Fuzzy Set Approach to Study Appositives and Its Impact Due to Positional Alterations

E. Mike Dison, T. Pathinathan

Abstract—Computing with Words (CWW) and Possibilistic Relational Universal Fuzzy (PRUF) are the two concepts which widely represent and measure the vaguely defined natural phenomenon. In this paper, we study the positional alteration of the phrases by which the impact of a natural language proposition gets affected and/or modified. We observe the gradations due to sensitivity/feeling of a statement towards the positional alterations. We derive the classification and modification of the meaning of words due to the positional alteration. We present the results with reference to set theoretic interpretations.

Keywords—Appositive, computing with words, PRUF, semantic sentiment analysis, set theoretic interpretations.

I. INTRODUCTION

In many circumstances, human judgments are influenced by the way that they perceive things. Human perception is reflected in rational decisions. Zadeh introduced the theory of fuzzy sets [15] that the modern day computer systems understand. Fuzzy sets provide a fundamental and concise theoretical framework for processing rational subjectivities. Rational subjectivities very often have the structure of fuzzy natural language propositions. The theory of CWW [7], [17] was developed in order to quantify the ill-defined subjective natural language proposition. In addition to that, concepts such as type-2 fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets, fuzzy multisets and fuzzy numbers are also employed to represent the imprecision. The natural language proposition comprises of linguistic variables along with the other basic units of sentence. A linguistic variable plays an important role in providing different sets of meaning for the different users. Different meanings of a word are obtained from the database called WordNet. WordNet as a lexical source acts as a knowledge database where the word and its classification of meaning are elaborately stored. Hedges are used to modify the linguistic variables which primarily refine the intensity of the meaning into different class [16].

Earlier it was linguistic variables, which modified the meaning of the natural language statements. Later, Zadeh extensively studied the linguistic hedges and its applications in the year 1972. The change in meaning based on the modifier by associating adjectives as hedges into word was extensively studied by Zadeh [16]. Later, he developed two categories of hedges such as; (i) very, more or less, much, essentially, slightly, etc (ii) technically, essentially, practically, etc. [16].

The above two categories of hedges had been applied in various real life situations on controlling fuzzy inference system. In this paper, we propose a new fuzzy set approach to study the positional alternations with opposite words associated together.

Pathinathan and Ponnivalavan [8] introduced pentagonal fuzzy number in the year 2014. Also, they studied reverse order pentagonal fuzzy number in the year 2015. Further, Pathinathan et al. [13] extended the generalized form of the pentagonal fuzzy number with geometrical illustration. Later on, Pathinathan and Rajkumar [10] used pentagonal fuzzy number to sieve out the poor in Nalanda District, Patna, India. Also, Pathinathan et al. [11] applied pentagonal fuzzy number to study the farmers’ issues on choosing best suitable crop for cultivation in Villupuram District, Tamil Nadu, India. In this paper, we introduce a new type of non-linear pentagonal fuzzy number and upside down non-linear pentagonal fuzzy number to study the positional alteration among the phrases. Also, in this paper, we employ an unambiguous function introduced by Piegat and Plucinski [14] to capture the relative measure among the different $\alpha - cut$ levels.

Through this article, we discuss the concept of positional alteration among the phrases which adds weightage on the words to have different meaning. More specifically, the concept of positional alteration commonly referred as appositives, is hard to quantify. The change in the meaning is highly influenced by the words associated with the other words. The co-occurrence of words has influence over the meanings. Sometimes even the words having opposite meaning will occur together and lead to a new meaning. Through this paper, we have tried to establish a new type of fuzzy relationship between two linguistic variables which may have contradictory implications inherent in them. Throughout this paper, we adopt $A$ notation for representing the fuzzy set, whereas in our papers [7]-[13], we have used $\sim$ below the letter A.

The paper is organized as follows. Section II provides the basic preliminaries on semantic ordering relation and appositives. Section III gives a detailed account of linear pentagonal fuzzy number with illustration. Section IV introduces new type of fuzzy numbers, namely non-linear pentagonal fuzzy number and upside down non-linear pentagonal fuzzy number along with the geometrical representation. Section V discusses the new combined non-linear pentagonal fuzzy number along with the concepts of $\alpha - cut$ representation, center of gravity, succession function and algebraic sum for the newly introduced non-linear and...
upside down non-linear pentagonal fuzzy number. Section 6 provides a new fuzzy set approach to study the linguistic variables which has opposite meaning due to positional alterations. Section VII presents an illustrative example for the newly proposed fuzzy set approach and followed by conclusion in Section VIII.

II. BASIC PRELIMINARIES

A. Appositives

An appositive is a noun or noun phrase that has another noun associated with it to create a new identity in a different way.
P1: My brother, a friend to all young people
P2: A boy who is friendly with all young people, is my brother

Proposition P1 implies brother gets more weightage than the relation he has with all young people, whereas, in proposition P2, brother gets less importance and the relation between him and other young people gets more importance. By considering the above two situations, both the nouns get difference in importance when their positions gets altered.

B. Lexical Association [4]

In some cases, the succession (association) of two words does not make any sense, e.g. road chair. In this example, the succession (association) of two words road and chair does not make any sense. Proper lexical associations have been made by Nastase et al. [4], and Table I shows the various lexical group associations and its description.

<table>
<thead>
<tr>
<th>Description</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>H causes M</td>
<td>Flu virus</td>
</tr>
<tr>
<td>H is the effect by M</td>
<td>Exam anxiety</td>
</tr>
<tr>
<td>H is for M</td>
<td>Concert hall</td>
</tr>
<tr>
<td>M performs H</td>
<td>Student protest</td>
</tr>
<tr>
<td>M is acted upon by H</td>
<td>Metal separator</td>
</tr>
<tr>
<td>M benefits from H</td>
<td>Student discount</td>
</tr>
<tr>
<td>H is directed towards M</td>
<td>Outgoing mail</td>
</tr>
<tr>
<td>H is the location of M</td>
<td>Home town</td>
</tr>
<tr>
<td>H is located at M</td>
<td>Desert storm</td>
</tr>
<tr>
<td>H occurs every time M occurs</td>
<td>Weekly game</td>
</tr>
<tr>
<td>H occurs when M occurs</td>
<td>Morning coffee</td>
</tr>
<tr>
<td>H existed while M existed</td>
<td>2-hour trip</td>
</tr>
<tr>
<td>H occurs as indicated by M</td>
<td>Stylish writing</td>
</tr>
<tr>
<td>H is made of M</td>
<td>Brick house</td>
</tr>
<tr>
<td>M is a measure of H</td>
<td>Heavy rock</td>
</tr>
</tbody>
</table>

In Table I, H represents the head noun and M represents the modifier which modifies the meaning of the phrase.

C. Semantics [2], [5], [6], [12]

Semantics is the study of meaning and its changes from one context to other. It focuses on the relation between signifiers like words, phrases, signs and symbols based on their denotation.

D. Linguistic Semantics [2], [5], [6], [12]

Linguistic semantics is the study of meaning that is used for understanding human expressions through natural language.

E. Semantically Ordering Relation [2], [5], [6], [12]

Partially ordering relation ≤ induced by the meaning of hedge called semantically ordering relation. It orders the elements of the algebra based on semantic characteristics of linguistic hedges. Elements of these algebras can be regarded as just linguistic terms and their relative meaning can be expressed in terms of the semantically ordering relation.

F. Zadeh’s Fuzzy Set [15]

A fuzzy set \( A \) of \( X \) is defined as the mapping \( \mu_A : X \rightarrow [0,1] \) and represented by the following pair as,

\[ A = \left\{ (x, \mu_A(x)) / x \in X \right\} \]  

where, \( x \) is an element in \( X \) and \( \mu_A(x) \) is a membership grade of the element \( x \) in \( X \).

G. Fuzzy Arithmetic [1]

In fuzzy set theory, the relation between two lexical terms has been described by interpreting them as a real valued function. Suppose that the linguistic proposition such as ‘x is close to y’ shows the information that x is a value which is very near to the other value y. Triangular, trapezoidal and pentagonal fuzzy numbers are the common type of fuzzy numbers to represent the linguistic impreciseness. The other vague notions which can be interpreted as fuzzy numbers are as follows; x and y are near each other, x is much smaller than y, x is much greater than y, x is relevant to y, x and y are almost equal, x and y are very far and x considerably larger than y, etc.

III. LINEAR PENTAGONAL FUZZY NUMBER

Let \( \tilde{A}_p = (a_1, a_2, a_3, a_4, a_5) \) be a pentagonal fuzzy number where \( a_1, a_2, a_3, a_4 \) and \( a_5 \) are the real numbers with \( a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \) and the membership function is defined as follows,

\[
\mu_p(x) = \begin{cases} 
0 & ; \text{ for } x < a_1 \text{ and } x > a_5 \\
\frac{x-a_1}{a_2-a_1} & ; \text{ for } a_1 \leq x \leq a_2 \\
\frac{x-a_2}{a_3-a_2} & ; \text{ for } a_2 \leq x \leq a_3 \\
1 & ; \text{ for } x = a_3 \\
\frac{a_5-x}{a_5-a_4} & ; \text{ for } a_4 \leq x \leq a_5 \\
\frac{a_5-x}{a_5-a_4} & ; \text{ for } a_4 \leq x \leq a_5 
\end{cases}
\]  

In (2), the membership function from \( a_1 \) to \( a_2 \) is a strictly
increasing continuous function and the membership function from \( a_1 \) to \( a_5 \) is also a strictly monotonic increasing continuous function with the variation in the \( \alpha - \) level. In general, the pentagonal fuzzy number takes 0.5 as its \( \alpha - \) level. Similarly, the membership function from \( a_5 \) to \( a_1 \) shows a strictly decreasing continuous function and the membership function from \( a_5 \) to \( a_1 \) shows the strictly decreasing continuous function with variation in the \( \alpha - \) level. The geometrical representation of the pentagonal fuzzy number is given by,

\[
\mu_{A_5}(x) = \begin{cases} 
\left(1 - \frac{(x-a+1)^2}{2m^2}\right) ; & \text{for } a-m+n \leq x \leq a-m \\
\left(1 - \frac{(x-a)^2}{2m^2}\right) ; & \text{for } a-m \leq x \leq a \\
1 - \frac{(a-x)^2}{2m^2} ; & \text{for } a \leq x \leq a+m \\
\left(1 - \frac{(a+m+n)-x)^2}{2n^2}\right) ; & \text{for } a+m \leq x \leq a+m+n 
\end{cases}
\]

In (39), the interval \([a-m-n,a-m]\) has a smooth continuous non-linear increasing function and the interval \([a-m,a]\) has smooth continuous non-linear increasing function with the variations in the \( \alpha - \) level 0.5. Whereas, the interval \([a,a+m]\) and \([a+m,a+m+n]\) represents smooth continuous non-linear decreasing functions with the variations in the \( \alpha - \) level 0.5. The geometrical representation of the non-linear pentagonal fuzzy number is given by,

IV. NON-LINEAR PENTAGONAL FUZZY NUMBER

This section introduces two types of non-linear pentagonal fuzzy number with geometrical illustration. Also, this section provides the new combined form of a non-linear pentagonal fuzzy number, where the two opposites are studied simultaneously. The new combined non-linear pentagonal fuzzy number establishes a new type of fuzzy relationship between two linguistic variables which may have contradictory implications inherent in them.

A. Non-Linear Pentagonal Fuzzy Number

Linear pentagonal fuzzy number was introduced by Pathinathan and Ponnivalavan in 2014 [8]. In linear pentagonal fuzzy number, the functions defined over the intervals are of linear type increasing continuous curves. The linear type curve shows the strictly increasing nature of the continuous curves which capture the impreciseness of the linguistic variable to a certain extent, whereas, in real life the linguistic variables (subjective opinions) are more complex, and the linear type of function is not sufficient to capture the impreciseness.

Let \( \tilde{A}_p = (a-m-n,a-m,a,a+m,a+m+n) \) be a non-linear pentagonal fuzzy number with ‘a’ be the centre and spreads into both the left and right side by the distance \( m+n \). Also, the membership function defined over the interval \([a-m-n,a+m+n]\) is a real valued function with \( a-m-n \leq x \leq a \leq a+m \leq a+m+n \). Then, the membership function of a non-linear pentagonal fuzzy number is given by,

\[
\mu_{\tilde{A}_p}(x) = \begin{cases} 
1 - \frac{(x-(a-m-n))^2}{2n^2} ; & \text{for } b-m-n \leq x \leq b-m \\
1 - \frac{(x-b)^2}{2m^2} ; & \text{for } b-m \leq x \leq b \\
0 ; & \text{for } x = b \\
1 - \frac{(b-x^2)}{2m^2} ; & \text{for } b \leq x \leq b+m \\
1 - \frac{(b+m+n)-x)^2}{2n^2} ; & \text{for } b+m \leq x \leq b+m+n 
\end{cases}
\]

where, \( b-m-n \leq b-m \leq b+m \leq b+m+n \) and \( b \) which is a real number with ‘m+n’ represents distance between the left spread and right spread. The geometrical representation of the upside down non-linear pentagonal fuzzy number is illustrated as follows:
V. COMBINED NON-LINEAR PENTAGONAL FUZZY NUMBERS

This section introduces the new combined non-linear pentagonal fuzzy number to study the phrases with contradictory words. The concept of α-cut representation of the newly introduced non-linear and upside down non-linear pentagonal is discussed along with the succession function.

Let \( A_p = [A_{pL}, A_{pR}] \) be defined as a new combined non-linear pentagonal fuzzy number with \( A_{pL} \) and \( A_{pR} \) represents the non-linear pentagonal fuzzy number and upside down non-linear pentagonal fuzzy number respectively.

\[
A_{pL} = (a - m - n, a - m, a, a + m, a + m + n) \\
A_{pR} = (b - m - n, b - m, b, b + m, b + m + n)
\]

The α-cut interval representation of the non-linear pentagonal fuzzy number in the region below the α-cut value 0.5 is given by,

\[
[x_{IL}, x_{IR}] = [\sqrt{2\mu - (a - m - n), (a + m + n) - \sqrt{2\mu}] 
\]

Similarly, the α-cut interval representation of the upside down non-linear pentagonal fuzzy number in the region above the α-cut value 0.5 is given by,

\[
[x_{IL}, x_{IR}] = [b + \sqrt{2\mu}, b - \sqrt{2\mu}] 
\]

Fig. 3 Upside Down Non-linear Pentagonal Fuzzy Number

Fig. 5 shows the geometrical illustration of the α-cut representation of the non-linear pentagonal fuzzy number and the upside down non-linear pentagonal fuzzy number.

Similarly, the α-cut representation of the upside down non-linear pentagonal fuzzy number by solving the respective quadratic polynomial in (4) is given by,

\[
X_{IL} = a + \sqrt{1 - 2m^2\mu} \\
X_{IR} = a - \sqrt{1 - 2m^2\mu} \\
X_{IL} = (a + m + n) - \sqrt{2n^2\mu} \\
X_{IR} = (a + m + n) - \sqrt{2n^2\mu}
\]
B. Center of Gravity

The center of gravity of a non-linear pentagonal fuzzy number is calculated by using the formula,

$$x^* = \frac{\int_{a}^{b} x \mu_{\alpha}(x) dx}{\int_{a}^{b} \mu_{\alpha}(x) dx} = a + \frac{(m+n) + (3m^2 + m^2 + n^2)}{(m+2n)(m+n)}$$

(17)

Similarly, the center of gravity of an upside down non-linear pentagonal fuzzy number is calculated by,

$$x^* = b$$

(18)

C. Succession Function

The relative distance measure of both below and above region of the $\alpha-cut$ value 0.5 is calculated by using the formula,

$$d = X_h - X_l$$

(19)

defined by the function,

$$y = f(\mu, x^*)$$

(20)

$$y = X_l + (X_h - X_l)(x^*)$$

(21)

where, $X_l$ represents the $\alpha-cut$ value of a quadratic curve spreads on the left of the core and below the region $\mu_{\alpha}(x) = 0.5$, $X_h$ represents the $\alpha-cut$ value of a quadratic curve spreads on the right side of the core and above the region $\mu_{\alpha}(x) = 0.5$ and $x^*$ is the center of gravity of the non-linear pentagonal fuzzy number.

D. Algebraic Sum [3]

Algebraic sum between two fuzzy sets $\tilde{A}$ and $\tilde{B}$ is denoted by,

$$\tilde{A} \oplus \tilde{B},$$

and the membership function is calculated as follows:

$$\mu_{\tilde{A} \oplus \tilde{B}}(x) = \mu_\tilde{A}(x) + \mu_\tilde{B}(x) - \mu_\tilde{A}(x)\mu_\tilde{B}(x)$$

(23)

VI. PROPOSED METHODOLOGY

This section introduces a new fuzzy set approach to examine two words which has opposite implications, but puts together to generate a new meaning word of greater aesthetic joy. The proposed methodology adopts non-linear pentagonal fuzzy number and upside down non-linear pentagonal fuzzy number to represent the two opposite implications.

Step 1. Define positive space and negative space of the two respective words which have opposite meanings.

Step 2. Form cluster centers of the two words based on their synonyms.

Step 3. Verify the semantic properties of the cluster centers with the individual terms present in each cluster.

Step 4. Deduce the subsethood measures of each cluster and their individual terms. Grade them in an order.

Step 5. Represent each cluster center as a real valued fuzzy function with grades of membership value, i.e., elements in non-linear and upside down non-linear pentagonal fuzzy number.

Step 6. Calculate $\alpha-cut$ value between each and every two cluster centers by using (5)-(12).

Step 7. Measure the relative distance between each cluster center based on the function defined in (21).

Step 8. By using algebraic sum, the membership value of combined words has been found.

VII. AN ILLUSTRATIVE EXAMPLE

Let us consider an example which represents the contradictory association of two words with positional alteration.

**அழகான ராட்சேய** (read as Azhagana Raatchasiye) -
Beautiful giant / Beautiful ugly lady

Giant / Beauty / Ugly beauty

Let X and Y be the two words with opposite meaning and X and Y has the following semantic meanings.

\[ X = \{x_1, x_2, x_3, x_4, x_5, ..., x_{10}\} \] (24)

Beautiful = \{attractive, pretty, handsome, good-looking, alluring, prepossessing, lovely, charming, delightful, appealing, engaging, winsome, ravishing, gorgeous, stunning, arresting, glamorous, bewitching, beguiling, graceful, elegant, exquisite, decorative, magnificent, beauteous, comely, fair, dazzling, opulent, extremely beautiful, admirable, enchanting\}

\[ Y = \{y_1, y_2, y_3, y_4, y_5, ..., y_{27}\} \] (25)

Giant = \{great, stupendous, huge, enormous, massive, prodigious, colossal, abhorrent, humongous, gigantic, elephantine, jumbo, bumper, whooping, ginormous, mountainous, abominable, unnatural, whaling, substantial, outrageous, monstrous, grotesque, appalling, heinous, wicked, terrible\}

The semantic ordering among the synonyms has been done on clustering the whole set of meanings into five different clusters. Based on the semantic properties, the five clusters are defined separately with the group of synonyms. The following ordering represents the semantic relation among synonyms of the word ‘beautiful’. Also, the five clusters constitute the positive space of the word ‘beautiful’.

Charming (+) < Attractive (+++) < Beautiful (++++) < Gorgeous (++++) < Exquisite (+++++)

### Table II

<table>
<thead>
<tr>
<th>Word</th>
<th>Synonyms based on their semantic properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charming</td>
<td>{beguiling, bewitching, delightful, ravishing, enchanting}</td>
</tr>
<tr>
<td>Attractive</td>
<td>{appealing, alluring, engaging, comely, winsome, decorative, prepossessing}</td>
</tr>
<tr>
<td>Beautiful</td>
<td>{pretty, handsome, good-looking, beauteous, lovely, fair, admirable}</td>
</tr>
<tr>
<td>Gorgeous</td>
<td>{dazzling, elegant, graceful, opulent}</td>
</tr>
<tr>
<td>Exquisite</td>
<td>{delicate, magnificent, stunning}</td>
</tr>
</tbody>
</table>

### Table III

<table>
<thead>
<tr>
<th>Word</th>
<th>Closest synonyms based on their semantic properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Giant</td>
<td>{great, stupendous, huge, enormous, massive}</td>
</tr>
<tr>
<td>Prodigious</td>
<td>{colossal, abhorrent, humongous, gigantic}</td>
</tr>
<tr>
<td>Elephantine</td>
<td>{jumbo, bumper, whooping, ginormous, mountainous}</td>
</tr>
<tr>
<td>Abominable</td>
<td>{unnatural, whaling, substantial, outrageous}</td>
</tr>
<tr>
<td>Monstrous</td>
<td>{grotesque, appalling, heinous, wicked, terrible}</td>
</tr>
</tbody>
</table>

For the word ‘beautiful’, the following set represents the cluster centers.

\[ W_1 = \{\text{Charming, Attractive, Beautiful, Gorgeous, Exquisite}\} \]

Similarly, the word ‘giant’ has its following five clusters along with Table III which represents the closest synonym word to them. The following equation represents the five cluster points of negative space.

\[ W_2 = \{\text{Abominable, Monstrous, Giant, Prodigious, Elephantine}\} \]

Fig. 6 represents the two set of words with near synonyms grouped together as X and Y. Also, Fig. 6 shows the subsethood relation among the near synonyms in each set. Then, the five cluster centers of both the words are represented by the membership gradation as:

\[
\mu_X(x) = \left\{ \frac{0}{c_1}, \frac{0.25}{c_2}, \frac{0.5}{c_3}, \frac{0.75}{c_4}, \frac{1}{c_5} \right\}
\] (26)

\[
\mu_Y(y) = \left\{ \frac{0}{c_1}, \frac{0.25}{c_2}, \frac{0.5}{c_3}, \frac{0.75}{c_4}, \frac{1}{c_5} \right\}
\] (27)

The above membership gradation has been represented in Fig. 7.

\[
\mu_X(x) = \left[0.45, 0.65\right]
\] (28)

Suppose that the word ‘beautiful’ and ‘giant’ has the following membership values,
Then, by (21), the succession function for the word ‘beautiful’ is calculated as follows,

$$v_{ij} = 0.6533, 0.0664$$ \hspace{1cm} (29)

Again, by (21), the succession function for the word ‘giant’ is calculated as follows,

$$v_{ij} = 0.7674, 0.6663$$ \hspace{1cm} (30)

Now, by using algebraic sum operation over the succession function obtained from the word ‘giant’, the membership value of the ‘giant’ is calculated as 0.9224. Also, it is observed that, by using the algebraic sum operation, the word ‘beautiful’ has the membership function 0.6764. The following membership value which shows the resultant meaning that arrived through one word operates on the other. Since the word ‘giant’ is acting upon the word ‘beautiful’, the word ‘beautiful’ gets more impact and has the positive influence which is captured by using the algebraic sum operation. The membership value obtained for the phrase ‘giant beauty’ is 0.9749.

VIII. CONCLUSION

Through this paper, we established a new fuzzy set approach to study the word association and its positional alternation. Through this proposed methodology, we have established a new type of fuzzy relationship between two linguistic variables which may have contradictory implications inherent in them. Furthermore, we observed the resultant value implies that the two words which have opposite implications, put together to generate pure aesthetic joy.

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REFERENCES


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