

Quantum Markov Modeling for Healthcare

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Abstract—A Markov model defines a system of states, composed by the feasible transition paths between those states, and the parameters of those transitions. The paths and parameters may be a representative way to address healthcare issues, such as to identify the most likely sequence of patient health states given the sequence of observations. Furthermore estimating the length of stay (LoS) of patients in hospitalization is one of the challenges that Markov models allow us to solve. However, finding the maximum probability of any path that gets to state at time t , can have high computational cost. A quantum approach allows us to take advantage of quantum computation since the calculated probabilities can be in several states, ending up to outperform classical computing due to the possible superposition of states when handling large amounts of data. The aid of quantum physics-based architectures and machine learning techniques are therefore appropriated to address the complexity of healthcare.

Keywords—Application, Hidden Markov models, Viterbi algorithm, quantum computing, quantum machine learning.

I. INTRODUCTION

THIS work explores machine learning algorithms in a quantum framework [1], [2], in particular, quantum machine learning (QML) through a real-world application. Comparison of classical learning algorithms and their quantum versions, have been made, and it shows clear differences in computational complexity and learning performance. Quantum theory is bringing a new metamorphosis in technology and information processing. We are now embracing the world of qubits, where the two computational basis are represented by $|0\rangle$ and $|1\rangle$, behaving separately or in superposition, where it is possible the combination of the two, unlike the bits, always either 0 or 1. Qubit is a quantum variable as it has characteristics both continuous and discrete. However, we only extract the binary details from a qubit, we can't know the full information of the superposition, only the combination of 0's and 1's it picks. This work focuses on the study of the length of stay (LoS), calculated on a circuit containing quantum gates. LoS may be used as an indicator of the perspective of healthcare activity in hospitals. A shorter stay of patients in hospitalization will reduce the cost per discharge and switch care from inpatient to less expensive arrangements. When a patient arrives, it is therefore advantageous to be able to foresee his/her stay, e.g, how many days in hospitalization, given the conditions he/she presents. Goal that we aim to achieve by predicting the LoS variable and thus building a

Markov model using the Viterbi algorithm and a quantum approach. In the Markov property, the state of a Markov chain is directly observed in each time step, hence the current state of a chain depends only on the previous state. The Markov chain is represented by the initial state and transition matrix, a stochastic matrix. A Markov chain can be depicted by a directed graph, a state diagram. The vertices of the graph are associated with states and each edge represents the probability of going from one state to the other. In a Markov chain where states are not observed directly, we obtain a hidden Markov model (HMM). The transition probabilities control the way the hidden state (number of stay days) at time t is chosen given the hidden state at time $t-1$. The quantum approach allows us to handle a large amount of data since the transition matrices, product of patient's conditions, can have very large dimensions, translating into a huge time consuming processing in the search space. We apply a quantum logic gate approach, since a basic quantum circuit operating on a small number of qubits can help achieve speed-up.

II. BACKGROUND AND RELATED WORK

Quantum data processing explores the significance of using quantum mechanics instead of classical practices to model information and information processing [3]. Nowadays, we raise the question of how much faster are quantum computers than standard computers for relevant problems, leading to the importance of computing speedup measures (e.g. criteria cost, criteria complexity) [4]. In information processing, low sample complexity is associated with efficient learning, and therefore low time complexity. According to studies, the best quantum learner has much less time complexity than a leading classical learner [5]. Most of machine learning problems presupposes manipulating and classifying huge numbers of vectors in high-dimensional spaces, and quantum computers are good at those kinds of tasks in large tensor product spaces, therefore quantum machine learning provides speed-ups over classical computers [6]. Quantum computation and quantum information share fundamental concepts from quantum mechanics to computer science [7]. Such as, when two waves encounter they overlap and interact, sometimes they build a big wave, sometimes they cancel each other, but usually it is a combination of both, it is when superposition occurs [8]. When groups of particles cannot be described separately due to interaction, we are present the quantum entanglement [9], [10]. And sometimes for efficient proposes we may need to operate in a small number of qubits, that is when we use a quantum logic gate [11]. In this work, we take advantage of some of these features, that we use in the design of the quantum circuit:

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$$\text{Hadamard} - H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (1)$$

$$\text{Pauli} - Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad (2)$$

$$\text{Pauli} - Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad (3)$$

Doing research concerning the length of stay (LoS) of patients in hospitalization, is not a novelty [12], [13]. Besides, using a discrete time Markov process for cardiovascular, neurological, respiratory, gastrointestinal, trauma and other diagnostic categories has already been created [14]. As well as using continuous-time Markov processes to analyze temporal trends of LoS [15]. Moreover, it has been a discussed problem [16] but it seems there is a lake of studies finding LoS in a quantum procedure. In fact, there are few quantum applications to real-world problems, particularly in the healthcare domain. What, given the existence of more and more clinical records, makes sense to start reasoning about an order of magnitude greater to solve issues that arise. Many of these concerns are solved by machine learning but these models can be very costly and time consuming [17].

III. EXPERIMENTS SETUP

To achieve the results we use a quantum simulator and a real quantum device. A quantum simulator shows information concerning an abstract mathematical function connected to a physical model [18]. A simulator should assess the relevance of the model and the accuracy in describing the real system in consideration. As known, quantum computers are conceived to outperform classical computers by running quantum algorithms [19].

A. Dataset

We use the Diabetes130US dataset available at the repository OpenML with dimension: 101.766 rows and 50 columns. The dataset presents 10 years (1999-2008) of clinical care at 130 US hospitals and integrated delivery networks. The data contains attributes such as patient number, race, gender, age, admission type, time in hospital (at least 1 day and at most 14 days), medical specialty of admitting physician, number of lab test performed, HbA1c test result, diagnosis, number of medication, diabetic medications, number of outpatient, inpatient, and emergency visits in the year before the hospitalization, etc.

B. Quantum Platform

We use the IBM Quantum Experience (QX) which offers the possibility to connect to an IBM quantum processor via the IBM Cloud. We can run algorithms and explore tutorials and simulations in a quantum computing environment. The experiments were implemented in the Python programming language using the Quantum Information Software Kit (QISKit), a software development kit (SDK) for working with the Open Quantum Assembly Language (OpenQASM) and

the IBM Q experience (QX). We set up the API and execute the program through a user IBM token and url. We use as backend the ibmqx5 (16 qubits), ibmqx4 (5 Qubits) and the ibmqx_qasm_simulator.

C. Algorithm Implementation

For this work, we selected one variable to study, insulin, and how this variable is related with our outcome variable, LoS.

- 1) create state space and initial state probabilities of the variable of interest (e.g. insulin) and of the outcome variable (e.g. time in hospital)
- 2) create transition probabilities Fig. 1 of each one
- 3) create a function that maps transition probability dataframe to markov edges and weights and create a graph object for each one
- 4) create a matrix of observation (emission) probabilities with the observable states and the hidden states
- 5) run the quantum Viterbi function

We begin by setting the quantum program specifications: the circuit name, quantum registers name and size, and the classical registers name and size (SPECS).

Algorithm 1: Quantum Viterbi

```

1 function qViterbi (pi, pt, pe, obs);
   Input : initial probabilities, transition matrix, emission
           matrix, observable states
   Output: path, delta, phi
2 qp ← QuantumProgram(specs=SPECS)
3 qc ← qp.get_circuit("qViterbi")
4 q ← qp.get_quantum_register("q")
5 c ← qp.get_classical_register("c")
6 states ← shape(pe)[0]
7 n ← shape(obs)[0]
8 path ← zeros(n)
9 delta ← zeros((states, n))
10 phi ← zeros((states, n))
11 delta[:, 0] ← pi * pe[:, obs[0]]
12 phi[:, 0] ← 0
13 qc.h(q[0])
14 for t ∈ {1, ..., n} do
15     qc.y(q[t])
16     for s ∈ states do
17         delta[s, t] ← max(delta[:, t-1] * pt[:, s]) * pe[s,
18             obs[t]]
19         phi[s, t] ← argmax(delta[:, t-1] * pt[:, s])+1
20     end
21     qc.u1(t/π, q[1])
22 end
23 path[n-1] ← argmax(delta[:, n-1])
24 for t ∈ {n-2, -1, -1} do
25     qc.z(q[t])
26     path[t] ← phi[int(path[t+1]), [t+1]]
27 end
28 qc.measure(q, c)
29 qp.set_api(token, url)
30 ibmqx ← qp.execute(["qViterbi"],
31     backend='ibmqx_qasm_simulator')
32 ibmqx4 ← qp.execute(["qViterbi"], backend='ibmqx4')
33 ibmqx4.get_data("qViterbi")
34 QASM_source ← qp.get_qasm("qViterbi")

```

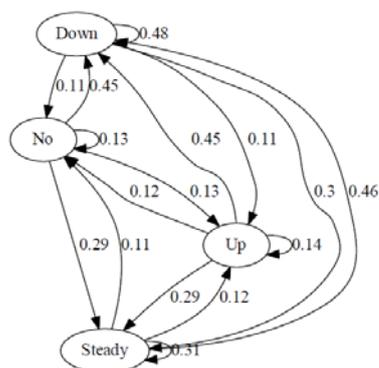


Fig. 1 Transition probabilities of insulin, four types of states: down, no, up and steady

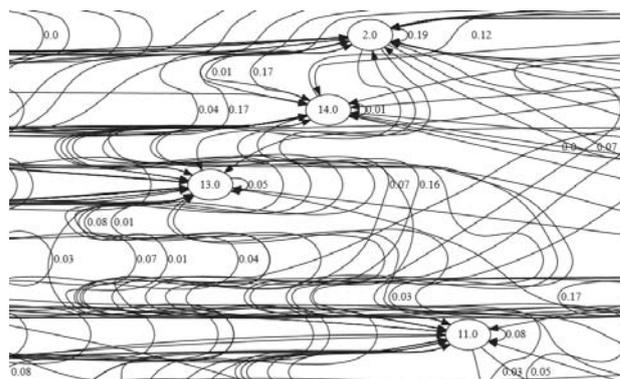


Fig. 2 Part of the emission probabilities of insulin states in 14 days of hospitalization

The Viterbi algorithm give us the best path, through a recursive optimal solution of estimating the state sequence of a discrete-time finite-state Markov process.

IV. RESULTS

A. Parameters of the Hidden Markov Model

According to the data set in consideration, observing the initial probabilities shown in Table I we can notice that the first days have higher values. Which allow us to assume that short stays in hospitalization are more probable than long stays.

TABLE I
 TIME IN HOSPITAL INITIAL PROBABILITIES OF EACH DAY

| days | probabilities |
|------|---------------|
| 1.0 | 0.139614 |
| 2.0 | 0.169251 |
| 3.0 | 0.174479 |
| 4.0 | 0.136824 |
| 5.0 | 0.097931 |
| 6.0 | 0.074082 |
| 7.0 | 0.057573 |
| 8.0 | 0.043148 |
| 9.0 | 0.029499 |
| 10.0 | 0.023014 |
| 11.0 | 0.018228 |
| 12.0 | 0.014229 |
| 13.0 | 0.011890 |
| 14.0 | 0.010239 |

Through the diagram we can analyze the probabilities of changing state, with the associated value in each arrow. Arrows pointing to the state itself indicate the likelihood of remaining the same. In this case, from highest to lowest, Down, followed by Steady and then Up and finally No. Which allows us to say that the most difficult state to change is when the insulin is going down.

As a way of uncovering the probabilities associated with the four states of insulin and days of hospitalization we show some of the complexity of the emission matrix diagram in Fig. 2.

In order to represent the quantum system we created, we present the Bloch sphere where we can visualize qubit states and gates. We placed the H gate, known as the Hadamard gate, on the qubit 0, then we add Pauli Y gate to the varying qubit t in the quantum register, afterwards we add the first physical

gate u1 to the qubit 1, and finally we add the Pauli Z gate to the varying qubit t.

B. Quantum Environment

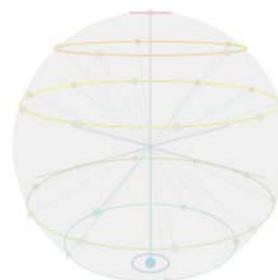


Fig. 3 Quantum sphere

In the appendix we present more details about the experimental setting on the quantum infrastructure we used.

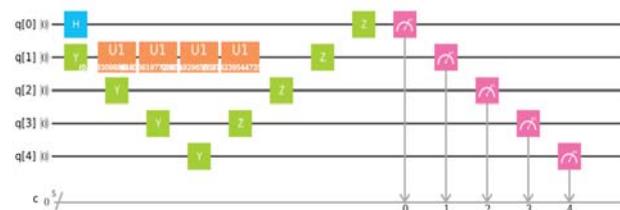


Fig. 4 Quantum circuit

V. DISCUSSION

Through the proposed algorithm it is possible to know how many days the patient should be in hospitalization given a sequence of insulin states by multiple observations. The purpose of the algorithm in the quantum version arises from the need to predict multiple episodes. Hidden Quantum Markov Models (HQMMs) [20] are suitable for learning models from data, some algorithms have been proposed [21]. The existence of new quantum technology leads us to take advantage of building classical algorithms (e.g.

Viterbi algorithm [22]) to a new computational approach and challenge [23]- [25].

VI. CONCLUSIONS AND FUTURE WORK

This work studied the outcome variable insulin in the diabetes diagnostic, which allows us to forecast the length of a patient stay in hospitalization according to the state of the insulin having into account all possible states. We achieve the results in a good computational time given by the integration of quantum methods in a classical algorithm. For further work, we aim to calculate the LoS considering all conditions of the patient, having in count all the risk factors.

APPENDIX

A. IBM QASM-Language Program

```
//OPENQASM 2.0
IBMQASM 2.0;
include "qelib1.inc";

qreg q[5];
creg c[5];

h q[0];
y q[1];
u1(0.318309886183791) q[1];
y q[2];
u1(0.636619772367581) q[1];
y q[3];
u1(0.954929658551372) q[1];
y q[4];
u1(1.273239544735163) q[1];
z q[3];
z q[2];
z q[1];
z q[0];
measure q[0] -> c[0];
measure q[1] -> c[1];
measure q[2] -> c[2];
measure q[3] -> c[3];
measure q[4] -> c[4];
```

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