Longitudinal Vibration of a Micro-Beam in a Micro-Scale Fluid Media

M. Ghanbari, S. Hossainpour, G. Rezazadeh

Abstract—In this paper, longitudinal vibration of a micro-beam in micro-scale fluid media has been investigated. The proposed mathematical model for this study is made up of a micro-beam and a micro-plate at its free end. An AC voltage is applied to the pair of piezoelectric layers on the upper and lower surfaces of the micro-beam in order to actuate it longitudinally. The whole structure is bounded between two fixed plates on its upper and lower surfaces. The micro-gap between the structure and the fixed plates is filled with fluid. Fluids behave differently in micro-scale than macro, so the fluid field in the gap has been modeled based on micro-polar theory. The coupled governing equations of motion of the micro-beam and the micro-scale fluid field have been derived. Due to having non-homogenous boundary conditions, derived equations have been transformed to an enhanced form with homogenous boundary conditions. Using Galerkin-based reduced order model, the enhanced equations have been discretized over the beam and fluid domains and solve simultaneously in order to obtain force response of the micro-beam. Effects of micro-polar parameters of the fluid as characteristic length scale, coupling parameter and surface parameter on the response of the micro-beam have been studied.

Keywords—Micro-polar theory, Galerkin method, MEMS, micro-fluid.

I. INTRODUCTION

THE solid-fluid interaction vibration is very important in the view point of structure performance and disaster prevention, for example in industries such as elevated water tanks and liquefied natural gas. Also, it is very important in the field of MEMS designing and manufacturing. Many works have been done on the behavior of liquid sloshing in circular cylindrical containers that are excited horizontally, considering the damping effect that liquid tanks have on the vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3]. Senda and Nakagawa have reported a linear analysis on the effect of vibration of the structure under excitation [1]-[3].
equations become incapable of explaining the micro scale fluid transport phenomena in the study of micro and nano-scale fluid mechanics. The theory of micro-polar fluids was introduced by Eringen [28], [29], which contains the effect of and couple stresses and local rotary inertia, and provides a mathematical model for behavior of certain liquids such as blood, polymer and so forth. Today, researches show that in a micro-scale fluid field, micro-polar fluid theory is a useful tool for modeling the flow in micro-devices [30]-[32].

II. GOVERNING EQUATIONS

The constitutive equations of the fluid field based on micro-polar theory considering the non-symmetric stress tensor \( \sigma^I = \{ \sigma^I_{ij} \} \) and the couple stress tensor \( C_{0I} = \{ C_{0I}^j \} \), are as following [31]:

\[
\sigma^I_{ij} = (-p + \lambda^I V_{ik,k}) \delta_{ij} + \mu^I (V_{ij} + V_{ji}) + k^I (V_{ij} - e_{ijk} G^I_{k})
\]

\[
C_{0I}^j = \alpha^I G^I_{ik,k} \delta_{ij} + \beta^I G_{ij,k} + \gamma^I G_{j,i}^I
\]

where the symbols denote; \( p \): hydrodynamic pressure, \( \lambda^I \): bulk viscosity coefficient, \( \mu^I \): classical shear viscosity coefficient, \( k^I \): vortex viscosity coefficient, \( \alpha^I, \beta^I, \gamma^I \): spin gradient viscosity coefficients, \( e_{ijk} \): Levi-Civita symbol, \( \delta_{ij} \): the Kronecker delta, \( V^I_{ij} \): velocity components and \( G^I_{ij} \): micro-rotation components.

Here, the coefficients satisfy following inequalities [31]:

\[
k^I > 0, 3\lambda^I + k^I + 2\mu^I > 0, 2\mu^I + k^I > 0
\]

\[
3\alpha^I + 2\gamma^I > 0, -\gamma^I < \beta^I < \gamma^I, \gamma^I > 0
\]

The balance equations of the fluid field based on micro-polar theory can be expressed as:

\[
\frac{\partial \rho^I}{\partial t} + (\rho^I V)_{ij} = 0
\]

\[
(\lambda^I + \mu^I)V_{ij,j} + (\mu^I + k^I) V_{ij,j} + k^I e_{ijk} G^I_{k,j} - p_j + \rho^I f^I_{j} = p^I \dot{V}_i
\]

\[
(\alpha^I + \beta^I) G^I_{ij,j} + \gamma^I G^I_{ij,j} + k^I e_{ijk} V_{kj} - 2k^I G^I_{ij} + \rho^I c^I_{j} = \rho^I J^I \dot{G}^I_{ij}
\]

with new non-dimensional parameters \( CP, L_f \) as [31]:

\[
CP = \sqrt{\frac{\kappa^I}{2\mu^I + k^I}}, L_f = \frac{L}{l}, l = \sqrt{\frac{\gamma^I}{4\mu^I + 2k^I}}
\]

where \( L_c \) is a reference quantity of dimension, \( L_f \) characterizes the relationship between the geometric dimension of the flow \( L_c \) and the physical properties of the fluid. In the limiting case, for \( L_f \rightarrow \infty \), the micro-rotation vector is equal to one half of the vector of the fluid stream vorticity: Parameter \( CP, 0 \leq CP \leq 1 \), characterizes coupling between the vortex viscosity coefficient \( k^I \) and the shear viscosity coefficient \( \mu^I \). In other words, it represents the dependency of micro-rotations to macro-rotation (classical rotation) in the fluid field. If the value of the coupling parameter approximates zero, then the equations of the linear and angular momentums become independent of each other and the linear momentum transforms into the classical Navier-Stocks equations for Newtonian fluids.

The constitutive equations for an isotropic elastic beam based on micro-polar elasticity theory considering the stress tensor \( \sigma^B = \{ \sigma^B_{ij} \} \) and the couple stress tensor \( C_{0B} = \{ C_{0B}^j \} \) are as follows [33]:

\[
\sigma^B_{ij} = (\beta^B e_{ik}^B) \delta_{ij} + (2\mu^B + k^B) e_{ij}^B + k^B e_{ijk} (r^B_k - G^B_k)
\]

\[
C_{0B}^j = \alpha^B G_{ik,k}^B \delta_{ij} + \beta^B G_{ij,k}^B + \gamma^B G_{j,i}^B
\]

And these equations for a piezoelectric material are as follows [33]-[35]:

\[
\sigma_{ij}^{pl} = (\lambda^B e_{ik}^B) \delta_{ij} + (2\mu^B + k^B) e_{ij}^{pl} + k^B e_{ijk} (r_{ik}^{pl} - G_{ki}^{pl}) - \epsilon_{ijk}^{pl} E_k
\]

\[
C_{0ij}^{pl} = \alpha^B G_{ik,k}^{pl} \delta_{ij} + \beta^B G_{ij,k}^{pl} + \gamma^B G_{j,i}^{pl}
\]

where \( \lambda^B, \mu^B, \lambda^{pl}, \mu^{pl} \) are classical Lame type constants of the micro-beam and piezoelectric material, respectively. \( \alpha^B, \beta^B, \gamma^B, k^B \) and \( \alpha^{pl}, \beta^{pl}, \gamma^{pl}, k^{pl} \) are new micro-polar
constants due to couple stresses. $G^b, G^{pi}_b$ are micro-rotation components of the micro-beam and piezoelectric material. $e^{pi}_k$ are piezoelectric constants. $E^{pi}_k$ is an electric field due to the voltage applied to the piezoelectric material. $r^b_i, r^{pi}_i$ are macro-rotation components of the micro-beam and piezoelectric material while $e^b_i, e^{pi}_i$ are the macro-strain tensors that have the following relations with the micro-beam displacement $U^b$ and the piezoelectric material displacement $U^{pi}$:

$$r^b_i = \frac{1}{2} \varepsilon_{ijk} U^{b}_{kj}, r^{pi}_i = \frac{1}{2} \varepsilon_{ijk} U^{pi}_{kj}$$

(15)

$$e^b_i = \frac{1}{2} (U^{b}_{ij} + U^{b}_{ji}), e^{pi}_i = \frac{1}{2} (U^{pi}_{ij} + U^{pi}_{ji})$$

(16)

where $\varepsilon_{ijk}$ is permutation symbol. So, the conservation equations for the elastic beam and piezoelectric material based on micro-polar theory can be obtained as [33]-[36]:

$$(\lambda^b + \mu^b) U^{b}_{ij, j} + (\lambda^b + k^b) U^{b}_{ij, ij} + k^b e_{ijk} G^{pi}_{kj} + \rho^b f^b_i = \rho^b \ddot{U}^b_i$$

(17)

$$(\alpha^b + \beta^b) G^{b}_{ij, j} + \gamma^b G^{b}_{ij, ij} + k^b e_{ijk} U^{b}_{jk} - 2k^b G^{pi}_{b} + \rho^b c^b_i = \rho^b J^b \ddot{G}^b_i$$

(18)

$$(\lambda^{pi} + \mu^{pi}) U^{pi}_{ij, j} + (\mu^{pi} + k^{pi}) U^{pi}_{ij, ij} + k^{pi} e_{ijk} G^{pi}_{kj} + \rho^{pi} f^{pi}_i - \varepsilon_{ijk} E^{pi}_{kj} = \rho^{pi} \ddot{U}^{pi}_i$$

(19)

$$(\alpha^{pi} + \beta^{pi}) G^{pi}_{ij, j} + \gamma^{pi} G^{pi}_{ij, ij} + k^{pi} e_{ijk} U^{pi}_{jk} - 2k^{pi} G^{pi}_{b} + \rho^{pi} c^{pi}_i = \rho^{pi} J^{pi} \ddot{G}^{pi}_i$$

(20)

With the following technical elastic properties [33]:

$$E^b = \frac{(2 \mu^b + k^b)(3 \lambda^b + 2 \mu^b + k^b)}{(2 \lambda^b + 2 \mu^b + k^b)}, G^b = \frac{2 \mu^b + k^b}{2}$$

(21)

$$\nu^b = \frac{\lambda^b}{2 \lambda^b + 2 \mu^b + k^b}$$

$$\nu^{pi} = \frac{\beta^b + \gamma^b}{2 \mu^b + \nu^{pi}}$$

(22)

$$G^b = \frac{2 \mu^b + k^b}{2 \mu^b + k^b}$$

The following assumptions are considered in order to simplify (7)-(9) and (17)-(20):

1. The fluid is assumed to be incompressible.
2. There are no body forces and body couples acting along x direction.
3. The beam is considered an isotropic elastic beam with constant properties.
4. There are no couple stresses in longitudinal vibration of the beam.
5. There is no Pressure gradient in x direction.

Consequently (7)-(9) and (17)-(20) are simplified to:

$$(\mu^f + k^f) V_{i, ij} + k^f e_{ijk} G^f_{ij} = \rho^f \ddot{V}_i$$

(23)

$$\gamma^f G^f_{ij, j} + k^f e_{ijk} V_{ijk} - 2k^f G^f_{i} = \rho^f J^f \ddot{G}^f_i$$

(24)

$$(2\mu^b) U^b_{ij, j} = \rho^b \ddot{U}^b_{ij}$$

(25)
(2\mu^n)^{ij}U_n^{pi} - \varepsilon_{ij}^{nE}E_n^{pi} = \rho^nU_n^{pi} \tag{26}

By supposing \((E_{pi}A_{pi} + E_{pi}A_{pi}) < E_{pi}, A_{pi}\) in which \(E_{pi}, A_{pi}\) are the Young modulus of beam, cross section area of beam, Young modulus of micro-plate, and cross section area of the micro-plate, respectively, and the fluid field can be considered one-dimensional. Considering the Cartesian coordinate system shown in Fig. 1, and neglecting the effect of electric field on piezoelectric layers, equations of motion of the fluid field and also equation of motion governing longitudinal displacement of the micro-beam can be simplified to:

\[ \frac{\partial^2 v}{\partial t^2} = (\mu + k_f) \frac{\partial^2 v}{\partial z^2} + k_f \frac{\partial g}{\partial z} \tag{27} \]

\[ \frac{\partial^2 g}{\partial t^2} = -2k_f g - k_f \frac{\partial v}{\partial z} + \gamma \frac{\partial^2 g}{\partial z^2} \tag{28} \]

\[ (EA)_n \frac{\partial^2 u}{\partial x^2} - \tau A\delta(x - L_b) = \left( f \rho A)_{eq} + m_0\delta(x - L_b) \right) \frac{\partial^2 u}{\partial t^2} \tag{29} \]

\[ \rho^\pi A^\pi + \rho^b A^b = (\rho A)_{eq}; \\pi^\pi A^\pi + \pi^b A^b = (EA)_{eq} \]

where \(A^\pi = t^\pi b, A^b = tb\). \(v\) and \(u\) are the velocity component of the fluid and displacement component of the micro-beam in \(x\) direction, \(g\) is the micro-rotation component of the fluid in \(y\) direction, \(\tau\) is the wall shear stress of the fluid acting on the lower and upper surfaces of the micro plate, \(m_0\) is the mass of the micro-plate, \(A^s\) is the area of the upper and lower surfaces of the micro-plate, and \(L_b\) is the length of the micro-beam, respectively. It should be noted that, for simplicity, the inertial force owing to the mass of the micro-plate and the shear force due to physical properties of the fluid are assumed to be singular distributed loads with zero load intensity through the whole length of the beam and infinite intensity at its end.

Accompany boundary conditions of (27) and (28) at \(z = 0\) and \(z = h\) are:

\[ v(0,t) = 0, v(h,t) = \frac{\partial u}{\partial t} \bigg|_{t=h} = \alpha(t) \tag{31} \]

\[ g(0,t) = -s\left( \frac{\partial v}{\partial z} \bigg|_{z=0} \right) = A(t), g(h,t) = -s\left( \frac{\partial v}{\partial z} \bigg|_{z=h} \right) = B(t) \tag{32} \]

where \(s\) is a constant such that \(0 \leq s \leq 1\) and is called surface parameter. In the case when \(s = 0\) (strong concentration), the microstructure does not rotate with respect to the surface. The case when \(s = 0.5\) (weak concentration), the micro-structures spin is equal to fluid vorticity [37]. \(\alpha(t)\) and \(B(t)\) are the time varying functions.

As the boundary conditions are time varying, \(v(z,t)\) and \(g(z,t)\) are introduced as follows:

\[ v(z,t) = u(z,t) + \alpha(t) \frac{z}{h} \tag{33} \]

\[ g(z,t) = \gamma R(z,t) + (\frac{B(t) - A(t)}{h})z + A(t) \tag{34} \]

Using (33) and (34), the governing equations of motion of the fluid field can be rewritten as:

\[ 3(\gamma R(z,t)) = \frac{\partial w}{\partial t} - \left( \frac{\mu + k_f}{\rho f} \right) \frac{\partial^2 w}{\partial z^2} \tag{35} \]

\[ \left( \frac{k_f}{\rho f} \right) \frac{\partial R}{\partial z} + \frac{\alpha(t)z}{h} + \left( \frac{k_f}{\rho f} \right)(A(t) - B(t)) \tag{36} \]

Therefore the accompanying boundary conditions of (31) and (32) take the following homogenous form:

\[ u(0,t) = u(h,t) = 0 \tag{37} \]

\[ \gamma R(0,t) = \gamma R(h,t) = 0 \tag{38} \]

Boundary conditions of (29) are:

\[ u(0,t) = 0 \tag{39} \]

\[ N(l,t) = \left( E_{pi}A_{pi} + E_{pi}A_{pi} \right) \frac{\partial u}{\partial x} - A\varepsilon_{31}E_z \]

\[ = (EA)_{eq} \frac{\partial u}{\partial x} - f(t) = 0 \Rightarrow \frac{\partial u}{\partial x} = f(t) \tag{40} \]

where \(E_z = \frac{2V_{pi}}{t_{pi}}\) is the uniform electric field in \(z\) direction due to the voltage \((V_{pi})\) applied to the piezoelectric layers, and \(\varepsilon_{31}\) is the equivalent piezoelectric coefficient for the one-dimensional problem. Due to the time variability of the boundary conditions, by introducing \(u(x,t)\) as:
\( u(x,t) = \psi(x,t) + \alpha(t) \)  

Equation (29) can be rewritten as:

\[
\lambda(\psi(x,t)) = \left( (\rho A)_{eq} + m_{ij} \delta(x - L) \right) \left( \frac{\partial^2 \psi}{\partial t^2} + \frac{\alpha(t)}{h} \right)
\]

\[-(EA)_{eq} \frac{\partial^2 \psi}{\partial x^2} + \tau A^t \delta(x - L) \]

With the corresponding homogenous boundary conditions:

\[
\psi(0,t) = 0, \quad \left. \frac{\partial \psi}{\partial x} \right|_{x=0} = 0
\]  

III. NUMERICAL SOLUTIONS

Many modern mathematical models pose challenges when used in numerical simulations, due to complexity and large dimension. Model order reduction reduces the computational complexity of these problems. In this work, we have applied Galerkin based reduce order model to solve the coupled equations of solid and fluid media. Galerkin methods are a class of methods for converting a continuous operator such as a differential equation to a discrete problem. In this method, we express an unknown function as a linear combination of a set of shape functions. The accuracy of Galerkin method depends on number and type of the shape functions. A Galerkin approximation precision depends on number and type of the shape functions. The accuracy of Galerkin method depends on number and type of the shape functions.

In this work approximate solutions for \( w(z,t) \), \( \Re(z,t) \) and \( \psi(x,t) \) are searched in the following form:

\[
w(z,t) \equiv w_M(z,t) = \sum_{j=1}^{M} a_j(t) \Omega_j(z) \quad (44)
\]

\[
\Re(z,t) \equiv \Re_M(z,t) = \sum_{j=1}^{P} b_j(t) \phi_j(z) \quad (45)
\]

\[
\psi(x,t) \equiv \psi_M(x,t) = \sum_{j=1}^{N} q_j(t) \phi_j(x) \quad (46)
\]

where, \( \Omega_j(z) \), \( \phi_j(z) \) and \( \phi_j(x) \) are the shape functions which satisfy geometrical boundary conditions (37), (38), and (43), respectively. The shear stress of the fluid acting on the microplate is expressed in the following form:

\[
\tau = (\mu' + k^f) \frac{\partial \psi}{\partial x} \bigg|_{x=0} + k^f \frac{\partial \psi}{\partial x} \bigg|_{x=L} + (\mu' + k^f) \frac{\partial \psi}{\partial x} \bigg|_{x=0} + k^f [\psi_{h,t} + B(t)] (47)
\]

in which

\[
\alpha(t) = \tilde{f}(t) L_0 + \sum_{j=1}^{N} q_j(t) \phi_j(L_0) \quad (48)
\]

\[
A(t) = -s \left( \frac{\partial w}{\partial x} \bigg|_{x=0} + \frac{\alpha(t)}{h} \right), \quad B(t) = -s \left( \frac{\partial w}{\partial x} \bigg|_{x=L} + \frac{\alpha(t)}{h} \right) \quad (49)
\]

Substituting the \( w_M(z,t) \), \( \Re_M(z,t) \), \( \alpha(t) \), \( A(t) \), and \( B(t) \) into (47), it can be rewritten as:

\[
\tau = (\mu' + k^f) \left( \sum_{j=1}^{M} a_j(t) \Omega_j(z) + \frac{1}{h} \sum_{j=1}^{N} q_j(t) \phi_j(L_0) \right) \quad (50)
\]

Substituting (44)-(46) and (50) into (35), (36) and (42), and using the Galerkin method, the following reduced order models can be obtained:

\[
\sum_{j=1}^{N} M_j^{(1)} q_j + \sum_{j=1}^{N} C_j^{(1)} q_j + \sum_{j=1}^{N} K_j^{(1)} q_j + \sum_{j=1}^{N} S_j^{(1)} \phi_j = P_i^{(1)}; i = 1, 2, ..., M
\]

\[
\sum_{j=1}^{N} M_j^{(2)} q_j + \sum_{j=1}^{N} C_j^{(2)} q_j + \sum_{j=1}^{N} K_j^{(2)} q_j + \sum_{j=1}^{N} S_j^{(2)} \phi_j = P_i^{(2)}; i = 1, 2, ..., P
\]

\[
\sum_{j=1}^{N} M_j^{(3)} q_j + \sum_{j=1}^{N} C_j^{(3)} q_j + \sum_{j=1}^{N} K_j^{(3)} q_j + \sum_{j=1}^{N} S_j^{(3)} \phi_j = P_i^{(3)}; i = 1, 2, ..., N
\]

where the corresponding coefficients are:

\[
M_j^{(1)} = \frac{\phi_j(L_0)}{h} \int_0^h \Omega_j(z) \rho(z) dz, \quad C_j^{(1)} = 0, \quad K_j^{(1)} = 0,
\]

\[
S_j^{(1)} = \int_0^h \Omega_j(z) \Omega_j(z) dz
\]

\[
D_j^{(1)} = \left( \mu' + k^f \right) \int_0^h \Omega_j(z) \Omega_j(z) dz + \frac{k^f}{\rho h} \int_0^h \Omega_j(z) \rho(z) dz
\]
\[ F_{ij}^{(1)} = 0, E_{ij}^{(1)} = -\frac{k_f}{\rho_j'} \int_0^h \phi_j'(z) \Omega_k(z) dz, \]
\[ P_l^{(1)} = -\frac{L_0 f(t)}{h} \int_0^h \Omega_k(z) dz, \quad M_{ij}^{(2)} = -\frac{s \phi_i(L_0)}{h} \int_0^h \phi_j(z) dz, \]
\[ C_{ij}^{(2)} = \frac{k_f}{\rho_j' J_j'} (1 - 2s) \phi_j'(L_0) \int_0^h \phi_i(z) dz, \quad K_j^{(2)} = 0, \]
\[ S_{ij}^{(2)} = \left( -\frac{s}{h} (\Omega_j(h) - \Omega_j(0)) \right) \int_0^h \phi_j(z) dz - s \Omega_j(0) \int_0^h \phi_i(z) dz, \]
\[ D_{ij}^{(2)} = -\frac{2k_f s^2}{\rho_j' J_j'} \left( (\Omega_j(h) - \Omega_j(0)) \int_0^h \phi_j(z) dz - \Omega_j(0) \int_0^h \phi_i(z) dz \right) \]
\[ + \left( \frac{k_f}{\rho_j' J_j'} \right) \Omega_j(z) \phi_i(z) dz \]
\[ E_{ij}^{(2)} = \int_0^h \phi_j(z) \phi_i(z) dz, \]
\[ E_{ij}^{(3)} = \frac{k_f}{\rho_j' J_j'} \left( \phi_j(z) \phi_i(z) + \frac{\rho_j s}{h} \int_0^h \phi_j(z) dz \right) \]
\[ p_i^{(2)} = \frac{k_f L_0}{\rho_j' J_j'} (2s - 1) \phi_i'(t) \int_0^h \phi_j(z) dz + s \phi_i(L_0) \int_0^h \phi_j(z) dz, \]
\[ M_{ij}^{(3)} = m_0 \phi_i(L_0) \phi_j(L_0) + (\rho A) \int_0^{L_0} \phi_i(x) \phi_j(x) dx, \]
\[ C_{ij}^{(3)} = \left( A' (\mu' + k') - \frac{k_f}{h} A' \right) \phi_i(L_0) \phi_j(L_0), \]
\[ K_j^{(3)} = -EA_{eq} \int_0^{L_0} \phi_i'(x) \phi_j(x) dx, \]
\[ S_{ij}^{(3)} = 0, \quad D_{ij}^{(3)} = (A' (\mu' + k') - k_f A') \Omega_j(0) \phi_i(L_0), \]
\[ E_{ij}^{(4)} = 0, \quad E_{ij}^{(3)} = k_f A' \phi_j(h) \phi_i(L_0), \]
\[ p_i^{(3)} = \left[ m_0 \phi_i(L_0) + (\rho A) \int_0^{L_0} \phi_i(x) dx \right] \phi_i(L_0), \]
\[ \phi_i(z) = \sin \left( \frac{i \pi z}{h} \right), \]
\[ \phi_i(z) = \sin \left( \frac{i \pi z}{h} \right), \]
\[ \phi_i(z) = \sin \left( \frac{(2i - 1) \pi}{2L_b} z \right). \]

Frequency response of the micro-beam immersed in a micro-scale fluid is shown in Fig. 2. As illustrated, by increasing the number of the used shape functions \(N\), the obtained results converge, and for \(N = 6\), the obtained result is considered acceptable. As shown in Table II, for the case when the effect of the shear force is not considered, the first calculated natural frequency of the system for \(N = 6\) is the same as the first natural frequency of the cantilever beam having a concentrated mass at the free end with 1.1% error.

### TABLE I

<table>
<thead>
<tr>
<th>Geometrical and Material Properties of the Micro-Beram, Piezoelectric Layers and Micro-Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>properties</strong></td>
</tr>
<tr>
<td>Width</td>
</tr>
<tr>
<td>thickness</td>
</tr>
<tr>
<td>length</td>
</tr>
<tr>
<td>Young's modulus</td>
</tr>
<tr>
<td>density</td>
</tr>
<tr>
<td>( V_p )</td>
</tr>
<tr>
<td>( e_{31} )</td>
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</tbody>
</table>

### TABLE II

<table>
<thead>
<tr>
<th>Values of First Natural Frequency of the System (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 1 )</td>
</tr>
<tr>
<td>-----------------------------------------------------</td>
</tr>
<tr>
<td>4.96</td>
</tr>
</tbody>
</table>

![Fig. 2 Tip vibration amplitude of the micro-beam versus exciting frequency for different number of shape functions](image-url)
beam for different values of the voltage applied to the piezoelectric layers is shown in Fig. 4. As shown in Fig. 3, lower values of resonance frequency and amplitude are observed in fluids with higher values of coupling parameter. Analyzing the results, we can say that in fluids with higher value of coupling parameter, due to having higher value of vortex viscosity coefficient, micro-rotations are more dependent to macro-rotation. It led to higher damping and inertial effects on the longitudinal vibration of the micro-beam. In the case of classical fluid \( (CP=0) \), the results are in good consistence with the results obtained in [12]. As shown in Fig. 4, the resonance amplitude of the micro-beam is also a function of voltage applied to the piezoelectric layers. It means that applying higher voltages to the piezoelectric layers the resonance amplitude of the micro-beam increases. It should be noted that the exciting frequency of the piezoelectric layers should be in the range of the first natural frequency of the micro-beam.

Effects of the characteristic length scale and surface parameter of the micro-scale fluid, on dynamic response of the micro-beam are investigated and shown in Figs. 5 and 6. The results of Fig. 5 show that in fluids with lower value of length scale parameter, dependency of micro-rotations to macro-rotation is more considerable than in fluids with higher values of length scale. Consequently, fluids with lower values of length scale have more damping and inertial effects on vibration of the micro-beam than fluids with higher values of length scale. Fig. 6 shows that in the case of strong concentration damping effect of fluid is more considerable than in the case of weak concentration. As we know, slip boundary conditions decreases damping effect of fluid, consequently in the case of no-spin boundary condition, the fluid has more damping effect than in the case of spin boundary conditions.

V. Conclusion

In this paper, the longitudinal vibration of a micro-beam in interacting with a micro-scale fluid media was investigated. A
mathematical model was proposed and the coupled governing equations of motion of fluid field and longitudinal vibration of the micro-beam were derived. The obtained equations were discretized and solved using a Galerkin-based reduced order model. It was shown that physical properties of the micro-scale fluid have dissipative and inertial effects on the dynamic response of the micro-beam. The effect of coupling parameter and length scale of the micro-scale fluid were studied and the results showed that in fluids with higher values of coupling parameter and lower values of length scale due to more dependence of micro-rotations to macro-rotations, inertial and damping effects of fluid are more considerable. The effect of surface parameter on dynamic response of the micro-beam was studied. We showed that lower values of resonance frequency and resonance amplitude and consequently more damping and inertial effects are observed in the case of weak concentration. Also, applying different voltages to piezoelectric layers showed that higher voltages result in higher amplitudes of vibration. It should be noted that, the presented mathematical model can be introduced as a micro-sensor for measurement of physical properties of micro-scale fluids.

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