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Simplified Equations for Rigidity and Lateral Deflection for Reinforced Concrete Cantilever Shear Walls

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Abstract**—**Reinforced concrete shear walls are the most frequently used forms of lateral resisting structural elements. These walls may take many forms due to their functions and locations in the building. In Palestine, the most lateral resisting forces construction forms is the cantilever shear walls system. It is thus of prime importance to study the rigidity of these walls. The virtual work theorem is used to derive the total lateral deflection of cantilever shear walls due to flexural and shear deformation. The case of neglecting the shear deformation in the walls is also studied, and it is found that the wall height to length aspect ratio (H/B) plays a major role in calculating the lateral deflection and the rigidity of such walls. When the H/B is more than or equal to 3.7, the shear deformation may be neglected from the calculation of the lateral deflection. Moreover, the walls with the same material properties, same lateral load value, and same aspect ratio, shall have the same of both the lateral deflection and the rigidity. Finally, an equation to calculate the total rigidity and total deflection of such walls is derived by using the virtual work theorem for a cantilever beam.

Keywords**—**Cantilever shear walls, flexural deformation, lateral deflection, lateral loads, reinforced concrete shear walls, rigidity, shear deformation, virtual work theorem

I. Introduction

THE lateral resistive structural systems for buildings consists of combination of vertical and horizontal elements. The horizontal elements are most often the roof decks; and it is called a horizontal diaphragm [1]. The horizontal diaphragm collects the lateral forces, and then distributes them to the vertical lateral resistive elements. If the horizontal diaphragm is considered to be flexible, then the distribution of the lateral loads to the vertical elements such as shear walls will be in a tributary area concept as shown in Fig. 1. However, if the horizontal diaphragm is rigid, then the distribution of the lateral loads will be by the rigidity of the vertical members such as shear walls as shown in Fig. 2.

The two general cases for the vertical shear wall diaphragm are the cantilever and the doubly fixed pier as shown in Fig. 3.

Fixity at both the top and the bottom of the shear wall usually affects deflection only when the wall is relatively short in length, B, with respect to the height, H, such walls or piers usually appear between wall openings.

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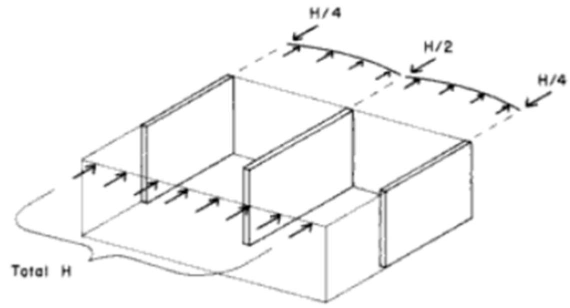

Fig. 1 Distribution of lateral load to vertical elements in flexible diaphragm [1]

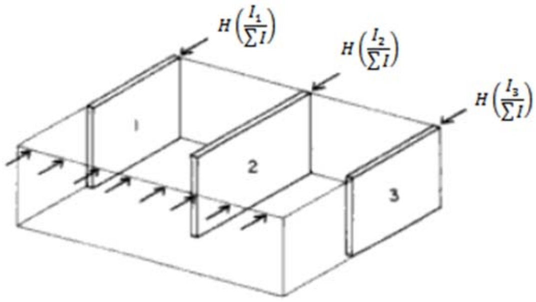

Fig. 2 Distribution of lateral load to vertical elements in rigid diaphragm [1]

II. Deflection of a Cantilever Wall

Because most concrete piers or cross walls act as short, deep beams, contributions to the displacement due to both flexure and shear must be considered as indicated in the following equation, where virtual work theorem is applied to a cantilever beam:

$$\Delta_{total} = \Delta_{flexure} + \Delta_{shear} \quad (1)$$

$$\Delta_{flexure} = \frac{PH^3}{3EI} \quad (2)$$

$$\Delta_{shear} = \frac{1.2PH}{GA} \quad (3)$$

where P is the lateral load, H the wall height (vertical dimension), E the modulus of elasticity, I the moment of inertia, and A the section area.

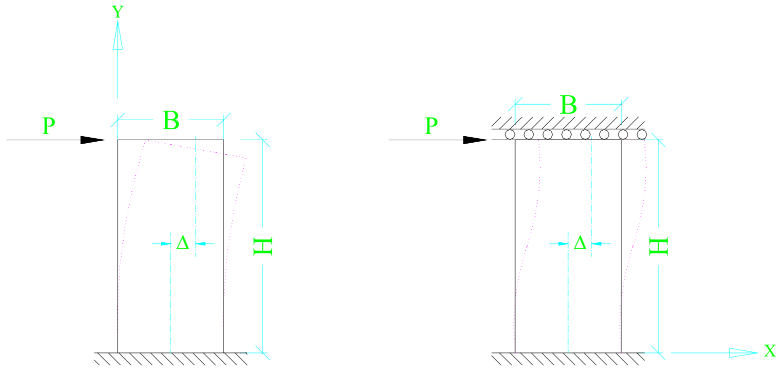

Fig. 3 Two idealized conditions of end support for vertical diaphragm: (a) Cantilever (b) Doubly fixed

The shear modulus can be expressed as [2]:

$$G = \frac{E}{2(1+\nu)} \tag{4}$$

where ν is the Poisson's ratio, and for concrete it will be 0.20.

The final equation for the lateral deflection of a cantilever concrete shear wall is:

$$\Delta_{total} = \left(\frac{P}{Et}\right)\left[4\left(\frac{H}{B}\right)^3 + 2.88\left(\frac{H}{B}\right)\right] \tag{5}$$

Equation (5) indicates that the total lateral deflection is a function of the wall height to length aspect ratio (H/B). This means that walls with the same aspect ratios and thicknesses, same material properties and lateral loads, shall have the same lateral deflection.

The ratio of the flexural deflection from the total cantilever wall deflection can be found by dividing the flexural deflection from (2) by the total deflection from (5), and then the contribution of flexural deformation is:

$$\frac{\Delta_f}{\Delta_{total}} = \frac{4\left(\frac{H}{B}\right)^2}{2.88+4\left(\frac{H}{B}\right)^2} \tag{6}$$

Using the same procedure, the contribution of the shear deformation is:

$$\frac{\Delta_s}{\Delta_{total}} = \frac{2.88}{2.88+4\left(\frac{H}{B}\right)^2} \tag{7}$$

From (6) and (7), the contribution of shear or flexural deformations to the total wall deformation is a function of wall aspect ratio. If the wall aspect ratio equals to 1, the contribution of flexural deformation will be 58% and the shear deformation contribution will be 42% from the total wall deflection, assuming elastic un-cracked section for the wall. Fig. 4 shows the relative contribution of shear and flexure deformation to total deformation drawn using (6) and (7) versus wall aspect ratio.

The shear deformation can be neglected if 5% difference due to shear deformation will be considered negligible, and when substituting this value in (7), the following result shall be found:

$$\frac{2.88}{2.88+4\left(\frac{H}{B}\right)^2} = 0.05 \rightarrow \frac{H}{B} = 3.7 \tag{8}$$

From (8), if the wall H/B ratio is less than 3.7, the shear deformation should be considered and be modeled using 2D area element or using Timoshenko beam element. Otherwise, the wall can be modeled as 1D Euler-Bernoulli beam element. The assumption of Euler-Bernoulli beam theorem is that any plane perpendicular to the neutral axis before bending will remain so after the beam is bent [3]. Timoshenko beam theorem accounts for the effect of the transverse shear deformation and takes into account the rotation between the cross section [4] and the bending line due to shear deformation. Therefore, the Euler-Bernoulli beam theorem underestimates the deflection because it models a stiffer beam. Because of that, beams with short length or expected to have large deflection have to be modeled with Timoshenko beam element.

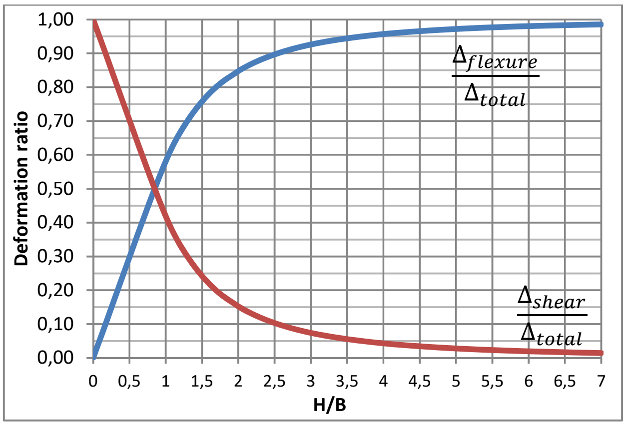

Fig. 4 Relative contribution of shear and flexure deformation to total deformation for cantilever wall

III. Effect of Concrete Compressive Strength on Lateral Deflection

The effect of concrete compressive strength f'_c will be studied in the range of normal concrete strength. The normal concrete strength is between 20 and 40 MPa. The relationship between modulus of elasticity and normal weight concrete according to ACI318M-14 is given as [5]:

$$E = 4700\sqrt{f'c} \tag{9}$$

where f'c is concrete compressive strength in MPa.

By substituting (9) into (5), the lateral cantilever normal concrete wall deflection will be:

$$\Delta_{total} = \left(\frac{P}{4700\sqrt{f'_c}t}\right)\left[4\left(\frac{H}{B}\right)^3 + 2.88\left(\frac{H}{B}\right)\right] \tag{10}$$

When the lower bound of f'_c was taken to be 20MPa, (10) will be:

$$\Delta_{total} = \left(\frac{4.76\times10^{-5}P}{t}\right)\left[4\left(\frac{H}{B}\right)^3 + 2.88\left(\frac{H}{B}\right)\right] \tag{11}$$

When the upper bound of f'_c was taken and equals to 40MPa, (10) shall be:

$$\Delta_{total} = \left(\frac{3.36\times10^{-5}P}{t}\right)\left[4\left(\frac{H}{B}\right)^3 + 2.88\left(\frac{H}{B}\right)\right] \tag{12}$$

For the same wall geometry, same lateral load, and when dividing (12) on (11), the effect of f'_c will be in the range of 1 to 1.42. Thus, decreasing the concrete compressive strength will increase the lateral deflection of a cantilever wall and this increase will be in the range of 1 to 1.42.

IV. Rigidity of a Cantilever Wall

Lateral loads are applied at different floor levels, where rigid floor systems act as diaphragms distribute the load to the cross walls and frames [6]. Horizontal forces will be distributed to the walls in inverse proportion to their capacity to deflect, or flexibility. Thus, a very flexible wall will resist only a small portion of the seismic force, while a stiffer wall will resist a large portion. In terms of stiffness, which is the reciprocal of flexibility, the lateral forces will be distributed in direct proportion to the relative stiffness of the resisting elements.

$$Rigidity,\ R = \frac{Load\ P}{Deflection\ \Delta} \tag{13}$$

The final equation for the rigidity of a concrete cantilever shear wall:

$$R = \frac{P}{\left(\frac{P}{Et}\right)\left[4\left(\frac{H}{B}\right)^3 + 2.88\left(\frac{H}{B}\right)\right]} \tag{14}$$

$$R = \frac{Et}{2.88}\cdot\frac{1}{\left[1.39\left(\frac{H}{B}\right)^3 + \left(\frac{H}{B}\right)\right]} \tag{15}$$

$$R_{flexure} = \frac{Et}{4}\left(\frac{H}{B}\right)^{-3} \tag{16}$$

$$R_{shear} = \frac{Et}{2.88}\left(\frac{H}{B}\right)^{-1} \tag{17}$$

Let the term $\frac{1}{\left[1.39\left(\frac{H}{B}\right)^3 + \left(\frac{H}{B}\right)\right]}$ in (15) known as A, the rigidity coefficient, then the final rigidity and the lateral deflection of a concrete cantilever shear wall will be:

$$R = \frac{Et}{2.88}\cdot A \tag{18}$$

$$\Delta_{total} = \frac{2.88P}{Et}\cdot\frac{1}{A} \tag{19}$$

To find out the rigidity or the total lateral deflection of any concrete cantilever shear wall, just use (18) and (19) with the A factor from Table I.

TABLE I
Rigidity Coefficients, A, for Concrete Cantilever Shear Wall

H/B	A	H/B	A
5.00	0.0056	2.25	0.0553
4.75	0.0065	2.00	0.0762
4.5	0.0076	1.75	0.1087
4.25	0.0090	1.50	0.1615
4.00	0.0108	1.25	0.2522
3.75	0.0130	1.00	0.4184
3.50	0.0158	0.75	0.7483
3.25	0.0196	0.50	1.4842
3.00	0.0247	0.25	3.6803
2.75	0.0316	0.15	6.4645
2.50	0.0413	0.10	9.8629

Fig. 5 shows the relative contribution of shear and flexure rigidity to total rigidity drawn using (15)-(17) versus wall aspect ratio (H/B). As it can be noticed from Fig. 5, when the wall aspect ratio H/B increases, the contribution of the shear rigidity to total cantilever wall rigidity increases too. On the other hand, when the wall aspect ratio H/B increases, the contribution of the flexure rigidity to total cantilever wall rigidity decreases.

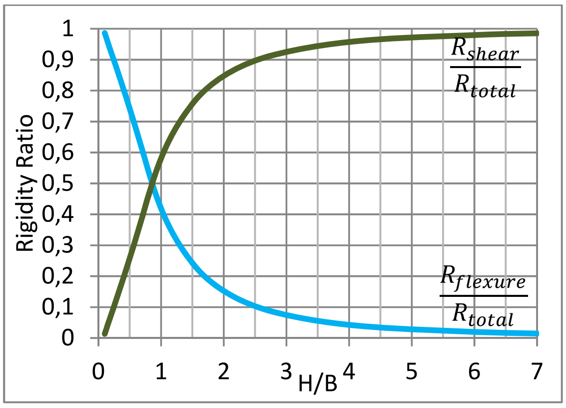

Fig. 5 Relative contribution of shear and flexure rigidity to total rigidity for cantilever wall

V. Rigidity of Cantilever Shear Wall with Central Window Openings

Many walls may contain openings such as door or window openings. Thus, there is a range in relative rigidity of these walls that extends from a solid wall to a frame. To study the effect of openings on the rigidity of a concrete cantilever shear wall, by using SAP2000 structural analysis program, 17 central squared window openings of varying sizes are suggested in a cantilever concrete wall with 3m length× 3m height×0.20m thickness, and this wall is made from concrete compressive strength equals to 24MPa. In this section the largest ratio of central window opening in a wall whose effect on the lateral stiffness is small and can be neglected will be identified. The results of the lateral deflection (Δ), the rigidity (R), and the rigidity ratio (R_S) are tabulated in Table II. For the naming of the models, C refers to the concrete wall and W refers to window opening. The stiffness ratio (R_S) is defined as the ratio of the lateral stiffness of a wall with opening divided by the lateral stiffness of the same wall without openings. The opening ratio (R_O) represents the opening area in the wall divided by the total wall side area.

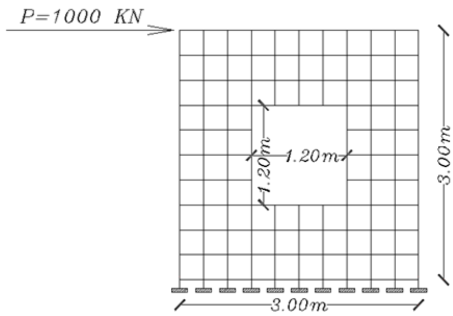

Fig. 6 C-W12 3×3m cantilever wall model with central window opening

Table II shows the final results for window openings patterns.

TABLE II
FINAL RESULTS FOR WINDOW OPENINGS PATTERNS

Model number	Opening size (m)	Opening Ratio (%) (m)	Δ (mm)	R kN/m × (10^4)	R_S ratio (%)
C-W0	0.00	0.00	1.47	67.89	100
C-W3	0.3×0.3	1.00	1.52	65.79	96.90
C-W4	0.4×0.4	1.87	1.52	65.70	96.78
C-W5	0.5×0.5	2.78	1.55	64.52	95.03
C–W6	0.6×0.6	4.00	1.66	60.24	88.73
C-W7	0.7×0.7	5.44	1.76	56.81	83.69
C-W8	0.8×0.8	7.11	1.86	53.76	79.19
C-W9	0.9×0.9	9.00	2.00	50.00	73.65
C-W10	1 ×1	11.11	2.21	45.24	66.65
C-W11	1.1×1.1	13.44	2.54	39.37	57.99
C-W12	1.2×1.2	16.00	2.84	35.21	51.87
C-W13	1.3×1.3	18.78	3.28	30.49	44.91
C-W14	1.4×1.4	21.78	3.90	25.64	37.77
C-W15	1.5×1.5	25.00	4.66	21.45	31.61
C-W16	1.6×1.6	28.44	5.66	17.66	26.02
C-W17	1.7×1.7	32.11	7.16	13.96	20.57
C-W18	1.8×1.8	36.00	9.05	11.05	16.27

Fig. 7 shows the relationship between R_s and R_o as expected. Increasing the size of opening will decrease the stiffness of the wall. If 5% reduction in the wall lateral stiffness is considered negligible, then the opening area in the wall give such a reduction in rigidity equals 3% of the total wall side area. Thus, central window opening can be neglected in modeling the walls when its area ratio to total wall side area is up to 3%. In the common practice the 3% opening area appears in the bathroom window openings. Typical squared window opening of size 1.30×1.30m which is commonly used in practice reduces the stiffness of 3×3m solid wall to about 50%. The rapid drop in stiffness can be noticed when using large opening ratios. When the opening ratio is around 17% from the total wall area, the wall will lose 50% of it is stiffness.

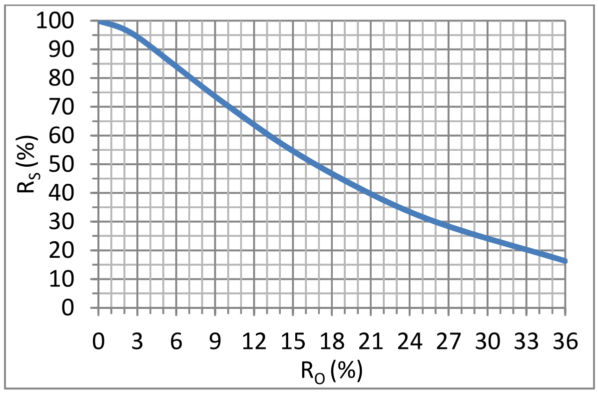

Fig. 7 Squared windows opening ratio versus stiffness ratio of 3×3m cantilever wall

VI. CONCLUSIONS

In this paper, analytical study has been done to investigate the rigidity and so the lateral deflection of the concrete cantilever shears walls. Moreover, simplified equations to calculate the concrete cantilever wall rigidity and the lateral deflection have been proposed.

The main conclusions from this study as the following:

- The rigidity of the solid reinforced concrete cantilever shear wall is a function of the wall height to length aspect ratio H/B. The H/B ratio is the most dominant factor in determining the deflection and the rigidity of RC walls.
- The walls with the same material properties and thickness and aspect ratios will have the same rigidity and lateral deflection.
- If 5% contribution in the shear deformation can be neglected, the shear deformation may be neglected when the wall aspect ratio is more than or equal to 3.7.
- Equations to calculate the lateral deflection and the rigidity of a cantilever concrete shear wall are derived.
- Finally, if the central window opening in concrete wall is up to 3% from the total side wall area, then it can be neglected safely from the model.

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