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Mixed Integer Programing for Multi-Tier Rebate with Discontinuous Cost Function

Y. Long, L. Liu, K. V. Branin

***Abstract*—**One challenge faced by procurement decision-maker during the acquisition process is how to compare similar products from different suppliers and allocate orders among different products or services. This work focuses on allocating orders among multiple suppliers considering rebate. The objective function is to minimize the total acquisition cost including purchasing cost and rebate benefit. Rebate benefit is complex and difficult to estimate at the ordering step. Rebate rules vary for different suppliers and usually change over time. In this work, we developed a system to collect the rebate policies, standardized the rebate policies and developed two-stage optimization models for ordering allocation. Rebate policy with multi-tiers is considered in modeling. The discontinuous cost function of rebate benefit is formulated for different scenarios. A piecewise linear function is used to approximate the discontinuous cost function of rebate benefit. And a Mixed Integer Programing (MIP) model is built for order allocation problem with multi-tier rebate. A case study is presented and it shows that our optimization model can reduce the total acquisition cost by considering rebate rules.

***Keywords*—**Discontinuous cost function, mixed integer programming, optimization, procurement, rebate.

I. Introduction

THE costs associated with the acquisition, receipt, movement, storage, use, maintenance, and disposal of a product or service are considered by procurement team. One challenge in the acquisition process is how to compare similar products from different suppliers and allocate orders among different products or services. For example, the procurement allocation problem is studied in [1] where a leader supplier firm must consider actions of their competitors. Instead of considering leader suppliers and their competitors differently as in [1], our work focuses on allocating orders among a group of exchangeable parts from multiple suppliers from the buyer's perspective. Two most important cost components considered by buyers during acquisition process are purchase cost and rebate. Usually suppliers provide rebate benefit if the order size during a period achieves a pre-defined threshold. In this study, we develop an optimization model to minimize the acquisition cost including purchase cost and rebate benefit. Other factors like receipt, movement, storage, use, maintenance, and disposal could be found in studies related to Total Cost of Ownership (TCO) [2].

Rebate policy could be very complex. First, rebate rules vary for different suppliers and usually are changing over time. It becomes more complex when one product is under several rebate policies. Moreover, sometimes supplier provides multiple tiers of rebate rates according to different trigger thresholds. There are quite a few studies focusing on different rebate policies [3]-[6]. Reference [3] developed a model for Total Quantity Discount (TQD) problem where the discounted prices apply to all goods bought from the supplier, not only to those goods exceeding the volume threshold. Reference [4] compared three types rebate rules: no rebate policy, a proportional rebate policy and a utility rebate policy. Procurement team may get the rebate policy in various formats from different suppliers. How to collect the rebate policy and transform them into a structured data format is key to the implementation of the model. To deal with different rebate policies, we first standardize the rebate policies and design a system collect different rebate polices automatically. For example, the TQD problem can be a special case collected by the system. Instead of looking through different policies and compare them manually, this study considers rebate policy collection and develops the optimization model which considers rebate policy and provide optimized order volume suggestion to procurement team.

Another challenge faced by buyers is that rebate is usually triggered by accumulating order during a time window. However, when buyers place order in the middle of time window, they are unsure whether the rebate benefit could be triggered and which level of rebate rates could be achieved at the end of the given time window. To deal with this challenge, one approach is to divide the plan horizon into several stages and use different models or strategies for different stages [7]-[9]. In [7], a three-stage procurement optimization problem is developed to deal with uncertainties on production shortages at the suppliers or competition from other firms. In this study, we developed two-stage optimization models for ordering allocation. One stage is to make acquisition decision at the beginning and middle of time window for rebate accumulation. The second stage is to make acquisition decision at last part of time window for rebate accumulation. The system can track the accumulate order information and use the optimization models to provide order allocation solution for buyers.

In our optimization models, we focus on rebate policy with multi-tiers, i.e., there are multiple trigger volumes and rebate rates. Buyers can get higher rebate rate when higher trigger volume is achieved. The discontinuous cost function of rebate

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benefit for multi-tier rebate policy is formulated. Piecewise linear function is used to approximate the discontinuous cost function of rebate benefit. MIP model is built for order allocation problem with multi-tier, multi-supplier. References [10] and [11] study a volume discount auction with piece-wise linear supply curves either consider a single product or assume the prices charged by a supplier for different commodities are independent. In our study, we considered the rebate interaction of different commodities under four scenarios.

In this article, we provide introduction and literature review on rebate in Section I. Section II is about the complexity of rebate policy under different scenarios and a system to track rebate benefit and support decision. In Section III, we formulated the two-stage optimization models to deal with rebate. Section IV includes a case study. Finally, conclusion is in Section V.

II. Rebate Policy

In this section, we will describe different scenarios of rebate, standardize the rebate policies and design a system collect different rebate polices automatically. Typically, supplier will provide rebates when buyers' orders are qualified for certain rules. However, the complexity of the rebate policies increases the difficulties for companies to make the optimal decisions. After we study different rebate policies of purchasing Hard Disk Drives, we find that the rebate policy can be partitioned into two parts, constraints and rebates rules.

Constraints are the requirements that buyers need to achieve to apply the rebates. Once the constraints are achieved, the corresponded rebates will be applied based on rebate rules. Rebate rules include a set of rebate benefits on a group of products qualified for rebate.

On a product perspective, each product involved in rebate policy is under one of three different scenarios. The simplest scenario is that a product is considered under both constraints and rebate rules. In this case, the product can contribute to achieve constraints, if the constraints are achieved, the rebates will also be applied to it. For example, the rebate policy says that if Product A's volume achieved Y Units, then rebate (X%) will be applied to Product A. A more complex scenario is that a product is considered under constraints but not rebate rules. In this case, the product can contribute to achieve the constraints. However, if the constraints are achieved, the rebates will not be applied to the product itself but other products. For example, the rebate policy says that Product A is counted toward the attainment but excluded from the rebate payment. The last scenario is that the product is considered under rebate rules but not constraints. In this case, the product itself does not contribute to achieve the constraints, but once the constraints are achieved, the rebates will be applied on it. For example, the rebate policy said that Product A is not counted toward the attainment but included in the rebate payment.

On a rebate policy perspective, each rebate policy is a combination of constraints and rebate rules. Within single rebate policy, it is an "AND" relationship among constraints, as well as rebate rules (as shown in Fig. 1). The rebates will be only applied if the constraints are qualified.

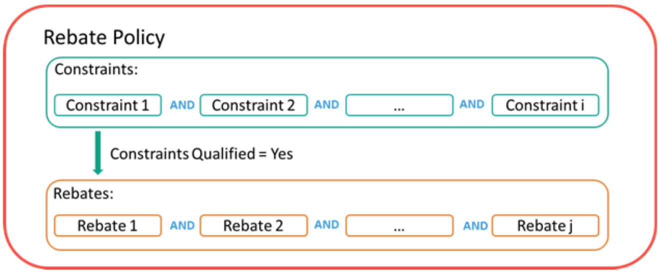

Fig. 1 Rebate Policy with Constraints Part and Rebates Part

If the whole rebate policy only contains a single trigger threshold and rebate rate, it is called "One-Tier Rebate Policy". If the whole rebate policy contains multiple trigger thresholds and rebate rates, it is called "Multi-Tier Rebate Policy.

A. User Interface to Collect Rebate Policies

It is found that rebate policies from different suppliers are different after we studied different rebate policies of purchasing process. And the rebate policy from one supplier is changing over time. Suppliers usually update their rebate rules regularly e.g., monthly or quarterly. To facilitate and standardize the process of updating rebate policy and transforming them to structured data, a user interface (Fig. 2) is designed to enter the rebate policy. Before users enter the constraints and rebates, they need to specify the rebate policy belongs to which Year, Quarter, Supplier, and Product Type. After that, users can start to enter the constraints, the left-hand side of the blue line is product description fields, these are for users to specify which products will be considered to meet constraints. By default, no products will be considered into constraint scope. On the right-hand side there are three fields to select, the first one is "In Scope", which allows the user to specify the products that they specified at the left-hand side whether will be considered in scope to meet the constraint or not. The second field is the constraint trigger type. There are three different trigger types which are units, USD and PB (drive petabytes). For example, if the trigger type is set to be units, the total units of products specified on the left-hand side need to meet certain units' threshold to enable rebates. The last field allows users to specify the period that the constraints will cover. For example, in Fig. 2, if user selects both 2017/Q4 and 2017/Q3, the accumulated order within these 2 quarters will be compared to the constraint threshold.

Fig. 3 is an example of a rebate policy with two constraints and one rebate rule. In one of the constraints, there are two tiers of trigger thresholds, and correspondingly there are two tiers of rebate rates.

In the example of multi-tier rebate policy in Fig. 3, suppose a supplier updates the HDD rebate policy for 2017 first quarter, the user can create a rebate policy with two constraints. The first constraint is that the total order of 'Drive Family 1' and 'Drive Family 2' drives should reach 40,000 units during the time window of 2017 Q1. The second constraint includes two tiers of thresholds, i.e., the total order of ‘Drive Family 1’ drive with 1.8TB Capacity during the time window should achieve 6,000 units (or 8,000 units). Only when both constraints are satisfied, the rebate on ‘Drive Family 1’ drive with 1.2TB Capacity is enabled. There are two tiers of rebate rates, 8% and 10%. Once the user finishes all information entering and confirms the rules, the rules will be collected and transformed into structured data so that later can be used in the optimization algorithm.

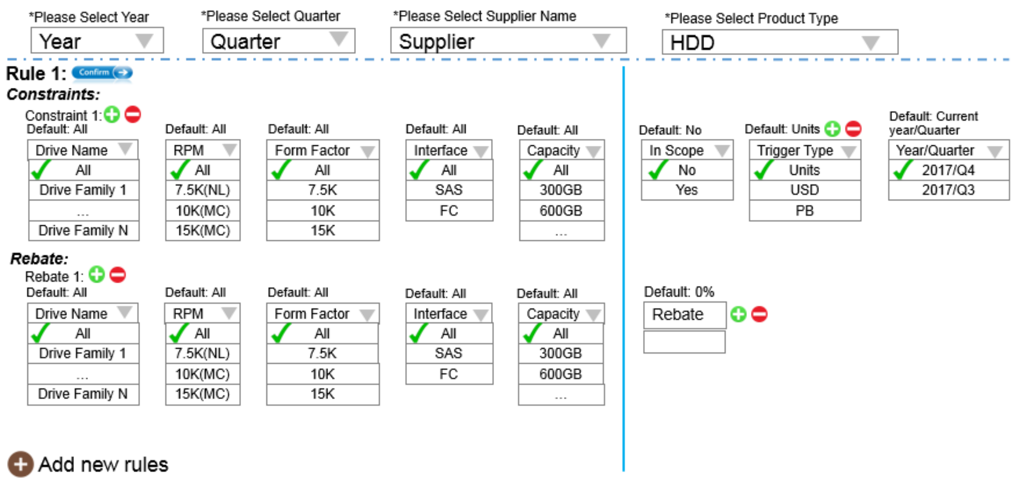

Fig. 2 User Interface of Rebate Collection

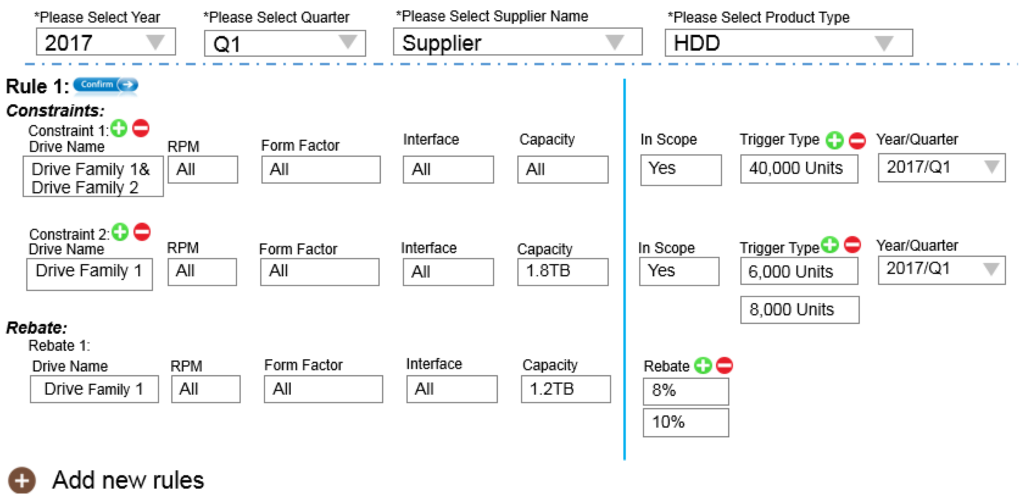

Fig. 3 Multi-Tier Rebate Policy Example

B. Acquisition Cost Optimization System

To support decisions on procurement, an acquisition cost optimization system (Fig. 4) is developed. It has three key phases. Phase I is data collection, model input data like rebate policies (include constraints and rebates) and purchase price are collected through a User Interface (UI). The historical orders are also collected. Phase II is to transform the data into a format that can be fitted into the model, then run the optimization model. Phase III is to present the final optimal solution which includes the optimal order volume, potential saving, and cost summary.

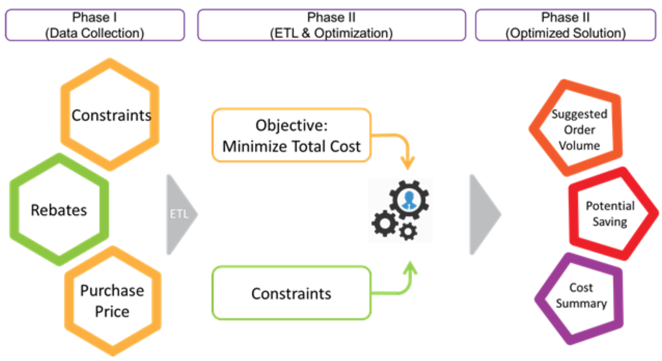

Fig. 4 Acquisition Cost Optimization System

TABLE I
NOMENCLATURE

x_k	real number decision variable. The order quantity of part k ($k \in K$)
$y_{k,q}^j$	binary decision variable. $y_{k,q}^j = 1$, if x_k falls in the j^{th} interval of rebate policy q; $y_{k,q}^j = 0$, if x_k doesn't fall in the j^{th} interval of rebate policy q
$z_{k,q}^s$	real number decision variable. $z_{k,q}^s > 0$ only when x_k falls in the (s-1)th or s^{th} interval.
K	the set of exchangeable parts
O	the order volume
p_k	purchase price of part k ($k \in K$)
$\bar{r}_k$	average rebate benefit of part k ($k \in K$)
l_k	maximum order quantity for part k ($k \in K$)
g_k	minimum order quantity for part k ($k \in K$)
Q	the set of rebate policies in consideration
q	one rebate policy in set Q ($q \in Q$). $q = \{C_q, D_q, E_q\}$
C_q	a set of parts which can be counted to trigger the rebate in policy q ($q \in Q$)
D_q	a set of parts to which the rebate applied in policy q ($q \in Q$)
E_q	a set of pairs of rebate rate and trigger threshold in policy q ($q \in Q$)
A_q	the total volume of accumulated order of all parts in C_q during the time window of rebate ($q \in Q$)
B_q	the total cost of accumulated order of all parts in D_q ($q \in Q$)
$R_{k,q}(x_k)$	the rebate benefit of part k in rebate policy q where the order volume of part k is x_k ($k \in K, q \in Q$)
$IA_{k,q}$	the indicator on whether $k \in C_q$ ($k \in K, q \in Q$)
$IR_{k,q}$	the indicator on whether $k \in D_q$ ($k \in K, q \in Q$)
r_q^i	the rebate rate of i^{th} tier in rebate policy q
$m_{k,q}$	The number of intervals in piecewise linear cost function of part k for rebate policy q ($k \in K, q \in Q$)

III. MODELING

In this session, we developed optimization models for order allocation problem considering multiple products and multiple rebate policies. Suppose a buyer plans to place an order o with quantity O, we built an optimization model to minimize the acquisition cost to help this buyer to satisfy order o. Several products from different suppliers with similar function can be used to satisfy this order o. These products are called exchangeable parts for order o, denoted as K_o. Since the optimization model is built and solved for each order separately in this paper, we simplify K_o into K. These exchangeable parts have different unit purchase price and are under different rebate policies. The buyer should make decision on how many part k for each $k \in K$ to buy to satisfy the order o and minimize the acquisition cost.

One widely applied rebate policy is that if the accumulated order of a certain part during a time window [T0, T] reaches a threshold, the supplier will provide a percentage of rebate for this part at the end of the time window T. Usually suppliers update rebate policy by month or quarter. T0 is defined as the start date of a month/quarter is and T is defined as the end date of a month/quarter. In this work, we assume that rebate of different parts from different suppliers has same time window for order accumulation. Rebate policies for different parts with different time window is not considered in this study.

One challenge faced by buyers is that they are unsure whether the accumulated order for one part can trigger rebate or not at the end of the time window when they make procurement decision in the middle of [T0, T]. In this study, we divided the time window [T0, T1] into two stages [T0, T1] and [T1, T]. We develop different models for these two stages.

A. Stage 1

During [T0, T1], the final rebate rate is uncertain. In this study, we define the potential rebate rate as $\bar{r}_k$. $\bar{r}_k$ can be estimated by calculating the average rebate benefit per unit based on historical rebate benefit for each exchangeable part.

We built a Linear Programming (LP) to minimize the acquisition cost.

Objective function

$$\min \sum_{k \in K} (p_k - \bar{r}_k) \cdot x_k \tag{1}$$

Subject to

$$\sum_{k \in K} x_k \geq O \tag{2}$$

$$g_k \leq x_k \leq l_k \quad \forall k \in K \tag{3}$$

$$x_k \geq 0, \forall k \in K \tag{4}$$

There is one type of decision variables in this model, x_k $\forall k \in K$. Equation (1) is the objective function of the optimization model. The total cost sums up the cost of all exchangeable parts. Equation (2) is the order constraint. The total order of exchangeable parts must satisfy the total required order quantity O. Equation (3) has two parts. It contains the availability constraint. This constraint is to ensure the order for one part is under the maximum limit of the suppliers can provide at one time. It also contains the minimum order constraint. This constraint is to balance the order among different exchangeable parts. The purpose is to avoid allocating 100% of the order on one part because the buyers want to maintain the relationship with all suppliers. Equation (4) is to ensure all decision variables are non-negative.

B. Stage 2

During [T1, T], the accumulated order has been tracked and whether there is a chance to trigger rebate is clearer. Suppose there is a set of polices Q about rebate in consideration, q ($q \in Q$) is one rebate policy in set Q. Q is collected through a user interface, standardized and stored. As explained in Section II, one rebate policy contains constraints and rebate rules. In this section, the trigger type in consideration is unit. i.e., the volume of accumulated order must satisfy the constraints. It can be easily extended to consider other trigger types like dollar or PB (drive petabytes) by multiplying volume by purchase price or capacity respectively. We start from a simple rebate policy with one constraint and one rebate rule. The rebate policy can be represented by $q = \{C_q, D_q, E_q\}$. There are three components in the q. C_q is a set of parts which can be counted to trigger the rebate. The total volume of accumulated order of all parts in C_q is denoted as A_q. D_q is a set of parts to which the rebate applied. The total cost of accumulated order of all parts in D_q is denoted as B_q. E_q is a set of pairs of rebate rate and trigger threshold. E_q= [$\{r_q^0, v_q^0\}$, $\{r_q^1, v_q^1\}$, …, $\{r_q^N, v_q^N\}$], where $v_q^0 \le v_q^1 \le \cdots \le v_q^N$, $r_q^0 \le r_q^1 \le \cdots \le r_q^N$. N is the number of tiers. If $v_q^1 \le A_q < v_q^2$, the rebate benefit confirmed is $r_q^1 \cdot B_q$.

We define $R_k(x_k)$ as the total rebate benefit of k. The total acquisition cost can be formulated as

$$\sum_{k\in K} p_k \cdot x_k - R_k(x_k) \tag{5}$$

$R_{k,q}(x_k)$ is the rebate benefit of part k in rebate policy q where the order volume of part k is x_k.

$$R_k(x_k) = \sum_{q\in Q} R_{k,q}(x_k) \tag{6}$$

Note that C_q and D_q are usually different. $IA_{k,q}$ is the indicator on whether $k \in C_q$. If $\in C_q$, $IA_{k,q}$=1, else $IA_{k,q}$=0. Similarly, $IR_{k,q}$ is the indicator on whether $k \in D_q$. If $k \in D_q$ is, $IR_{k,q}$=1, else $IR_{k,q}$=0.

There are four scenarios for each part. The cost function under the four scenarios described above is in Fig. 5.

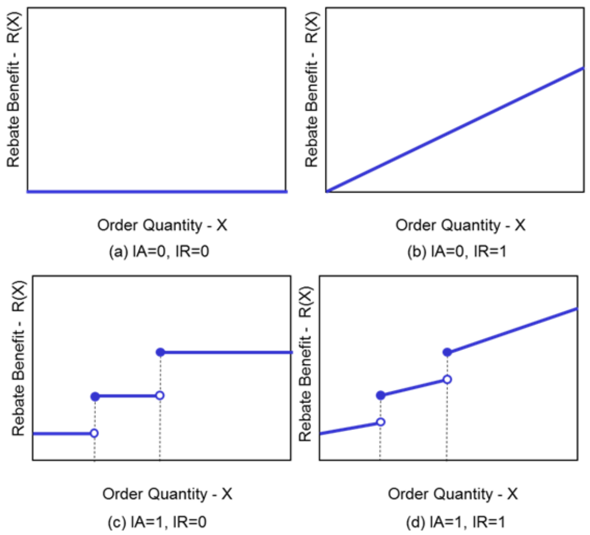

Fig. 5 Rebate benefit by X under different scenarios

1) $IA_{k,q}$=0, $IR_{k,q}$=0 (Fig. 5 (a)). In this case, no rebate benefit is related to k from rebate policy q.
2) $IA_{k,q}$=0, $IR_{k,q}$=1 (Fig. 5 (b)). In this case, part k is not required in constraint to trigger rebate. However, when rebate constraint is achieved, rebate will be applied to park k. The rebate on part k is counted as benefit rebate.
3) $IA_{k,q}$=1, $IR_{k,q}$=0 (Fig. 5 (c)). In this case, the rebate rate is not applied to part k. However, ordering part k helps satisfy the constraints and thus helps parts in D_q get rebate. The rebate on parts in D_q is counted as benefit rebate.
4) $IA_{k,q}$=1, $IR_{k,q}$=1 (Fig. 5 (d)). In this case, park k is counted toward to attainment in constraints, and thus it can help parts in D_q get rebate. Also, when rebate constraint is achieved, rebate rate is applied to park k. The two parts of savings are counted as benefit rebate.

Suppose the at the time t ($T1 \le t \le T$), for a multi-tier rebate policy, if the accumulated order required to trigger rebate A_q is $v_q^i \le A_q < v_q^{i+1}$. The gap to trigger next level is $G_q^1 = v_k^{i+1} - A_q$, $G_q^2 = v_q^{i+2} - A_q$, …., $G_q^{N-i} = v_q^N - A_q$. $R_{k,q}(x_k)$ can be formulated as

$$R_{k,q}(x_k) = r_q^i \cdot B_q \cdot IA_{k,q} + r_q^i \cdot p_k \cdot x_k \cdot IR_{k,q} \qquad 0 \le x_k < G_q^1 \tag{7}$$

$$= r_q^{i+1} \cdot B_q \cdot IA_{k,q} + r_q^{i+1} \cdot p_k \cdot x_k \cdot IR_{k,q} \qquad G_q^1 \le x_k < G_q^2 \tag{8}$$

$$= \ldots$$

$$= r_q^N \cdot B_q \cdot IA_{k,q} + r_q^N \cdot p_k \cdot x_k \cdot IR_{k,q} \qquad G_q^{N-i} \le x_k \tag{9}$$

In this study, we approximate the discontinuous cost function using piecewise linear function.

Fig. 6 is the cost functions approximated using piecewise linear function for scenario 3) and 4).

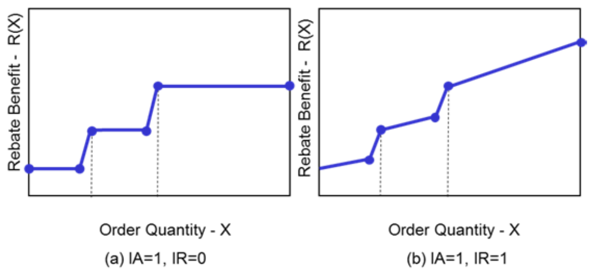

Fig. 6 Piecewise Linear Rebate Benefit

After applying the piecewise linear, suppose there are $m_{k,q}$ intervals and $m_{k,q}$+1 breakpoints, the break points are $b_{k,q}^1 = 0$, $b_{k,q}^2 = G_{k,q}^1 - \Delta$, $b_{k,q}^3 = G_{k,q}^1$, …., $b_{k,q}^{m_{k,q}+1} = M$. Δ is a small value, e.g., 1. M is a big value which is bigger than the order size O and the largest threshold of policy q. We define that $y_{k,q}^j$ is a binary decision variable. $y_{k,q}^j = 1$ if x_k falls in the jth interval, and $y_{k,q}^j = 0$ if x_k doesn't fall in the jth interval. We define that $z_{k,q}^s$ is a decision variable. $z_{k,q}^s > 0$ only when x_k falls in the (s-1)th or sth interval.

$R_{k,q}(x_k)$ can be reformulated as below.

$$R_{k,q}(x_k) = z_{k,q}^1 \cdot R_{k,q}(b_{k,q}^1) + z_{k,q}^2 \cdot R_{k,q}(b_{k,q}^2) + \cdots z_{k,q}^{m_{k,q}+1} \cdot R_{k,q}(b_{k,q}^{m_{k,q}+1}) \tag{10}$$

where

$$y_{k,q}^1 + y_{k,q}^2 + \cdots + y_{k,q}^{m_{k,q}} = 1 \tag{11}$$

$$x_k = z_{k,q}^1 \cdot b_{k,q}^1 + z_{k,q}^2 \cdot b_{k,q}^2 + \cdots + z_{k,q}^{m_{k,q}+1} \cdot b_{k,q}^{m_{k,q}+1} \tag{12}$$

$$z_{k,q}^1 + z_{k,q}^2 + \cdots + z_{k,q}^{m_{k,q}+1} = 1 \tag{13}$$

$$z_{k,q}^1 \le y_{k,q}^1,\ z_{k,q}^2 \le y_{k,q}^1 + y_{k,q}^2,\ \ldots,\ z_{k,q}^{m_{k,q}} \le \tag{14}$$

$$y_{k,q}^{m_{k,q}} + y_{k,q}^{m_{k,q}-1},\ z_{k,q}^{m_{k,q}+1} \leq y_{k,q}^{m_{k,q}}$$

The optimization model for the 2nd stage can be formulated as:

$$min\ \sum_{k\in K} p_k \cdot x_k - \sum_{k\in K} R_k(x_k) = \sum_{k\in K} p_k \cdot x_k - \sum_{k\in K}\sum_{q\in Q} R_{k,q}(x_k) = \sum_{k\in K} p_k \cdot x_k - \sum_{k\in K}\sum_{q\in Q}[z_{k,q}^1 \cdot R_{k,q}(b_{k,q}^1) + z_{k,q}^2 \cdot R_{k,q}(b_{k,q}^2) + \cdots z_{k,q}^{m_{k,q}+1} \cdot R_{k,q}(b_{k,q}^{m_{k,q}+1})] \tag{15}$$

Subject to

$$\sum_{k\in K} x_k \geq O \tag{16}$$

$$g_k \leq x_k \leq l_k,\quad \forall k \in K \tag{17}$$

$$x_k \geq 0\ , \forall k \in K \tag{18}$$

$$y_{k,q}^j \text{ is 0 or 1}, \forall k \in K, q \in Q, j = 1,2,\dots,m_{k,q},$$
$$z_{k,q}^s \geq 0, \forall k \in K, q \in Q, s = 1,2,\dots,m_{k,q},$$

$$y_{k,q}^1 + y_{k,q}^2 + \cdots + y_{k,q}^{m_{k,q}} = 1\ , \forall k \in K, q \in Q \tag{19}$$

$$x_k = z_{k,q}^1 \cdot b_{k,q}^1 + z_{k,q}^2 \cdot b_{k,q}^2 + \cdots + z_{k,q}^{m_{k,q}+1} \cdot b_{k,q}^{m_{k,q}+1},\quad \forall k \in K, q \in Q \tag{20}$$

$$z_{k,q}^1 + z_{k,q}^2 + \cdots + z_{k,q}^{m_{k,q}+1} = 1\ , \forall k \in K, q \in Q \tag{21}$$

$$z_{k,q}^1 \leq y_{k,q}^1,\ z_{k,q}^2 \leq y_{k,q}^1 + y_{k,q}^2, \dots,\ z_{k,q}^{m_{k,q}} \leq y_{k,q}^{m_{k,q}} + y_{k,q}^{m_{k,q}-1},\ z_{k,q}^{m_{k,q}+1} \leq y_{k,q}^{m_{k,q}},\quad \forall k \in K, q \in Q \tag{22}$$

The optimization model can be easily extended for rebate policies with multiple constraints and multiple rebate rules. When a rebate policy input from user interface has *m* constraints and *n* rebate rules. We can save the rebate policy as *n* policies, i.e., in each *q*, there are *m* constraints and 1 rebate rule.

For a rebate policy with multiple constraints and 1 rebate rule, we can get one cost function and approximated it using piecewise linear function. We take the rebate policy in Fig. 3 with two constraints as an example. The rebate policy can be standardized and stored in the same format $q = \{C_q, D_q, E_q\}$. C_q is set with two lists. The two lists of parts are the attainment part for the two constraints. E_q= $[\{r_q^0, v_q^0\}, \{r_q^1, v_q^1\}, \dots, \{r_q^N, v_q^N\}]$ where v_q^i contains two thresholds. Only when the both thresholds are achieved, the rebate rate r_q^i can be applied. At the time of making ordering decision, for each exchangeable part *k*, the gaps to trigger rebate for each constraint and each tier is calculated. Suppose G1_1 is the gap between the accumulated order and threshold of constraint 1. G1_2 and G2_2 are the two gaps for two-tier rebate in constraint 2. In Fig. 7, we can get one cost function by considering both constraints. Piecewise linear function can be formulated for this unconscious cost function. Then we can model (15)-(22) for order allocation where rebate benefit follows these piecewise linear functions.

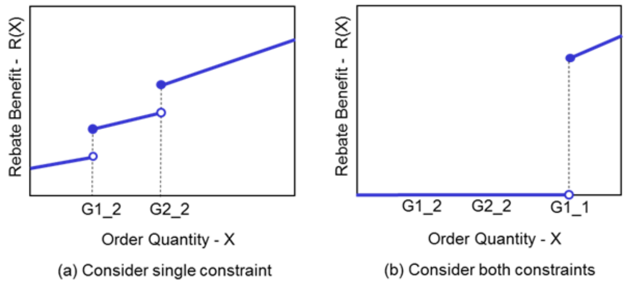

Fig. 7 Rebate Benefit of Multiple Constraint Rebate

IV. Experiment

In this section, we introduce a case study to show the benefit of considering rebate policies in the two-stage optimization modeling. Four parts are considered, i.e., PN0001, PN0002, PN0003, and PN0004. The purchase prices of these four parts are set to be \$100, \$102, \$25, \$24.5. There are two pairs of exchangeable parts. PN0001 and PN0002 are exchangeable pair 1. And PN0003, PN0004 are exchangeable pair 2. There are two types of orders. The first type can be satisfied by exchangeable pair 1. The order quantity follows a normal distribution $N(2000,1000^2)$. The second type of order can be satisfied by exchangeable pair 2. The order quantity also follows normal distribution $N(2000,1000^2)$. To balance order from suppliers and consider availability constraints, we assume the order from each exchangeable part should be ≤90% and ≥10% of the order quantity, i.e., $g_k = 10\%O$, $l_k = 90\%O, \forall k \in K$.

Two rebate policies are considered in this experiment:

- q_1= $\{C_{q1} = [PN0001, PN0003],\ D_{q1} = [PN0001],\ E_{q1} = [\{0,0\},\{1000,3\%\},\{5000,4\%\},\{7000,7\%\},\{8000,8\%\},\{9000,9\%\},\{10000,10\%\}]\}$
- q_2=$\{C_{q2} = [\ PN0002],\ D_{q2} = [PN0002],\ E_{q2} = [\{0,0\},\{3000,6\%\}]\}$.

We generate 1000 samples. In each sample, there are four orders, order quantity is randomly generated based on normal distribution. We divide the time window into two stages. In each stage, there are two orders. One order requires parts in exchangeable pair 1 to satisfy. The other order requires parts in exchangeable pair 2 to satisfy. Table II contains samples of two-stage orders generated randomly. When making order decision in stage 1, the order quantity in stage 2 is unknown.

TABLE II
Two-Stage Order Samples Generation

SN	Stage 1		Stage 2	
	Order for exchangeable pair 1 $N(2000,1000^2)$	Order for exchangeable pair 2 $N(2000,1000^2)$	Order for exchangeable pair 1 $N(2000,1000^2)$	Order for exchangeable pair 2 $N(2000,1000^2)$
1	2,196	1,614	2,292	1,365
2	2,339	598	-	1,890
…	…	…	…	…
1000	1,209	2,215	1,452	1,484

To evaluate the performance of our two-stage optimization models, we first consider three simple strategies to allocation order.

Strategy 1: Don't consider rebate information. Always buy the cheaper parts as more as possible, i.e., when type 1 order arrives, always buy 90% of PN0001, 10% of PN0002 because PN0001 is cheaper than PN0002. When type 2 order arrives, always buy 90% of PN0004, 10% of PN0003 because PN0004 is cheaper.

Strategy 2: Consider the expected rebate rate for parts which are qualified for rebate. For example, the expected order quantity of type 1 order is 4,000 (the mean of order quantity in stage 1 (2000) plus the mean of order quantity in stage 2 (2000)). According to q_1, PN0001's rebate rate is 3%. Its price after applying this rebate rate is \$100*(1-3%)=\$97. Similarly, according to q_2, PN0002's rebate is 6%. Its price after applying this rebate rate is \$102*(1-6%)=\$95.9. PN0002 is cheaper. When type 1 order arrives, always buy 90% of PN0002, 10% of PN0001. Since PN0003 and PN0004 has no rebate, when type 2 order arrives, always buy 90% of PN0004, 10% of PN0003.

Strategy 3 (best case): Assume we know the order quantity at the beginning of time window in each sample, i.e., when make decision for pair 1 at stage 1, we not only know the order quantity for pair 1 at stage 1, we also know the order quantity for pair 2 at stage 1 and the order quantity for pair1 and 2 at stage 2. In this case, without uncertainty, we can always make best decision. This strategy can't be applied in real procurement process because is it impossible to confirm all future orders with 100% certainty in advance. We use this strategy here as a benchmark.

We developed the two stages of optimization models in R. $\bar{r}_k$ is estimated by calculating the average rebate benefit per unit for each rebate policy. We used the rebate benefit and order decisions from Strategy 1 experiment to calculate $\bar{r}_k$. The average rebate benefit per unit is \$2.1 for PN0001and PN0003, while the average rebate benefit per unit is \$0 for PN0002, PN0004. We use these $\bar{r}_k$ values in Strategy 4.

Table III shows the average purchase cost, rebate benefit and total cost of the 1,000 samples in four different strategies. Total cost is purchase cost minus rebate benefit. As expected, Strategy 3 (best case) has the highest rebate benefit and lowest total cost. Two simple strategies, i.e., strategy 1 and 2, have obviously higher total cost. Strategy 4 uses our two-stage optimization model. Its total cost is the second lowest. We also display the rebate benefit and cost of different order strategies in Fig. 8. Although Strategy 4 (two-stage) has higher purchase cost compared with Strategy 1. However, its rebate benefit is 50% higher than Strategy 1. As a result, the total cost of Strategy 4 is lower than Strategy 1. Compare with Strategy 2, Strategy 4 has lower purchase cost and higher rebate benefit, and thus the total cost is lower than strategy 2.

We use the cost and rebate benefit in Strategy 3 (best case) as a benchmark and calculate the difference between the cost and benefit in other strategies from the benchmark. Fig. 9 displays the cost difference from benchmark. It shows that cost difference from our two-stage optimization model is significantly lower than other strategies. The cost difference from strategy 4 is 38% of cost difference from strategy 2, and 31% of cost difference from strategy 1. It indicates that the two-stage optimization models can help the buyer make better decision and thus reduce the total acquisition cost.

TABLE III
COST FROM DIFFERENT ORDER STRATEGIES

Strategy	Purchase Cost	Rebate Benefit	Total Cost
Strategy 1	\$ 504,749	\$ 8,553	\$ 487,643
Strategy 2	\$ 511,265	\$ 10,885	\$ 489,494
Strategy 3 best case	\$ 506,289	\$ 14,028	\$ 478,234
Strategy 4 two-stage	\$ 506,897	\$ 12,536	\$ 481,825

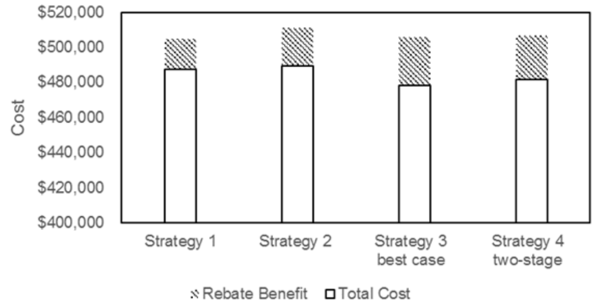

Fig. 8 Rebate Benefit and Cost of Different Order Strategy

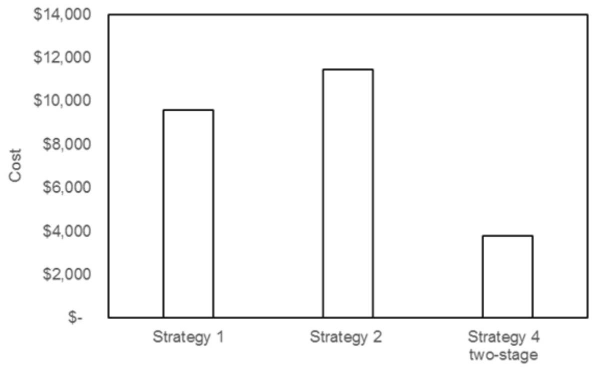

Fig. 9 Cost Difference from Benchmark

V. CONCLUSION

Procurement decision-makers can make a significant impact for their businesses by considering the TCO for products and services during the acquisition process. The complexity of rebate policies makes the evaluation of the TCO difficult, especially given uncertainty in future business, volumes, and ordering needs. However, by creating a two-stage optimization model, decision-makers can identify a strategy that results in substantially better decisions. This type of modeling can be integrated into an organization's procurement processes by ensuring reliable collection of purchase costs, rebate benefits, and enabling the modeling to be performed and referenced during key decisions points throughout the period when a product or service is acquired.

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