Abstract—One of the main concerns of miners is to increase the quality level of their products because the mining metals price depends on their quality level; however, increasing the quality level of these products has different costs at different levels of the supply chain. These costs usually increase after extractor level. This paper studies the coordination issue of a decentralized three-level supply chain with one supplier (extractor), one mineral processor and one manufacturer in which the increasing product quality level cost at the processor level is higher than the supplier and at the level of the manufacturer is more than the processor. We identify the optimal product quality level for each supply chain member by designing a revenue sharing contract. Finally, numerical examples show that the designed contract not only increases the final product quality level but also provides a win-win condition for all supply chain members and increases the whole supply chain profit.

Keywords—Three-stage supply chain, product quality improvement, channel coordination, revenue sharing.

I. INTRODUCTION

In today’s globalized economy, supply chain management is one of the most useful management practices for industries to increase their profit and competitiveness. There are two decision making systems in a supply chain: centralized and decentralized. The decision making system in most supply chain models is assumed to be decentralized. To improve overall performance of a supply chain, a coordination mechanism is needed. Different definitions and perspectives on supply chain coordination exist in the literature (refer to [1], [2]) for a comprehensive review of supply chain coordination. A supply chain is coordinated when the players make the decisions that are optimal for the whole supply chain. For coordinating a supply chain, contracts are designed among the decentralized decision makers to reduce the difference between outcome of a centralized decision and a decentralized decision. Different kinds of contracts such as commitment to purchase quantity [3], credit option [4], two-part tariff [5], revenue sharing [6], [7], buy back [8], sales rebate [9], and mail-in-rebate [10], have been used in supply chains as the ways improving supply chain performance.

Revenue sharing is one of the widely used contracts in a supply chain that is between an upper and lower level of supply chains, in which the upper level provides better selling condition to the lower level and then lower level shares a fraction of its revenue with the upper level.

Nowadays quality is one of the key competitive dimensions of industries, and quality management is one of the most important aspects in operations management. Industries are always trying to increase their profits, and mines are one of these industries but their profit enhancement has an extreme relation with their product’s quality. There are two common practices for improving product quality in industries: 1- Technology changing and 2- Practical policies. Therefore, one of the most important concerns of miners is to increase their profit through product quality level enhancement by operational approaches.

There are two streams of literatures related to the research in this paper. The first stream focuses on supply chain coordination under revenue sharing contract. Reference [11] provided a good survey on this contract. Reference [12] has proven that revenue sharing contracts for decentralized supply chains are beneficial in achieving coordination for various types of supply chains. Reference [13] used Stackelberg game to model the revenue sharing contract problem. They showed that the party that keeps more than half the revenue should serve as the leader of the Stackelberg game. Reference [14] presented a new revenue sharing contract embedding corporate social responsibility to coordinate a two level supply chain. Reference [15] proposed a revenue sharing contract to coordinate a two-stage supply chain with one manufacturer and two competing retailers. They illustrated that the provision of revenue sharing in the contract can increase supply chain performance than a price-only contract. Reference [16] proposed a game theory model for revenue-dependent revenue sharing contracts in which the supply chain revenue is shared among the members depends on the quantum of revenue generated. Reference [17] discussed on revenue sharing contracts for coordination in a supply chain with one manufacturer and two competing retailers in which demands are disrupted. They showed that it is necessary to adjust the original revenue-sharing contracts to demand disruptions. Reference [18] modeled the supply chain with revenue sharing contract and service requirement under supply and demand uncertainty. They found that in the coordinated supply chain under supply and demand uncertainty, the revenue sharing ratio for the supplier will be higher if the wholesale price...
remains the same, or the wholesale price will be higher if the revenue sharing ratio for the supplier keeps the same. Reference [19] studied supply chain coordination via revenue sharing contracts in two different supply chain structures. First, for a three-echelon supply chain with a loss-averse retailer, a loss-neutral distributor, and a loss-neutral manufacturer and second, for a two-echelon supply chain consisting of a loss-averse retailer and a loss-neutral distributor.

Another subset of literature related to this research is improving the quality level of products in a supply chain. Many efforts have been made to improve product quality in supply chains.

Reference [20] studied a single product distribution channel and proposed a mutually beneficial contract that simultaneously increases profit and improves quality. Reference [21] proposed a setting in which the manufacturer’s quality improvement effort affects the final products quality and developed two new cost-sharing contracts to coordinate quality improvement efforts of two members. Reference [22] presented a game-theory model to show how the manufacturer coordinates the supply chain with a quality assurance policy by revenue-sharing contract. Reference [23] discussed the potential coordinating power of revenue-sharing contracts in supply quality management. Reference [24] explored a joint pricing and product quality decision problem in a two level decentralized supply chain and compared different contract formats for this kind of supply chain. Reference [25] investigated a supply chain with a buyer and a supplier, where the buyer has the option to invest in the supplier’s quality improvement. However, a few researches have been done on the three level supply chain coordination with revenue sharing contract and in the three-level supply chain coordination research literatures mentioned above, all of them do not take the product quality improvement into account, and also none of above articles focus on mining metal supply chain coordination. Therefore, the aim of this study is to design a revenue sharing contract for a three level mining metal supply chain in order to not only coordinate the supply chain and provide win-win condition for all its members but also increase the final product quality level.

The rest of this paper proceeds as follows. In section II, the notations will be defined. Section III presents the supply chain descriptions and assumptions used in this paper. We investigate the decentralized supply chain model without coordination in Section IV. Section V develops a new revenue sharing contract for coordination of a supply chain. Section VI provides numerical examples to illustrate the proposed contract. Conclusions are provided in Section VII.

II. NOTATIONS

The following notations are used to describe the proposed model.

\[ i \] Index for supply chain levels; \( S \) for supplier, \( P \) for processor and \( M \) for manufacturer.

\[ j_0 \] Minimum acceptable product quality level

\[ P_i \] Selling price of unit product produced at the supply chain level \( i \) with quality level \( j \)

\[ d_i \] Increasing percentage of product quality level in supply chain level \( i \)

\[ C_i \] Constant production cost for a unit product in supply chain level \( i \)

\[ c_{d_{ij}} \] Cost coefficient for increasing product quality level with quality level \( j \) in supply chain level \( i \)

\[ \alpha_i \] Price increasing coefficient for product produced in supply chain level \( i \) per unit product quality improvement in supply chain level \( i \)

\[ \beta_i \] Price increasing coefficient for product produced in supply chain level \( i \) per unit product quality improvement in supply chain levels before \( i \)

\[ \gamma_i \] Quality improvement cost increasing coefficient in supply chain level \( i \) per unit product quality improvement in supply chain levels before \( i \)

\[ \pi_i \] Supply chain level \( i \) profit

\[ \phi_1 \] Processor's revenue share, \( 0 < \phi_1 < 1 \)

\[ \phi_2 \] Manufacturer's revenue share, \( 0 < \phi_2 < 1 \)

In this paper \( d_i \), \( \phi_1 \) and \( \phi_2 \) are decision variables.

III. MODEL DESCRIPTION AND ASSUMPTIONS

A decentralized three-stage mining metal supply chain in which minerals will convert to concentrate after extraction is assumed in this paper. The considered supply chain is consisting of a supplier (extractor), a processor, and a manufacturer. The first level extracts minerals and sells them to the second level, which then processes the minerals and sells the mineral concentrate after processing to the manufacturer, which then produces mineral products such as pellets and ingots and sells them to the customer. The price of products produced in each level depends on the quality of those products. Therefore, all of these supply chain members try to increase their product’s quality. Product quality level improvement is not mandatory for supply chain members, but the supplier must supply raw material with a minimum quality level \( j_0 \). Product quality improvement for each member requires more cost but because of different production processes in each level, this cost enhancement is different for each member and it is assumed to be nonlinear ascending.

![Fig. 1 Mining metal three level supply chain](image-url)
The product quality improvement cost increases from supplier to manufacturer due to the increasing complexity of production processes from supplier to manufacturer. It is necessary to say a real assumption that in this type of supply chain, increasing the quality of the product at each supply chain level creates added value for both that level (with coefficient $\alpha$) and next levels (with coefficient $\beta$), but increases the next levels product quality improvement costs (with coefficient $\gamma$); for example, the cost for increasing five quality levels from level 50 to level 55 is more than the cost for increasing five quality levels from level 30 to level 35.

In our study, we assume that the supply chain is asymmetric and its members do not share their cost information among themselves. All members are in full capacity production and all their products will be sold, therefore, consideration of the demand parameter in the problem is neglected. Also, the shipment costs are not considered in this model due to equality in coordinated and non-coordinated mode.

IV. DECENTRALIZED SUPPLY CHAIN WITHOUT COORDINATION

We assume that all members in the considered supply chain try to improve their products quality, but in a decentralized supply chain they try to maximize their own profit. Considering the model assumptions, the supplier's profit for each unit product extraction is:

$$\pi_S = P_{g_S} (1 + \alpha_S d_S) - C_S - cd_{g_S} d_S^2 \ (1)$$

where the first part denotes a unit extracted material selling price with minimum quality level plus selling price enhancement due to product quality level improvement by supplier ($d_S$). The second part is constant extraction cost for a unit product in supplier level. Similar to previous studies [26], [27], the third part shows the supplier’s cost for increasing the product quality level.

The processor’s profit for processing one unit product is:

$$\pi_P = P_{g_P} (1 + \beta_P d_P + \alpha_P d_P) - P_{g_S} (1 + \alpha_S d_S)$$

$$- C_P - cd_{g_P} d_P^2 (1 + \gamma_P d_P) \ (2)$$

where similar to (1), the first part shows a unit processed material selling price with minimum quality level plus selling price enhancement due to product quality level improvement by processor ($d_P$) and supplier ($d_S$). The second part is the purchasing price of a unit extracted product from the supplier. The third part is the constant processing cost for a unit product in a processor and the fourth part shows the processor’s cost for increasing the product quality level.

The manufacturer’s profit for manufacturing one unit product is:

$$\pi_M = P_{g_M} (1 + \beta_M (d_S + d_P) + \alpha_M d_M) - C_M$$

$$- P_{g_S} (1 + \beta_P d_S + \alpha_P d_P) - cd_{g_M} d_M^2 (1 + \gamma_M (d_S + d_M)) \ (3)$$

where the first part represents a unit manufactured product selling price with minimum quality level plus selling price enhancement due to product quality level improvement by manufacturer ($d_M$) and its previous levels ($d_S + d_P$). The second part is the purchasing price of a unit processed product from the processor. The third part is the constant manufacturing cost for a unit product in the manufacturer level. The fourth part shows the processor’s cost for increasing the product quality level.

As mentioned before, all members in a decentralized supply chain try to maximize their profit, so the members’ optimal decisions will be as follows:

**Proposition 1.** The optimum increasing percentage of product quality level by the supplier in considered decentralized supply chain is:

$$d_S^* = \frac{\alpha_S P_{g_S}}{2cd_{g_S}} \ (4)$$

**Proof.** Since the $\pi_S$ is concave in $d_S$, there exists a unique optimal product quality level improvement $d_S$ that maximizes supplier’s profit because second derivative of equation $\pi_S$ is negative.

$$\frac{\partial \pi_S}{\partial d_S^2} = -2cd_{g_S} < 0$$

Therefore, the optimum amount of $d_S$ can be obtained as:

$$\frac{\partial \pi_S}{\partial d_S} = 0 \rightarrow \alpha_S P_{g_S} - 2cd_{g_S} d_S = 0 \rightarrow d_S^* = \frac{\alpha_S P_{g_S}}{2cd_{g_S}}$$

This completes the proof.

**Proposition 2.** The optimal percentage of product quality level improvement by the processor in considered decentralized supply chain is:

$$d_P^* = \frac{\alpha_P P_{g_P}}{2cd_{g_P} (1 + \gamma_P d_S^*)} \ (5)$$

**Proof.** Since the $\pi_P$ is concave in $d_P$, there exists a unique optimal product quality level improvement $d_P$ that maximizes the processor's profit because the second derivative of function $\pi_P$ is negative.
\[
\frac{\partial \pi_p}{\partial d_p} = -2cd_{p_0}(1 + \gamma_p d_s) < 0
\]

Therefore, the optimum \( d_p \) can be obtained as follows:

\[
d_p^* = \frac{\alpha_P P_{Mr}}{2cd_{p_0}(1 + \gamma_p d_s)}
\]

This completes the proof.

**Proposition 3.** The optimal percentage of product quality level improvement by the manufacturer in considered a decentralized supply chain is:

\[
d_M^* = \frac{\alpha_M P_{Mj}}{2cd_{p_0}(1 + \gamma_M (d_s + d_p))}
\]

**Proof.** Since the \( \pi_M \) is concave in \( d_M \), there exists a unique optimal product quality level improvement \( d_M \) that maximizes the manufacturer's profit because the second derivative of function \( \pi_M \) is negative.

\[
\frac{\partial^2 \pi_M}{\partial d_M^2} = -2cd_{Mj}(1 + \gamma_M (d_s + d_p)) < 0
\]

Therefore, the optimal value of \( d_M \) can be obtained as:

\[
\frac{\partial \pi_p}{\partial d_p} = 0 \rightarrow \alpha_M P_{Mj} - 2cd_{Mj} d_M(1 + \gamma_M (d_s + d_p)) = 0
\]

\[
\rightarrow d_M^* = \frac{\alpha_M P_{Mj}}{2cd_{Mj}(1 + \gamma_M (d_s + d_p))}
\]

This completes the proof.

Therefore, the optimal value of total supply chain profit without coordination can be written as:

\[
\pi^* = \pi_S^* + \pi_P^* + \pi_M^*
\]

And the product quality improvement percentage for the final product without coordination can be calculated as:

\[
d_T^* = d_S^* + d_P^* + d_M^*
\]

V. SUPPLY CHAIN COORDINATION WITH REVENUE SHARING

Since increasing the quality level of the product by the supplier increases the processor and manufacturer's profit, they share a portion of this profit enhancement with the supplier. Based on the designed revenue sharing contract, whenever the supplier increases the quality of his product \( (d_s') \) more, he will receive more profit from the processor. Considering this revenue sharing contract, the supplier's profit for each unit product extraction is:

\[
\pi_s' = \phi_P P_{Mj} \beta_s d_s^* + \phi_P \phi_P P_{Mj} \beta_s d_s' - C_s
\]

\[
+ P_{Mj} (1 + \alpha_s d_s') - cd_{s'}^2
\]

where the first and second parts show a portion of the processor's profits which the supplier received from the processor due to increasing the product quality improvement \( (d_s') \). It is clearly understandable that if a supplier does not increase his product quality level, he will receive no shared profit from the processor. The other parts of (9) are similar to (1).

According to the presented revenue sharing contract, when the processor delivers product with higher quality to the manufacturer, he will share his profit more with the processor. But some percent of this product quality improvement is done by supplier and the rest of product quality improvement is done by processor, therefore processor shares a portion of the profit received from the manufacturer which is related to the supplier's product quality improvement with the supplier. Hence, considering the above contract descriptions, the processor’s profit for processing one unit product under the proposed revenue sharing contract is:

\[
\pi_p' = \phi_P P_{Mj} \beta_s (1 - \phi_s) d_s + d_p - C_p
\]

\[
+ P_{Mj} (1 + (1 - \phi_s) \beta_s d_s + \alpha_s d_p) - P_{Mj} (1 + \alpha_s d_s)
\]

\[
- cd_{p_0} (1 + \gamma_p d_s^*)
\]

where the first part is a portion of the manufacturer's profit which the processor received from the manufacturer minus a part of it that the processor gives to the supplier. The second part denotes a processed unit material selling price with minimum quality level plus selling price enhancement due to product quality level improvement by processor \( (d_p') \) and supplier \( (d_s') \), minus a part of it that the processor gives to the supplier proportionate to \( (d_s') \). Based on the designed contract, even if the processor does not like to increase his product quality, it is beneficial for him to motivate the supplier to improve product quality. The other parts of (10) are similar to (2).

According to the proposed contract, a manufacturer will share a part of his profit resulting from product quality improvement in the previous supply chain levels with the processor, so the manufacturer’s profit for manufacturing one unit product based on the presented contract can be written as:
\[ \pi'_M = P_{M_0} (1 + (1 - \phi_2)\beta_M (d_S + d_P) + \alpha_M d_M) - C_M - P_{M_0} (1 + \beta_P d_S + \alpha_P d_P) - cd_{M_0} d_M^2 (1 + \gamma_M (d_S + d_M)) \]  

(11)

where the first part shows a unit manufactured product selling price with minimum quality level plus selling price enhancement because of product quality level improvement by the manufacturer \((d'_M)\) and its previous levels \((d'_S + d'_P)\) minus a part of it that manufacturer gives to processor proportionate to \((d'_S + d'_P)\). The other parts of (11) are similar to (3).

After considering the proposed revenue sharing contract in the supply chain, again all members try to maximize their own profit due to the decentralization of the supply chain. Therefore, the members’ optimal decisions can be written as follows:

**Proposition 4.** The optimum increasing percentage of product quality level by the supplier after considering the revenue sharing contract in supply chain will be:

\[ d'_S = \frac{\phi_1 \beta_P P_{M_0} + \phi_2 \beta_M P_{M_0} + \alpha_S P_{S_0}}{2cd_{S_0}} \]  

(12)

**Proof.** \( \pi'_S \) is concave in \( d'_S \) and there exists a unique optimal product quality level improvement \( d'_S \) that maximizes supplier’s profit because the second derivative of equation \( \pi'_S \) is negative.

\[ \frac{\partial \pi'_S}{\partial d'_S} = -2cd_{S_0} < 0 \]

Therefore, the optimal \( d'_S \) can be obtained as follows:

\[ \frac{\partial \pi'_S}{\partial d'_S} = 0 \rightarrow \phi_1 \beta_P P_{M_0} + \phi_2 \beta_M P_{M_0} + \alpha_S P_{S_0} = 0 \]

\[ -2cd_{S_0} d'_S = 0 \]

\[ \rightarrow d'_S = \frac{\phi_1 \beta_P P_{M_0} + \phi_2 \beta_M P_{M_0} + \alpha_S P_{S_0}}{2cd_{S_0}} \]

This completes the proof.

**Proposition 5.** The optimal percentage of product quality level improvement by the processor in an assumed decentralized supply chain after considering revenue sharing contract is:

\[ d'_P = \frac{\phi_1 \beta_M P_{M_0} + \alpha_P P_{M_0}}{2cd_{M_0} (1 + \gamma_P d'_S)} \]  

(13)

**Proof.** \( \pi'_p \) is concave in \( d'_p \) and there exists a unique optimal product quality level improvement \( d'_p \) that maximizes the processor’s profit because the second derivative of function \( \pi'_p \) is negative.

\[ \frac{\partial \pi'_p}{\partial d'_p} = -2cd_{M_0} (1 + \gamma_P d'_S) < 0 \]

Therefore, the optimum \( d'_p \) can be obtained as follows:

\[ \frac{\partial \pi'_p}{\partial d'_p} = 0 \rightarrow \phi_2 \beta_M P_{M_0} + \alpha_P P_{M_0} - 2cd_{M_0} d'_p (1 + \gamma_P d'_S) = 0 \]

\[ \rightarrow d'_p = \frac{\phi_2 \beta_M P_{M_0} + \alpha_P P_{M_0}}{2cd_{M_0} (1 + \gamma_P d'_S)} \]

This completes the proof.

**Proposition 6.** The optimal percentage of product quality level improvement by the manufacturer in the considered decentralized supply chain based on the designed contract is:

\[ d'_M = \frac{\alpha_M P_{M_0}}{2cd_{M_0} (1 + \gamma_M (d'_S + d'_P))} \]  

(14)

**Proof.** Completely similar to the proof of proposition 3. Therefore, the optimal value of total supply chain profit without coordination can be written as:

\[ \pi'_T = \pi'_S + \pi'_P + \pi'_M \]  

(15)

And optimal product quality improvement percentage for final product can be calculated as follows:

\[ d'_T = d'_S + d'_P + d'_M \]  

(16)

The other decision variables are \( \phi_1 \) and \( \phi_2 \), that we obtain their optimal values by numerical examples due to model complexity.

**VI. NUMERICAL EXAMPLES**

In order to illustrate that the \( \pi'_p \) function is concave in \( \phi_i \) and the \( \pi'_M \) function is concave in \( \phi_2 \), we give an example by using the parameters: \( C_s = 150; \ C_p = 250; \ C_M = 350; \ P_{M_0} = 20; \ P_{M_0} = 30; \ P_{P_0} = 40; \ \alpha_s = 3; \ \alpha_p = 5; \ \alpha_M = 7; \ \beta_p = 6; \ \beta_M = 20; \ \gamma_p = \gamma_M = 10; \ cd_{S_0} = 4; \ cd_{P_0} = 5; \ cd_{M_0} = 6. \) But first of all we have to obtain the optimal value of \( \phi_2 \) and then we can calculate the optimal value of \( \phi_1 \).
Fig. 2 Supply chain members profit for different values of $\phi_2$

In Fig. 2, the green, red, blue and black curves show the total, supplier, processor and manufacturer's profits, respectively, and it can be seen that the manufacturer’s function $\pi_M$ is concave in $\phi_2$ and the optimal value of $\phi_2$ using the assumed parameters values is 0.2002, which is recognizable by a blue line. Now, using the calculated $\phi_2$, we can obtain $\phi_1$. As can be seen in Fig. 3, the processor’s function $\pi_P$ is concave in $\phi_1$ (the blue curve) and the optimal value of $\phi_2$ using assumed parameters values is 0.3220, which is recognizable by the blue line.

A numerical example considering the mentioned parameters using MATLAB software is done and the results are presented in Table I.

It can be observed from the results presented in Table I that the designed contract cannot only increase all supply chain members and total supply chain's profit, but also increase the final product quality level. Also, since the profit of all supply chain members has increased, we can claim that the proposed channel coordination provides a win-win condition for all supply chain members.

**TABLE I**

<table>
<thead>
<tr>
<th>Coordination Variables mechanism</th>
<th>$d_S^*$</th>
<th>$d_P^*$</th>
<th>$d_M^*$</th>
<th>$d_T^*$</th>
<th>$\pi_S^*$</th>
<th>$\pi_P^*$</th>
<th>$\pi_M^*$</th>
<th>$\pi_T^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without coordination</td>
<td>7.500</td>
<td>0.197</td>
<td>0.299</td>
<td>7.996</td>
<td>95</td>
<td>674.80</td>
<td>4480.18</td>
<td>5249.98</td>
</tr>
<tr>
<td>Coordination with revenue sharing contract</td>
<td>21.233</td>
<td>0.145</td>
<td>0.108</td>
<td>21.486</td>
<td>1673.52</td>
<td>3398.44</td>
<td>9510.64</td>
<td>14582.60</td>
</tr>
<tr>
<td>Changes percentage</td>
<td>183.17%</td>
<td>-26.4%</td>
<td>-63.8%</td>
<td>1661.6%</td>
<td>112.2%</td>
<td>177.7%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

VII. CONCLUSIONS

Mine industries like other industries try to increase their profit and keep their customers in competitive market. Product quality improvement is the miner's most important key to survive in competitive market. The two main practices for increasing product quality in industries are technological change and practical policies; thus, one of the main concerns of miners is to increase profit through product quality level improvements via operational approaches. This paper studied the coordination issue of a decentralized three-level supply chain with one supplier, one processor and one manufacturer. Due to different product quality improvement costs of supply chain members, a revenue sharing contract is designed and the optimal product quality level for each of them obtained. Finally, the numerical examples illustrate that the proposed revenue sharing contract either increases the final product quality level or provides a win-win condition for all supply chain members and increases the whole supply chain profit.

The author's suggestions for future researches include considering the quality dependent demand and environmental impacts for increasing product quality in the model.

REFERENCES


