Sixth-Order Two-Point Efficient Family of Super-Halley Type Methods

Ramandeep Behl, S. S. Motsa

Abstract—The main focus of this manuscript is to provide a highly efficient two-point sixth-order family of super-Halley type methods that do not require any second-order derivative evaluation for obtaining simple roots of nonlinear equations, numerically. Each member of the proposed family requires two evaluations of the given function and two evaluations of the first-order derivative per iteration. By using Mathematica-9 with its high precision compatibility, a variety of concrete numerical experiments and relevant results are extensively treated to confirm the theoretical development. From their basins of attraction, it has been observed that the proposed methods have better stability and robustness as compared to the other sixth-order methods available in the literature.

Keywords—Basins of attraction, nonlinear equations, simple roots, Super-Halley.

I. INTRODUCTION

Efficient solution techniques are required for finding simple roots of nonlinear equation \( f(x) = 0 \), which partake of scientific, engineering and various other models. One of the best known one-point optimal method is classical Newton’s method [1], [2]. With the advancements of computer algebra, researchers [3]-[8], from the worldwide proposed three-point sixth-order methods that are known as the extensions of Newton’s method at the expense of additional evaluations of functions, derivatives and changes in the points of iterations.

But, the body structures of these three-point sixth-order methods are not simple as compared to two-point methods [9], [10]. Further, it is not easy to find two-point methods whose order of convergence greater than four [11].

Therefore, our primary aim is to develop a new highly efficient two-point sixth-order family of super-Halley type methods, that do not require any second-order derivative. It is also observed that the body structures of our proposed methods are simpler than the existing three-point sixth-order methods. Further, our proposed methods are more effective in all the tested examples to the existing methods available in the literature. The dynamic study of our proposed methods which is given in Section V, to cross verify the theoretical aspects.

II. DEVELOPMENT OF TWO-POINT SIXTH-ORDER METHODS

In this section, we intend to develop several new families of sixth-order super-Halley type methods. For this purpose, we consider the following well-known third-order super-Halley method [1], [2]

\[
x_{n+1} = x_n - \frac{f(x_n)}{2f'(x_n)} \left( \frac{2(f'(x_n))^2 - f(x_n)f''(x_n)}{(f'(x_n))^2} \right). \tag{1}
\]

Further, we consider \( y_n = x_n - \frac{f(x_n)}{f'(x_n)} \) a Newton’s iterate. With the help of Taylor series, we expand the function \( f(y_n) \) about a point \( x = x_n \) as follows:

\[
f''(x_n) \approx \frac{2(f'(x_n))^2f(y_n)}{(f'(x_n))^2}. \tag{2}
\]

Similarly, expanding the function \( f'(y_n) = f'(x_n - \frac{f(x_n)}{f'(x_n)}) \) about a point \( x = x_n \) by Taylor series, we have \( f''(y_n) \approx f''(x_n) + f'(x_n)(y_n - x_n) \), which further yields

\[
f''(x_n) \approx \frac{f''(x_n) + f'(x_n)(y_n - x_n)}{f'(x_n)}. \tag{3}
\]

From (2) and (3), we have

\[
f''(x_n) \approx \frac{2(f'(x_n))^2f(y_n)}{f(x_n)(y_n - x_n)} + \frac{f'(x_n)(y_n - x_n)}{f'(x_n)}. \tag{4}
\]

Using this approximate value of \( f''(x_n) \) in formula (1) and using the weight function on the second step, we get a modified family of methods free from second-order derivative as follows:

\[
\begin{align*}
y_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} \left( \frac{2f'(x_n)f'(x_n) + f(x_n)f''(x_n) - 2f'(x_n)f(y_n)}{(f'(x_n))^2} \right), \tag{5}
\end{align*}
\]

where the weight function \( L_f \) is sufficient differential function in a neighborhood of \((1, 0)\) with \( u = \frac{f'(x_n)}{f(x_n)} = 1 + O(e_n) \) and \( v = \frac{f(y_n)}{f(x_n)} = O(e_n) \). Theorem III indicates that under what choices on the weight function which is proposed in (5), the order of convergence will reach at six without using any more functional evaluations.

III. ORDER OF CONVERGENCE

Theorem I: Let a sufficiently smooth function \( f : D \subseteq \mathbb{R} \to \mathbb{R} \) have a simple zero \( \xi \) in the open interval \( D \). Then, the iterative scheme defined by (5) has sixth-order convergence when it satisfies the following conditions

\[
\begin{align*}
L_{00} &= 1, & L_{01} &= \frac{1}{2}, & L_{02} &= -\frac{1}{4}, & L_{11} &= \frac{1}{2}, & L_{20} &= \frac{3}{8}, \\
L_{12} &= -\frac{1}{2} - 4L_{21} - 4L_{30}, & L_{03} &= -9 + 12L_{21} + 16L_{30}, \\
L_{04} &= -8(9 + L_{13} + 3L_{22} + 4L_{31} + 2L_{40}),
\end{align*}
\]

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where \( L_{ij} = \frac{\partial^2 f}{\partial u^i \partial v^j} f(u, v) \). It satisfies the following error equation

\[
e_{n+1} = \frac{-c_2}{12} \left[ 4\left(9 + 2L_{13} + 12L_{21} + 12L_{22} + 48L_{30} + 24L_{31} + 16L_{40}\right) c_3^3 - 2\left(2L_{11} + 24L_{21} + 6L_{22} + 48L_{30} + 12L_{31} + 8L_{40} - 12\right) c_2^2 c_3 + 3\left(3 + 2L_{21} + 4L_{30}\right) c_5^2 - 12c_2 c_4 \right] e_n^6 + O(e_n^8).
\]

(7)

**Proof:** Let \( \xi \) be a simple zero of \( f(x) \). With the help of Taylor’s series, we get the following expansion of \( f(x_n) \) and \( f'(x_n) \) around \( x = \xi \)

\[
f(x_n) = f'(\xi) (x_n - \xi) + \frac{f''(\xi)}{2!} (x_n - \xi)^2 + \frac{f'''(\xi)}{3!} (x_n - \xi)^3 + \frac{f^{(4)}(\xi)}{4!} (x_n - \xi)^4 + O\left((x_n - \xi)^5\right)
\]

(8)

and

\[
f'(x_n) = f'(\xi) + f''(\xi) (x_n - \xi) + \frac{f'''(\xi)}{2!} (x_n - \xi)^2 + \frac{f^{(4)}(\xi)}{3!} (x_n - \xi)^3 + \frac{f^{(5)}(\xi)}{4!} (x_n - \xi)^4 + O\left((x_n - \xi)^5\right)
\]

(9)

respectively. From (8) and (9), we obtain

\[
y_n = c_2^2 e_n^2 - 2(c_2^2 - c_3) e_n^3 + (4c_2^3 - 7c_2 c_3 + 3c_4) e_n^4 + (20c_2^3 - 8c_2^3 - 6c_3 - 10c_2 c_4 + 4c_5)e_n^5 + O(e_n^6).
\]

(10)

By using (10) and with the help of Taylor series, we get the following expansions of \( f(y_n) \) and \( f'(y_n) \) about \( x = \xi \)

\[
f(y_n) = f'(\xi) (y_n - \xi) + \frac{f''(\xi)}{2!} (y_n - \xi)^2 + \frac{f'''(\xi)}{3!} (y_n - \xi)^3 + \frac{f^{(4)}(\xi)}{4!} (y_n - \xi)^4 + O\left((y_n - \xi)^5\right)
\]

(11)

and

\[
f'(y_n) = f'(\xi) + f''(\xi) (y_n - \xi) + \frac{f'''(\xi)}{2!} (y_n - \xi)^2 + \frac{f^{(4)}(\xi)}{3!} (y_n - \xi)^3 + \frac{f^{(5)}(\xi)}{4!} (y_n - \xi)^4 + \frac{f^{(6)}(\xi)}{5!} (y_n - \xi)^5 + O\left((y_n - \xi)^6\right)
\]

(12)

By using (8)-(12), we obtain

\[
h = \frac{f(y_n)}{f'(x_n)} = 1 + 2c_2 c_n + (-2 c_2^2 + 3c_3) e_n^2 + (-4c_2 c_3 + 4c_4) e_n^3 + (4c_2^3 - 3c_2 c_3 - 6c_2 c_4 + 5c_5) e_n^4 + (22c_2 c_3 - 8c_2^2 - 4c_2 c_3 + 5c_2 c_4 - 2c_5) e_n^5 + O(e_n^6).
\]

(13)

and

\[
k = \frac{f(y_n)}{f'(x_n)} = c_2 e_n + (-3c_2^2 + 2c_3) e_n^2 + (8c_2^2 - 10c_2 c_3 + 3c_4) e_n^3 + (-30c_2^2 + 37c_2^3 c_3 - 8c_3 - 14c_2 c_4 + 4c_5) e_n^4 + (48c_2^3 - 118c_2^3 c_3 + 51c_2^3 c_4 - 22c_3 c_4 + c_5(55c_2^2 - 18c_3) + 5c_3)e_n^5 + O(e_n^6).
\]

(14)

Since it is noteworthy from the above mention equations namely, (13) and (14), \( u = 1 + p \) and \( v = O(e_n) \). Then, from these equations, we get the remainder \( p = u - 1 \) and \( v \) are infinitesimal with the same order of \( e_n \). Therefore, we can expand weight function \( L_f(u, v) \) in the neighborhood of \( (1, 0) \) by Taylor series expansion up to fourth-order terms as follow

\[
L_f(u, v) = L_{00} + L_{10}p + L_{01}v + \frac{1}{2!}(L_{20}p^2 + 2L_{11}pv + L_{02}v^2) + \frac{1}{3!}(L_{30}p^3 + 3L_{21}p^2v + 3L_{12}pv^2 + L_{03}v^3) + \frac{1}{4!}(L_{40}p^4 + 4L_{31}p^3v + 6L_{22}p^2v^2 + 4L_{13}pv^3 + L_{04}v^4) + O(e_n^6).
\]

(15)

Using (8)-(15), in scheme (5), we get

\[
e_{n+1} = (1 - L_{00}) e_n - c_2(L_{01} + 2L_{10}) e_n^2 + \sum_{l=3}^{6} M_l e_n^l,
\]

(16)

where \( M_l = M_l(c_2, c_3, \ldots, c_6)L_{ij} \), for \( 0 \leq i, j \leq 4 \).

We will get at least third-order convergence if we insert the following values of \( L_{00} \) and \( L_{01} \) in (16),

\[
L_{00} = 1, \quad L_{01} = -2L_{10}.
\]

(17)

Further, using (17) into \( M_3 = 0 \), we find two independent relation as follows:

\[
(L_{02} + 4(2L_{10} + L_{11} + L_{20})) = 0, \quad (1 + 4L_{10}) = 0
\]

(18)

Solving the equations defined in (18) for \( L_{11} \) and \( L_{10} \), we have

\[
L_{11} = -\frac{1}{4}(-2 + L_{02} + 4L_{20}), \quad L_{10} = -\frac{1}{4}
\]

(19)

By inserting (17) and (19) into \( M_4 = 0 \), we obtain

\[
(L_{03} + 8(3 + 6L_{12} + L_{13} + 24L_{20} + 24L_{21} + 3L_{22} + 24L_{30} + 4L_{31} + 2L_{40})) = 0
\]

(20)

Further, solve the above equation namely, (20) for \( L_{02} \) and \( L_{03} \), we get

\[
L_{02} = 4L_{20} - 3, \quad L_{03} = -(12 + 6L_{12} + 12L_{21} + 8L_{30})
\]

(21)

By substituting (17), (19) and (21) into \( M_5 = 0 \), we obtain

\[
\begin{cases}
(3 - 8L_{20}) = 0, \\
\begin{cases}
-5 + 2L_{12} + 16L_{20} + 8L_{21} + 8L_{30} = 0, \\
[L_{04} + 8(3 + 6L_{12} + L_{13} + 24L_{20} + 24L_{21} + 3L_{22} + 24L_{30} + 4L_{31} + 2L_{40}) = 0.
\end{cases}
\end{cases}
\]

(22)

Solving the above equation for \( L_{20} \), \( L_{12} \) and \( L_{04} \), we further yield

\[
\begin{cases}
L_{20} = \frac{3}{8}, \\
L_{12} = -\frac{1}{2}(1 + 8L_{21} + 8L_{30}), \\
L_{04} = -8(9 + L_{13} + 3L_{22} + 4L_{31} + 2L_{40})
\end{cases}
\]

(23)

We can easily obtain the following error equation, by using (17), (19), (21) and (23) into (16)

\[
e_{n+1} = \frac{-c_2}{12} \left[ 4(9 + 2L_{13} + 24L_{21} + 12L_{22} + 48L_{30} + 24L_{31} + 16L_{40}) c_2^2 - 2(L_{13} + 24L_{21} + 6L_{22} + 48L_{30} + 12L_{31} + 8L_{40} - 12)c_2 c_3 + 3(3 + 2L_{21} + 4L_{30}) c_5^2 - 12c_2 c_4 \right] e_n^6 + O(e_n^8).
\]

(24)
This reveals that our proposed scheme (5) has sixth-order of convergence while using only four functional evaluations (viz $f(x_n)$, $f'(x_n)$, $f'(y_n)$, and $f'(y_{n+1})$) per full iteration. Hence, this completes the proof of above Theorem III.

IV. Special Cases

In this section, we discuss some interesting special cases of weight function $L_f(u, v)$, which are defined as follows:

(1) For $L_{21} = 0$, $L_{30} = 0$, $L_{13} = 0$, $L_{22} = 0$ and $L_{31} = 0$ in (15), we get the following weight function

$$L_f(u, v) = 1 - \frac{p}{4} + \frac{3v^2}{16} + L_{40} \frac{4}{4} + \frac{4 + 4p + 3p^2}{8} - \frac{3 + 7p + 3p^2}{4} u^3 - 2L_{40} v^4,$$

(25)

where $L_{40}$ is a free variable and for the sake of simplicity $p = u - 1$. With the help of this disposable parameter, we can easily obtain various different types of weight functions as well as two-point sixth-order methods.

(2) For $L_{21} = 0$, $L_{40} = 0$, $L_{13} = 0$, $L_{22} = 0$ and $L_{31} = 0$ in (15), we obtain

$$L_f(u, v) = 1 - \frac{p}{4} + \frac{3v^2}{16} + L_{30} \frac{3}{3} + \frac{p + 1}{2} u + \frac{1 + 8L_{30}p + 3v^2}{6} + \frac{16L_{30} - 9(1 + p)}{6} v^3,$$

(26)

where $L_{30}$ is a free variable.

(3) We consider the following weight function, which satisfies all the conditions defined in theorem III

$$L_f(u, v) = \frac{1}{16u^2} (22u^3v - 3u^4v + v(6 + 8u) - u^2(19v + 12v^2 + 36u^3 - 11) - 1).$$

(27)

V. Numerical Experiments

In this section, we will check the validity and efficiency of theoretical results. Therefore, we apply our methods for $(L_{40} = 0 \& L_{40} = \frac{9}{16})$ in scheme (25) and for $(L_{30} = 0 \& L_{31} = \frac{3}{4})$ in scheme (26) are denoted by $OM1$, $OM2$, $OM3$, and $OM4$, respectively, to solve some nonlinear equations given in Table I. We compare them with a three-point sixth-order method proposed by Sharma and Guha [3], method (3) for $(a = 2)$ denoted by (SG). In addition, we also compare our schemes with a method namely, method (5) for $(c=60, d=1, r=0)$ called (WM) which is given by Wang and Liu in [5]. Finally, we will also compare them with a two-point family of sixth-order methods that is very recently proposed by Guem et al. [11], between them we will choose their best expression (3.8 and 3.12) denoted by (KM1 and KM2), respectively. For better comparisons of our proposed methods, we have given three comparison tables in each example: one is corresponding to absolute error in Table II, the second one is with respect to number of iterations in Table III and third one is regarding their computational order of convergence in Table IV respectively. All computations have been performed using the programming package Mathematica 9 with multiple precision arithmetic. The meaning of $a(-b)$ is $a \times 10^{-b}$ in Table II. We use $\epsilon = 10^{-34}$ as a tolerance error. The following stopping criteria are used for computer programs:

$$i) |x_{n+1} - x_n| < \epsilon \text{ and } (ii) |f(x_{n+1})| < \epsilon.$$
### Table I
#### Test Problems

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>LG.</th>
<th>SG.</th>
<th>WM</th>
<th>K1</th>
<th>K2</th>
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<th>OM3</th>
<th>OM4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1 (x) = xe^{-x} - \sin x^2 + 3 \cos x + 5$</td>
<td>1.0</td>
<td>5.996</td>
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### Table II
#### Comparison of Different Sixth-Order Methods with the Same Total Number of Functional Evaluations (TNFE=12)

<table>
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### Table III
#### Comparison of Different Sixth-Order Methods with Respect to Number of Iterations

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### Table IV
#### Computational Order of Convergence of Different Sixth-Order Methods

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<th>K2</th>
<th>OM1</th>
<th>OM2</th>
<th>OM3</th>
<th>OM4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1 (x) = xe^{-x} - \sin x^2 + 3 \cos x + 5$</td>
<td>1.0</td>
<td>5.996</td>
<td>5.996</td>
<td>5.996</td>
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</tr>
<tr>
<td>$f_2 (x) = e^{-x} + \sin x$</td>
<td>3.13</td>
<td>5.997</td>
<td>5.997</td>
<td>5.997</td>
<td>6.000</td>
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<tr>
<td>$f_3 (x) = (x - 2)^2 - \log x - 3x^2$</td>
<td>36.98</td>
<td>5.997</td>
<td>5.997</td>
<td>5.997</td>
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</tr>
<tr>
<td>$f_4 (x) = \cos x - x$</td>
<td>0.79</td>
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<tr>
<td>$f_5 (x) = \tan^{-1}(x^2 - x)$</td>
<td>1.00</td>
<td>6.000</td>
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<tr>
<td>$f_6 (x) = e^{-x^2 + x - 1}$</td>
<td>2.00</td>
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REFERENCES
