_1^{rdq} and x_1^{smp} are shown in Fig. 3. Comparing $\mathcal{V}(x_1^{rdq})$ with $\mathcal{V}(x_1^{smp})$, note that the dispersion of time response of the states is reduced by taking constraint (10) into account. Time responses of both variances of x_2^{rdq} and x_2^{smp} are shown in Fig. 4. It can be observed from Fig. 4 that the variance of x_2^{smp} is reduced to a greater extent than the one of x_2^{rdq}. Time responses of both norms of $||\mathcal{E}(x^{rdq})||$ and $||\mathcal{E}(x^{smp})||$ are shown in Fig. 5. We can see from Fig. 5 that $||\mathcal{E}(x^{smp})||$ converges to zero much faster than $||\mathcal{E}(x^{rdq})||$. Time responses of both norms of $||\mathcal{V}(x^{rdq})||$ and $||\mathcal{V}(x^{smp})||$ are shown in Fig. 6. It can be observed from Fig. 6 that $||\mathcal{V}(x^{smp})||$ is reduced to a greater extent than $||\mathcal{V}(x^{rdq})||$.

All Figs. 1-6 reveal that the stochastic model predictive control method is useful for linear discrete-time control systems with random dither quantization subject to state constraints. Consequently, we are able to verify the feasibility and effectiveness of the stochastic model predictive control method by numerical simulations.

VI. Conclusion

In this study, we have examined the effectiveness of stochastic model predictive control method for linear discrete-time systems with the random dither quantization. Thus, the feasibility of optimal control problems subject to probabilistic state constraints was investigated by numerical simulations. It was shown that the optimal control problems subject to probabilistic constraints can be reduced to quadratic programming problems with deterministic constraints that are solvable using a conventional algorithm. The obtained results on numerical simulations reveal that the stochastic model predictive control method proposed here exhibits much better performance than the nominal random dither quantization control method.

It is known that not only uncertain disturbances but also time delays may cause instabilities and lead to more complex analysis [20]-[25]. The control problem of random dither systems with time delays is also a possible future work.

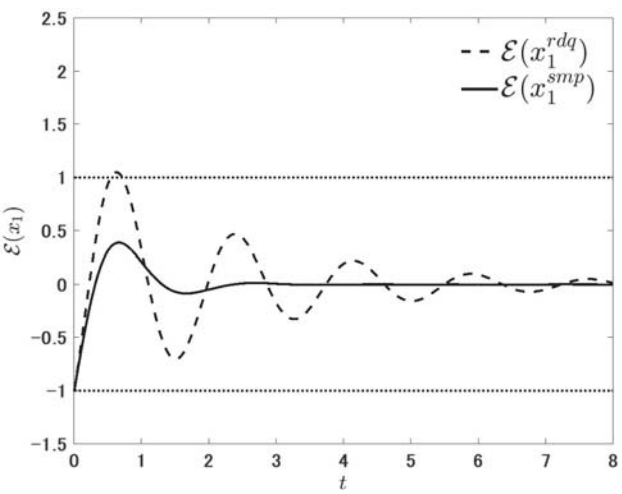

Fig. 1 Time responses of $\mathcal{E}(x_1^{rdq})$ and $\mathcal{E}(x_1^{smp})$.

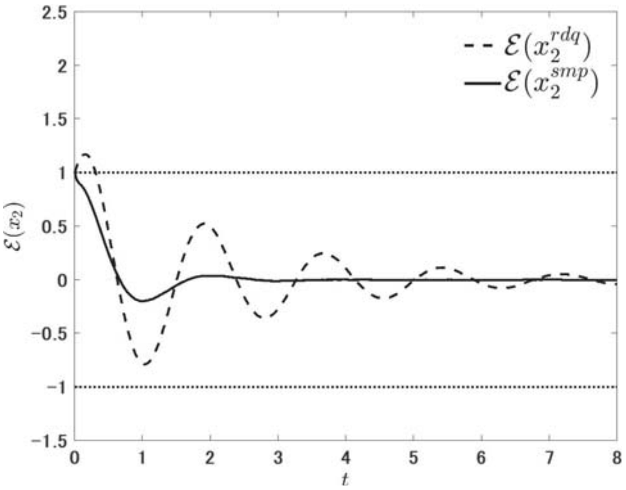

Fig. 2 Time responses of $\mathcal{E}(x_2^{rdq})$ and $\mathcal{E}(x_2^{smp})$.

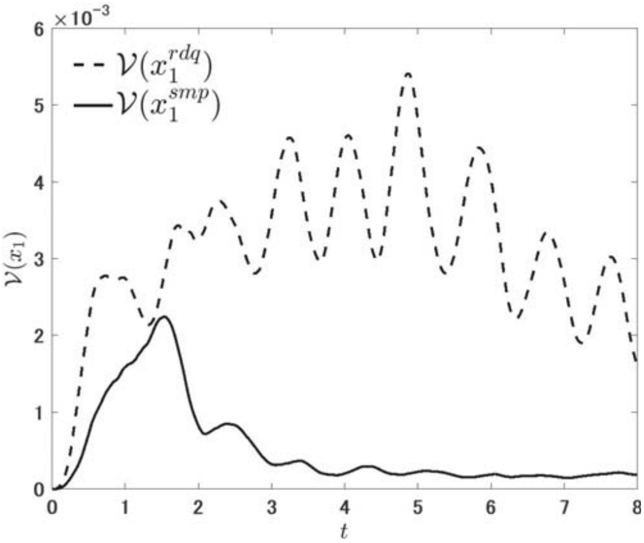

Fig. 3 Time responses of $\mathcal{V}(x_1^{rdq})$ and $\mathcal{V}(x_1^{smp})$.

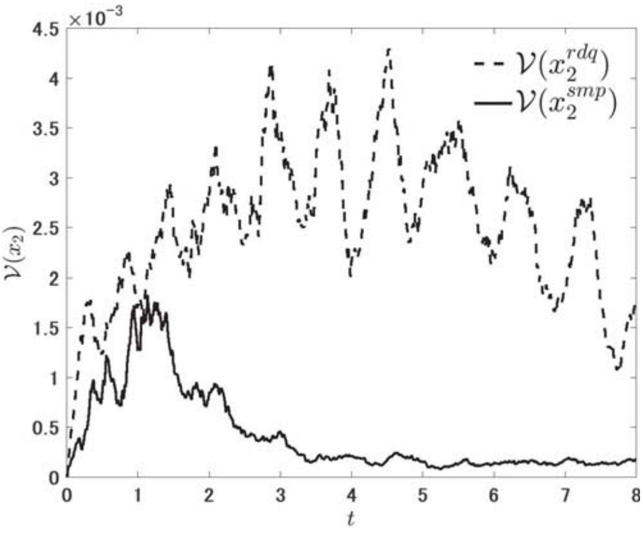

Fig. 4 Time responses of $\mathcal{V}(x_2^{rdq})$ and $\mathcal{V}(x_2^{smp})$.

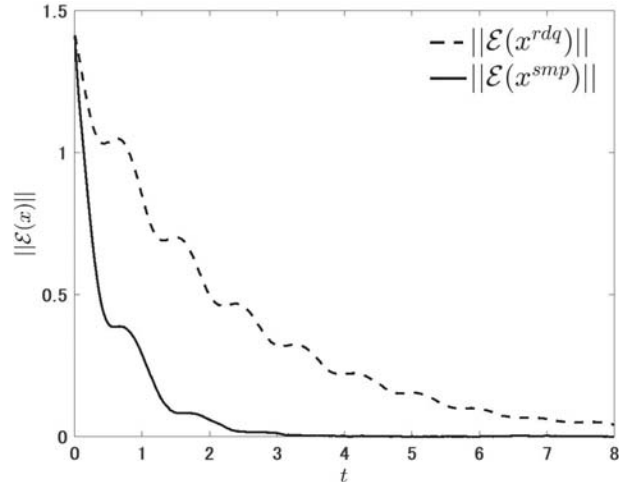

Fig. 5 Time responses of $||\mathcal{E}(x^{rdq})||$ and $||\mathcal{E}(x^{smp})||$.

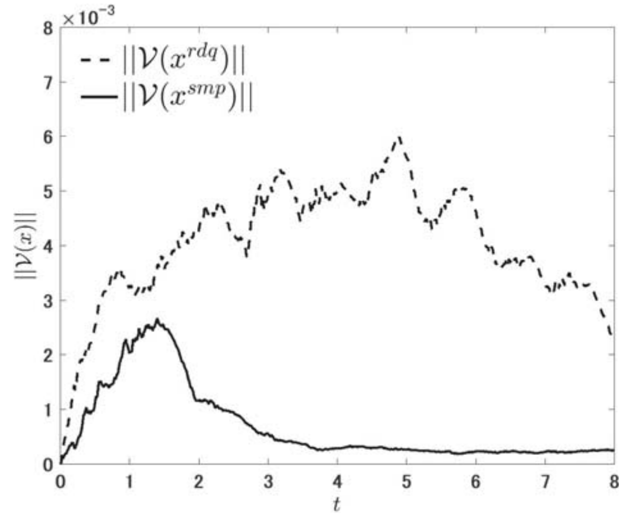

Fig. 6 Time responses of $||\mathcal{V}(x^{rdq})||$ and $||\mathcal{V}(x^{smp})||$.

REFERENCES

[1] R. Morita, S. Azuma, T. Sugie, Performance Analysis of Random Dither Quantizers in Feedback Control Systems, *SICE Journal of Control, Measurement, and System Integration*, Vol. 6, No. 1, pp. 21-27, 2013.

[2] T. Hashimoto, Y. Yoshioka, T. Ohtsuka, Receding Horizon Control with Numerical Solution for Thermal Fluid Systems, *Proceedings of SICE Annual Conference*, pp. 1298-1303, 2012.

[3] T. Hashimoto, Y. Yoshioka, T. Ohtsuka, Receding Horizon Control with Numerical Solution for Spatiotemporal Dynamic Systems, *Proceedings of IEEE Conference on Decision and Control*, pp. 2920-2925, 2012.

[4] T. Hashimoto, Y. Takiguchi and T. Ohtsuka, Output Feedback Receding Horizon Control for Spatiotemporal Dynamic Systems, *Proceedings of Asian Control Conference*, 2013.

[5] T. Hashimoto, Y. Yoshioka and T. Ohtsuka, Receding Horizon Control for Hot Strip Mill Cooling Systems, *IEEE/ASME Transactions on Mechatronics*, Vol. 18, No. 3, pp. 998-1005, 2013.

[6] T. Hashimoto, Y. Yoshioka and T. Ohtsuka, Receding Horizon Control With Numerical Solution for Nonlinear Parabolic Partial Differential Equations, *IEEE Transactions on Automatic Control*, Vol. 58, No. 3, pp. 725-730, 2013.

[7] T. Hashimoto, Y. Takiguchi and T. Ohtsuka, Receding Horizon Control for High-Dimensional Burgersf Equations with Boundary Control Inputs, *Transactions of the Japan Society for Aeronautical and Space Sciences*, Vol. 56, No.3, pp. 137-144, 2013.

[8] R. Satoh, T. Hashimoto and T. Ohtsuka, Receding Horizon Control for Mass Transport Phenomena in Thermal Fluid Systems, *Proceedings of Australian Control Conference*, pp. 273-278, 2014.

[9] T. Hashimoto, Receding Horizon Control for a Class of Discrete-time Nonlinear Implicit Systems, *Proceedings of IEEE Conference on Decision and Control*, pp. 5089-5094, 2014.

[10] T. Hashimoto, Optimal Feedback Control Method Using Magnetic Force for Crystal Growth Dynamics, *International Journal of Science and Engineering Investigations*, Vol. 4, Issue 45, pp. 1-6, 2015.

[11] T. Hashimoto, R. Satoh and T. Ohtsuka, Receding Horizon Control for Spatiotemporal Dynamic Systems, *Mechanical Engineering Journal*, Vol. 3, No. 2, 15-00345, 2016.

[12] T. Hashimoto, I. Yoshimoto, T. Ohtsuka, Probabilistic Constrained Model Predictive Control for Schrödinger Equation with Finite Approximation, *Proceedings of SICE Annual Conference*, pp. 1613-1618, 2012.

[13] T. Hashimoto, Probabilistic Constrained Model Predictive Control for Linear Discrete-time Systems with Additive Stochastic Disturbances, *Proceedings of IEEE Conference on Decision and Control*, pp. 6434-6439, 2013.

[14] T. Hashimoto, Computational Simulations on Stability of Model Predictive Control for Linear Discrete-time Stochastic Systems, *International Journal of Computer, Electrical, Automation, Control and Information Engineering*, Vol. 9, No. 8, pp. 1385-1390, 2015.

[15] T. Hashimoto, Conservativeness of Probabilistic Constrained Optimal Control Method for Unknown Probability Distribution, *International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering*, Vol. 9, No. 9, pp. 11-15, 2015.

[16] T. Hashimoto, A Method for Solving Optimal Control Problems subject to Probabilistic Affine State Constraints for Linear Discrete-time Uncertain Systems, *International Journal of Mechanical and Production Engineering*, Vol. 3, Issue 12, pp. 6-10, 2015.

[17] T. Hashimoto, Solutions to Probabilistic Constrained Optimal Control Problems Using Concentration Inequalities, *International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering*, Vol. 10, No. 10, pp. 441-446, 2016.

[18] T. Hashimoto, Stability of Stochastic Model Predictive Control for Schrödinger Equation with Finite Approximation, *International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering*, Vol. 11, No. 1, pp. 12-17, 2017.

[19] T. Hashimoto, Stochastic Model Predictive Control for Linear Discrete-time Systems with Random Dither Quantization, *International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering*, Vol. 11, No. 2, pp. 130-134, 2017.

[20] T. Hashimoto, T. Amemiya and H. A. Fujii, Stabilization of Linear Uncertain Delay Systems with Antisymmetric Stepwise Configurations, *Journal of Dynamical and Control Systems*, Vol. 14, No. 1, pp. 1-31, 2008.

[21] T. Hashimoto, T. Amemiya and H. A. Fujii, Output Feedback Stabilization of Linear Time-varying Uncertain Delay Systems, *Mathematical Problems in Engineering*, Vol. 2009, Article ID. 457468, 2009.

[22] T. Hashimoto and T. Amemiya, Stabilization of Linear Time-varying Uncertain Delay Systems with Double Triangular Configuration, *WSEAS Transactions on Systems and Control*, Vol. 4, No.9, pp.465-475, 2009.

[23] T. Hashimoto, Stabilization of Abstract Delay Systems on Banach Lattices using Nonnegative Semigroups, *Proceedings of the 50th IEEE Conference on Decision and Control*, pp. 1872-1877, 2011.

[24] T. Hashimoto, A Variable Transformation Method for Stabilizing Abstract Delay Systems on Banach Lattices, *Journal of Mathematics Research*, Vol. 4, No. 2, pp.2-9, 2012.

[25] T. Hashimoto, An Optimization Algorithm for Designing a Stabilizing Controller for Linear Time-varying Uncertain Systems with State Delays, *Computational Mathematics and Modeling*, Vol.24, No.1, pp.90-102, 2013.